The Capsize Band Concept Revisited

Nikolaos Tsakalakis, Jakub Cichowicz and Dracos Vassalos

The Ship Stability Research Centre, Department of Naval Architecture and Marine Engineering, University of Strathclyde, Glasgow, UK

ABSTRACT

A concept for analytical representation of the capsize rate, a measure directly related to damage ship survivability, has attracted attention ever since the first attempts were made to explain the behaviour of a damaged ship in waves. Attempts in the late 1990s helped to enhance understanding and facilitate characterisation of phenomena pertaining to capsize probability and time to capsize in given environments and loading conditions, but a consistent verifiable formulation is still lacking. In this respect, pursuing an analytical approach to express the capsize rate offers many advantages, time efficiency being amongst the most important. In an era when stability/survivability calculations are required to be carried out in real time, there is a need for a model combining accuracy close to that of time-domain simulations whilst relying on hydrostatic models, catering for uncertainty and capsize boundaries in the process. This study is an attempt to establish a new methodology for survivability assessment by means of a multivariable analytical model based on numerical simulations, validated against the results of physical model tests.

KEYWORDS

Damage Stability; Capsize Band; Critical Wave Height; Ro-Pax;

INTRODUCTION

The concepts of capsize boundary and capsize band lie at the core of damage survivability assessment of ships. The s-factor used to derive the Attained Index of subdivision corresponds to the 50% probability of survival in damaged condition and in a sea state characterised by what is called *critical significant wave height*. Hs_{crit} is nothing else than a capsize boundary – a wave height at which the capsize rate (P_f) equals 0.5. The capsize band, in turn, reflects the marginal nature of the capsize phenomena and by analogy to statistics it can be interpreted as a confidence interval about Hs_{crit}. In fact, the capsize band is not a confidence interval in a strict sense[1] – it is rather a measure of dispersion of capsizes, separating sea states in which the capsize rate (i.e. the conditional probability of capsize given H_S) is very low from those in which the rate is very high. In other words, the capsize band emphasizes a well-known fact that there is no distinct boundary separating safe from unsafe sea states; instead there is rather a transition zone within which capsize is possible. The presence of the band also implies that although there must be sea states at which the vessel will never capsize and that there must be sea states

[1] With significant wave height at the instance of capsize being a random variable, the confidence interval would simply be a band of wave heights containing most of the area under the p(*Hs*|capsize) probability density function curve. Instead, boundaries of the capsize band are expressed with the use of the following equalities: $(H_S)_{low} = H_S\big|_{P_f(H_S)=\alpha}$ and $(H_S)_{high} = H_S\big|_{P_f(H_S)=1-\alpha}$, where α is some (small) number.

at which she would inevitably capsize, due to limited resolution of physical or numerical experiments the lower and upper boundaries can only be expressed by means of limits. Such asymptotic nature requires the use of some threshold values of P_f outside of which occurrence of capsize will either be virtually impossible or practically certain. Making use of analogy to statistics again, such limiting sea states corresponding to threshold values of P_f, can be interpreted as confidence limits.

Although the capsize rate, P_f, is a function of many variables, such as sea state, loading condition and damage characteristics, it has been observed that in all cases it follows a clear and recurring trend. This has triggered the pursuit for its analytical representation that could be used in parametric studies on capsize phenomena in order to derive universal formulae for probability of capsize and corresponding time to capsize.

Understandably, such studies require a vast number of experiments to be performed, which sets particular limits on the achievable resolution and accuracy of the results. In this paper, the authors present a brief account of the current state-of-the-art, discuss advantages and shortcomings and propose an alternative approach, which can offer significant reduction of effort (normally expended in numerical simulations and model experiments) whilst retaining comparable accuracy of the outcome.

APPROACH

Software Tools

Numerical experiments supporting this work have been carried out with PROTEUS3, the in-house developed software that has been successfully employed over many years in a number of research and commercial projects. It has been referenced a number of times, benchmarked against experimental data and other numerical codes successfully and has aided greatly in our understanding of capsize phenomena in damage conditions. OriginPro8 – a powerful statistical package – was used for processing of the results, parametric studies and development of the methodology.

