

Experimental and Numerical Study on Predicting Method of Parametric Rolling in Regular Head Seas

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Abstract: The methods to be used for direct stability assessment of parametric rolling are now under development by the International Maritime Organization (IMO) in the second generation intact stability criteria. In order to provide a reliable numerical method for predicting parametric rolling, firstly, free running experiments and partially restrained free running experiments were conducted to examine the effect of surge motion