Ship Models

Two models of Ro-Pax vessels have been studied for the purpose of this paper, of 89 and 170 metres in length between perpendiculars. The first ship (EUGD01-R2) is being extensively tested numerically at the moment for the on-going EU Project GOALDS that aims to re-engineer the probabilistic rules formulation for damage survivability of passenger ships. Physical model experiments are scheduled for this ship later in the course of the project. The larger Ro-Pax has been used in previous research projects, including HARDER, the foundation of the current probabilistic regulatory framework for damage stability. Results of physical model experiments carried out on this vessel are being used for validation of the numerical code.

Table 1: Main Particulars of Models

Model	PRR01	EUGD01-R2	
Passengers	1420	622	
L_{OA}	194.3	97.9	m
L_{BP}	176	89	m
Breadth	25	16.4	m
Deepest subdivision loadline	6.55	4	m
Depth to bulkhead deck	9.1	6.3	m
Displacement	16,558	3,445	tn
Service speed	21.0	19.5	kn

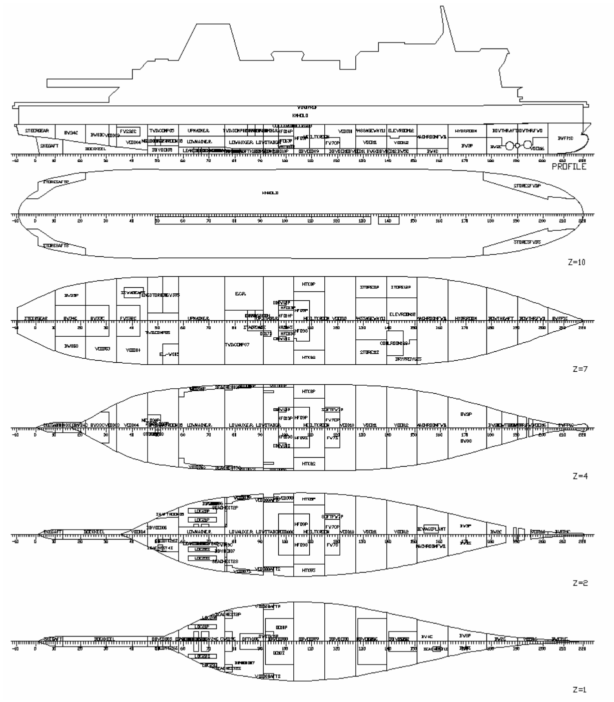

Fig. 1: Subdivision of PRR01 from NAPA

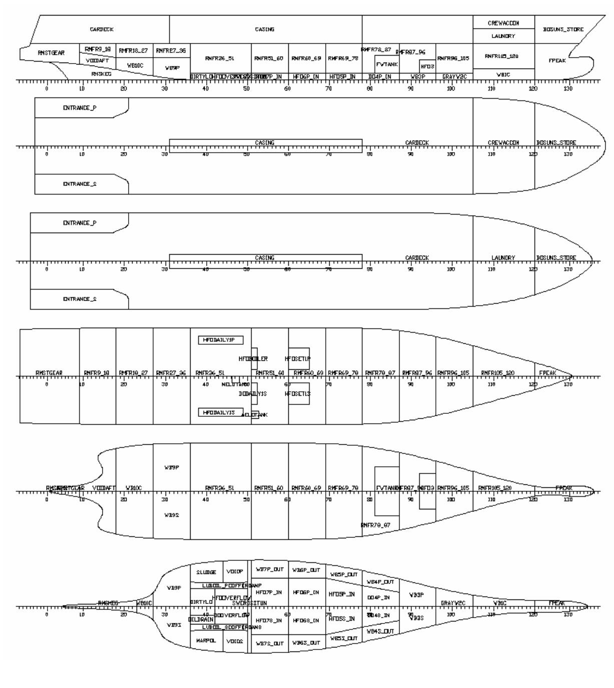

Fig. 2: Subdivision of EUGD01-R2 from NAPA

The two chosen ships cover different regions of the design space to ensure universal application of the results. The PRR01 was designed for transport of, primarily, vehicles across short routes such as the English Channel, actually converted to carry a number of passengers in addition to that during the building stage. The second ship was designed for transport of a small number of both passengers and vehicles within an island archipelago in short-crested, choppy seas.

Numerical Experiments

Accurate representation of the capsize rate characteristic across the entire capsize band, requires adequate resolution. Therefore, it was deemed necessary to use at least 10 measurements within the transition zone, performed by increasing Hs in small steps, varying from 0.1m to 0.25m depending on the width of the capsize band. For each wave height, Pf was determined on a basis of at least 20 wave realisations to maintain at least 5% resolution. The larger ship was tested in seven and the smaller in five different loading conditions, including variations of draught and KG. Additionally, the survivability of the smaller vessel was studied in two distinct damages and various wave spectra. Waves were modelled using JONSWAP spectrum of slope (height to length ratio) equal to 1/20 and 1/25, respectively. Each realisation was limited to 1,800 seconds, which is the maximum time currently required by regulations for evacuation of a vessel. Complete time history of the motions and water accumulation (including water on Ro-Ro deck) was measured and recorded. No wind effect was included in the experiments. All simulations started with the ship in the damage equilibrium position.

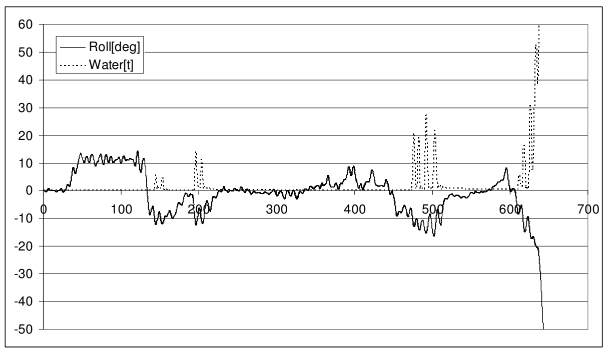

Fig. 3: Time history of Roll motion and water accumulation as recorded from PROTEUS3 for a capsize case.

Numerical code Validation

Given the relative ease of use of numerical tools it is possible to carry out hundreds of simulations in a short period of time, given that results can be verified. Within the present study, the outcome of numerical software was benchmarked against available experimental data from project HARDER (availability of data was one of the reasons for selecting PRR01 as sample ship). Comparison between numerical and experimental results show satisfactory agreement (fig. 4).

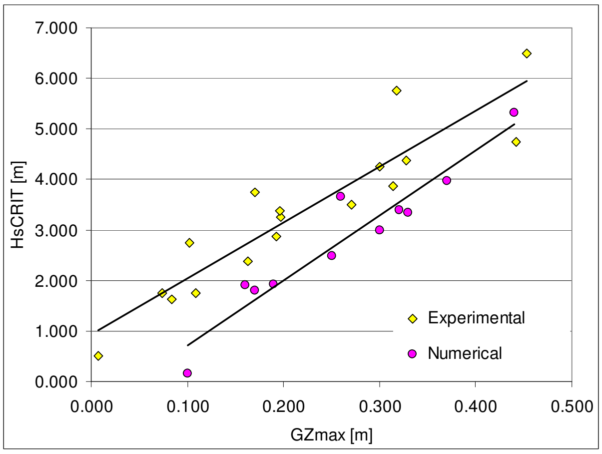

Fig. 4: Experimental versus numerical results for model PRR01

It should be noted here that the quantitative agreement between the results was considered of minor importance with emphasis being put on the observed trends. However large any discrepancies might be regarded, it is obvious that both sets of data bear large uncertainties.. Nevertheless, for the purpose of this work it was decided that as long as the differences are systematic an exact match is not required and no further numerical model calibration was performed, particularly as observations show that numerical results err on the side of safety.

PROBABILITY OF CAPSIZE

Capsize rate

The term capsize rate (P_f) is used to denote the approximation[2] of the probability of capsize of a damaged ship, given loading conditions and sea state. Predictably, for a given number of realisations[3], capsize rate will vary from 0 for very small[4] to 1 for very large waves. Between minimum H_S for which $P_f = 0$ and maximum H_S for which $P_f = 1$, P_f can take any value ranging from 0 to 1. Following adopted convention (Vassalos et al, 1997), critical wave height corresponds to the significant wave height for which capsize rate is 0.5.

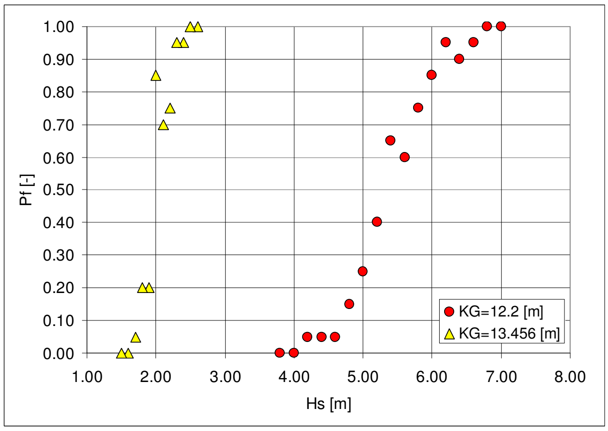

Fig.5: Capsize rate values for different loading conditions

Disregarding the experimental errors, it is obvious from figure 5 is that data follow a specific pattern throughout the range. The evident trend common for all the observations made across the entire H_S range led previous attempts to approach this characteristic by making use of its similarity to the integral of a normal Gaussian distribution – Cumulative Density Function (CDF) (Jasionowski et al, 2007). A major advantage of such approach is that the normal distribution is a well known function and statistical tools can be readily applied to the recorded data in order to find an interval around critical H_S, which could be interpreted as capsize band by use of standard deviation of the derivative of capsize rate. The biggest downside of this method is that it

[2] This follows the classical definition of probability, expressed as the ratio of favourable experiment outcomes over the total number of trials. It would become a probability of capsize (conditional on loading condition and wave parameters) if the number of trials approached infinity.

[3] A time series of seakeeping either by means of numerical simulations or physical model tests

[4] Relative to the critical significant wave height

requires numerical differentiation of recorded data, i.e. it involves computation of the derivative of the capsize rate, P_f. As differentiation of infrequent data unequally distributed along the H_S range may introduce large uncertainties, the approach is practically limited to large[5] data sets.

Non-Linear Regression

Exhaustive pursuit for a more convenient functional representation of the capsize rate resulted in a parametrically defined sigmoid function that turned out to be an attractive alternative to the Gaussian distribution. Bolzmann's sigmoid allows direct regression of measured rates, without the need for prior numerical differentiation. The resulting function can be differentiated easily afterwards to derive the requisite information on the capsize band. In its most general form the function is given by means of four parameters: A_1, A_2, x_0 and d_x.

$$y(x) = \frac{A_2 + (A_1 - A_2)}{1 + e^{\frac{x - x_0}{dx}}} \tag{1}$$

Where:

A_1 : asymptotic lower limit

A_2 : asymptotic upper limit

x_0 : ordinate of centre of symmetry

d_x : time constant[6]

By nature of the capsize rate observations, the first two parameters can be constrained to 0 and 1, respectively, which leaves just two parameters requiring estimation and allows for, after some basic manipulation, the expression of P_f as a function of H_S, x_0 and d_x (2). The derivative of P_f with respect to H_S is given as in (3)

$$P_f(H_S) = \frac{e^{\frac{H_S - x_0}{dx}}}{1 + e^{\frac{H_S - x_0}{dx}}} \tag{2}$$

$$\frac{dP_f}{dH_S} = \frac{e^{\frac{H_S - x_0}{dx}}}{dx\left(1 + e^{\frac{H_S - x_0}{dx}}\right)^2} \tag{3}$$

Figures 6 and 7 depict an example of Bolzmann's sigmoid fitted to the experimental data as well as residuals of fitting. Statistical data describing goodness of fit are presented in Tables 2 and 3.

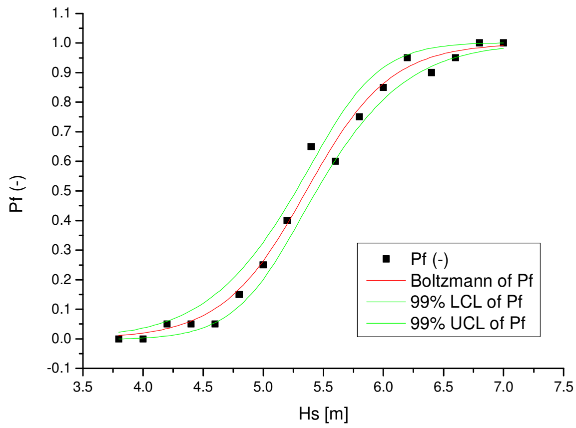

Fig. 6: Fitted sigmoid and 99% confidence boundaries

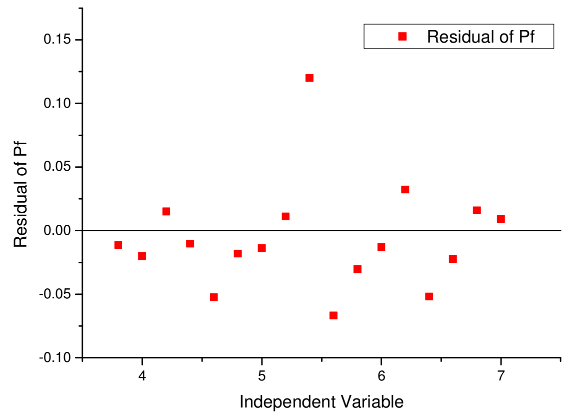

Fig. 7: Residuals of Pf sigmoid fitting

[5] Word *large* in this context refers rather to computational or experimental effort than actual, numerical size of the data.

[6] The parameter *dx* is referred to by analogy to dynamic system response to step input. In context of current application is a span parameter (related to slope at inclusion point).

Table 2: Parameters of sigmoid regression to Pf for T=6.25 m, KG=12.200 m, even keel.

Parameters		
	Value	Standard Error
A1	0	0
A2	1	0
x0	5.35778	0.02832
dx	0.34893	0.02503

Table 3: Statistics of sigmoid regression

Statistics	
Number of Points	17
Degrees of Freedom	15
Reduced Chi-Sqr	0.00192
Residual Sum of Squares	0.02873
Adj. R-Square	0.98814

Results of employing this technique to data deriving from numerical simulations performed at different KGs are presented in figure 8. It can be readily seen that increasing KG causes a shift of P_f characteristics towards lower sea states with a more rapid transition from low to high capsize rates (probability distribution becoming narrower as KG increases). This implies that as survivability decreases the transition from the region considered safe to that considered as unsafe is faster. The performance of this particular probability distribution's parameters against other ship characteristics can be established in the same manner, with the scope to detect any dependencies between survivability and specific design variables.

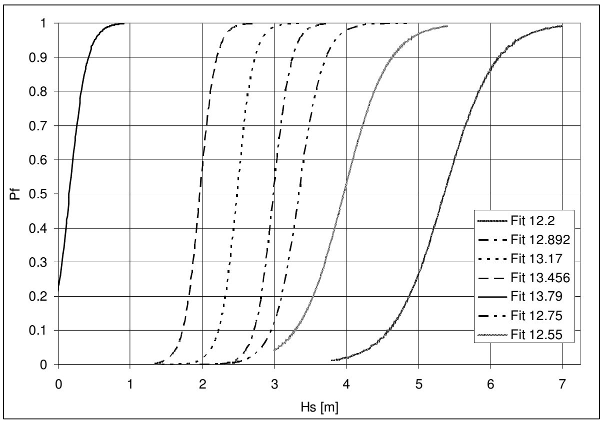

Fig. 8: Capsize rate for various critical significant wave heights

Estimation of the capsize band

The previous observation can be quantitatively confirmed by use of critical significant wave height and capsize band parameters. The first quantity is associated with x_0 parameter of the regression's sigmoid function whereas the latter can be easily calculated using equation (1). By analogy to statistics the capsize band can be interpreted as the range of the probability distribution, spreading either side of the capsize boundary (P_f = 0.5), symmetrically. In a more straightforward interpretation limits of the capsize band simply determine boundaries outside which capsize rate is either so high or so low that capsize in given H_S is either certain or unlikely, beyond upper and below lower limits, respectively. In order to determine such limits, it is convenient to take some small number α, and find those values of H_S, which satisfy the following conditions:

$$(H_S)_{low} = H_S\big|_{P_f(H_S)=\alpha} \tag{4}$$

And

$$(H_S)_{high} = H_S\big|_{P_f(H_S)=1-\alpha} \tag{5}$$

The boundaries $(H_S)_{low}$ and $(H_S)_{high}$ can be calculated using the inverse P_f function, given as:

$$H_S(P_f) = x_0 + dx \cdot \ln\left(\frac{P_f}{1-P_f}\right) \qquad (6)$$

Lower and higher limits of the capsize band, given as $H_S(P_f = a)$ and $H_S(P_f = 1 - a)$ are equal to:

$$H_S(P_f = \alpha) = x_0 + dx \cdot \ln\left(\frac{\alpha}{1-\alpha}\right) \qquad (7)$$

And

$$H_S(P_f = 1-\alpha) = x_0 + dx \cdot \ln\left(\frac{1-\alpha}{\alpha}\right) \qquad (8)$$

The following figure demonstrates these limits, calculated with the parameter.α = 0.05.

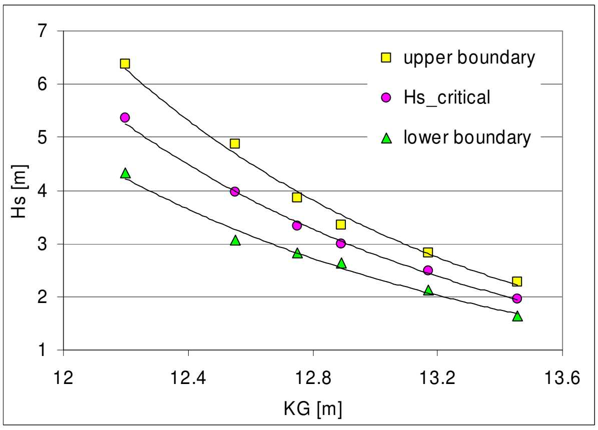

Fig. 9: Capsize band Vs KG

Parameterisation

Attempts to derive a simple analytical function to represent capsize boundaries and capsize band revealed new possibilities for parameterisation of the formula to populate a family of functions, which could be used as a universal tool for survivability assessment in both design and operational stages. In case of the sigmoid, the two defining parameters, i.e. x_0 and d_x can be expressed by means of wave characteristics (other than H_S, which is explicitly present in the P_f formulae) or parameters related to loading condition, damage extent etc. Understandably, parametric studies require extensive and systematic simulation (testing) effort but some rough examples may be presented here. They may also shed some light on sensitivity problems associated with these studies. A single-variable parameterisation of the sigmoid's x_0 and d_x using KG as a parameter is presented in figure 10.

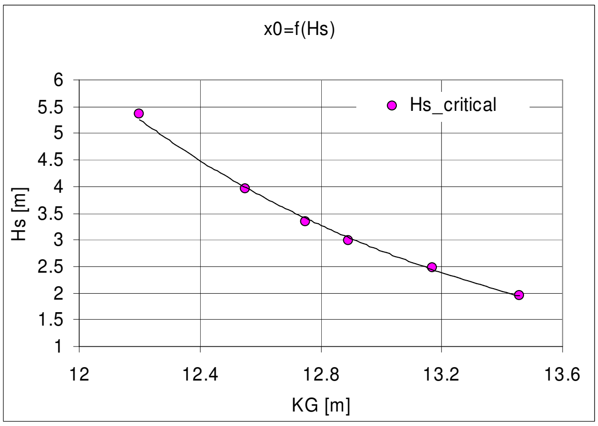

Fig. 10: Plot of critical significant wave height (capsize boundary) Vs KG (intact ship)

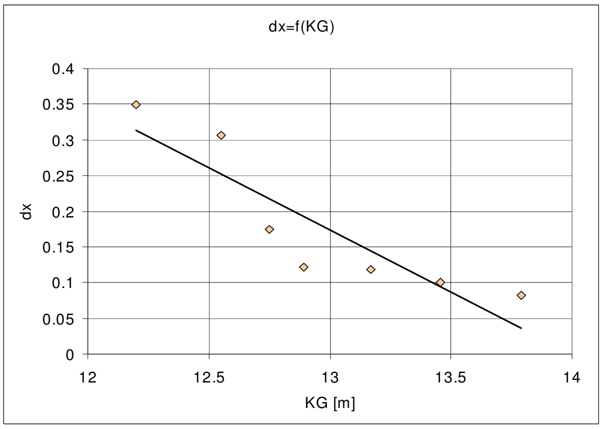

Fig. 11: Bandwidth parameter Vs KG (intact ship)

Obviously, the family of sigmoids describing the capsize rate should be populated with as many parameters as necessary, including also those specific to the damaged ship, e.g. residual freeboard, water head on a car deck etc. to enhance its functionality. For the purpose of this work, the parameters investigated are associated with the intact ship characteristics, leaving aside damage-related quantities, until more research output is available. The following figure shows an example of decomposition of critical significant wave height with respect to (intact) *GM* and wave slope λ.

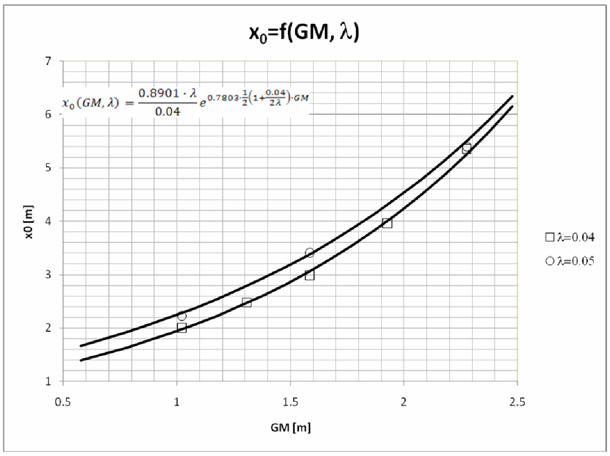

Fig. 12: Bi-variate parameterisation of critical significant wave height

LINEAR APPROXIMATION

However convenient the sigmoid regression is to use, it also comprises some significant drawbacks. To start with, something that is particularly evident in cases of very narrow capsize band is that the goodness of fit depends strongly on the quality of data in the proximity of tail asymptotes. Unfortunately. due to limited resolution of experimental data, these regions bear the highest uncertainty.

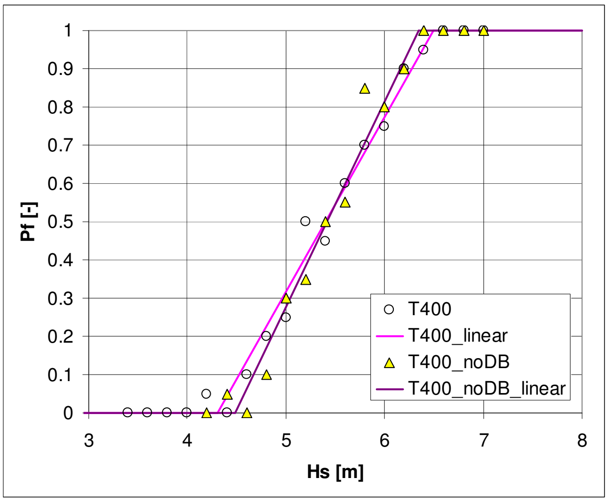

Fig. 13: Linear regression for different damage cases (EUGD01-R2)

Assuming that the data in proximity of the critical value, lying in the middle of the range of P_f should be the most reliable, an attempt has been made to simplify the approach and to use linear regression instead of non-linear, with encouraging results. It can be noticed that some cases demonstrate higher goodness of fit for linear regression than for a sigmoid. In order to achieve this, though, the tails of the series needed to be omitted as that is where the non-linear behaviour is dominant. However, it was observed that removing "tails" from the data set has no major impact on the result.

Table 4: Sigmoidal Vs linear regression

	Sigmoid	Linear
Number of Points	19	13
Degrees of Freedom	17	11
Reduced Chi-Squares	0.00211	
Residual Sum of Squares	0.03595	0.03247
Adj. R-Square	0.98703	0.97626

This, as demonstrated in table 4 by comparison of the residual sum of squares for one sample dataset, makes this approach really attractive. A major concern whilst using linear regression is related to the capsize band and its analogy to the confidence interval. It is obvious that relying on statistical measures of goodness of fit may overshadow the fact that linear regression does not bring any information about the "tails" of the capsize rate distribution and therefore any prediction of capsize band based on this method should be approached carefully. However, closer examination of the linear regression and its affiliation with the sigmoid reveals some important virtues. Linear regression of the data close to x_0 will actually result to the tangent of the sigmoid at the inclusion point (x_0, 0.5). Therefore, for the linear regression parameters α (slope) and β (intercept) the following relation holds:

$$\alpha = \left.\frac{dP_f}{dH_S}\right|_{H_S=x_0} = \frac{e^{\frac{H_S-x_0}{dx}}}{dx\left(1+e^{\frac{H_S-x_0}{dx}}\right)^2} = \frac{1}{4dx} \quad (9)$$

$$y(x_0) = \alpha x_0 + \beta = \frac{1}{2} \quad (10)$$

The parameters for the bandwidth and centre of symmetry of the sigmoid function can be derived directly from the linear regression formula:

$$dx = \frac{1}{4\alpha} \quad (11)$$

$$x_0 = \frac{0.5-\beta}{\alpha} \quad (12)$$

Finally, since all the parameters required for the sigmoid representation can be evaluated on the basis of a linear fit, it is sufficient to apply linear regression to the observations and once x_0 and d_x are estimated, the capsize band limits can be calculated with the use of equations 7 and 8, respectively.

Table 5: Impact of slope estimate on capsize band and H_S critical.

	Fitted	Estimate 1	Estimate 2
0.05	1.28691	0.99266	1.16301
0.5 (H_{Scrit})	1.68031	1.70087	1.68481
0.95	2.07372	2.40909	2.20661
band	0.78681	1.41643	1.0436

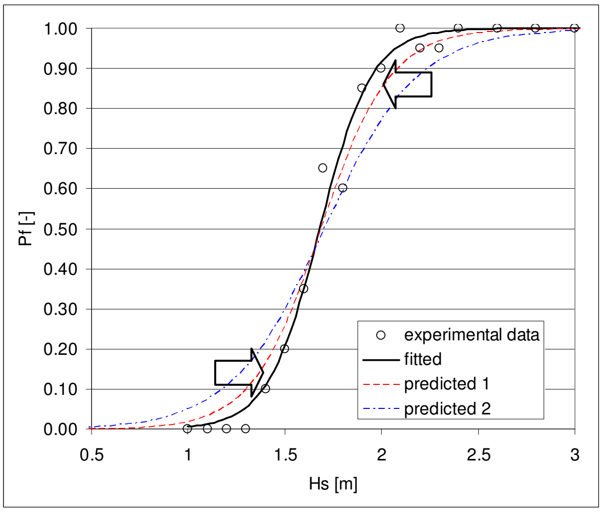

Fig. 14: Fit convergence - accurate estimate of slope at x_0 results in closer match.

Such approach, based on linear regression, has some rather serious implications. First of all, it allows use of formulae derived for the sigmoid curve, well representing observed phenomena, but without the necessity of non-linear (least-squares) regression. Furthermore, as discussed earlier, experimental results in close proximity to 0 and 1 asymptotes are expected to suffer due to large uncertainties and in general, they require higher resolution. On the contrary, points corresponding to moderate capsize rates are usually following the trend better. An approach based on linear regression makes it possible to disregard those regions entirely or just the parts that might be ambiguous. In the latter case (partial reduction) it is important that the remaining data preserve basic characteristics of the distribution, such as symmetry around x_0. Given that sufficient resolution is available around the x_0 region, the resulting sigmoid function should be very accurate. The benefit of this approach is that one could derive an approximate capsize band, having nothing more than 2 measurements of the capsize rate, as long as they are different than 1 and 0 – ideally – and the smaller measurement corresponds to lower H_S. Of course, this should only be treated as an indication and a more accurate calculation of the slope of the probability distribution at its centre would have to be available for reliable results.

CONCLUSIONS

This paper presents an alternative approach to the representation of the behaviour of a damaged ship in waves. The approach adopted for analytical approximation of the capsize band has both benefits (speed) and drawbacks (uncertainty) but some compromise is not only inevitable but also necessary in most engineering applications – particularly those that are exceptionally labour intensive and costly.

The characteristics of the probability distribution that describes the behaviour of Ro-Ro ships in boundary conditions have been identified and an analytical model describing the capsize band has been developed.

Furthermore, the way to utilise the outcome to predict the critical wave height has been demonstrated. In addition, the capability to facilitate these characteristics in the design process as constrains and/or objectives has been discussed.

Lastly, the merits of having an analytical approach to describe such a complex phenomenon are indisputable. The amount of realisation performed numerically for this work is counted in thousands, so the amount of work saved by such an approach is massive.

The presented analytical approach offers the necessary flexibility to integrate this with more complex models for prediction of time to capsize, which in turn can be associated with number of people to successfully evacuate and finally risk from flooding etc.

REFERENCES

HARDER (1999-2003). "Harmonization of Rules and Design Rational". Project funded by the European Commission, DG XII-BRITE, 2000-2003.

Jasionowski, A., Vassalos, D., Guarin, L., "Time-Based Survival Criteria for Passenger Ro-Ro Vessels", 6th International Ship Stability Workshop, Webb Institute, 2002

Jasionowski, A., Vassalos, D., Scott, A., Ship Vulnerability to flooding, 3rd international conference for maritime safety, Berkeley, California, 2007

Vassalos, D., Turan, O. and Pawlowski, M.: "Dynamic Stability Assessment of Damaged Ships and Proposal of Rational Survival Criteria", Journal of Marine Technology, Vol. 34, No. 4, pp 241-269, October 1997.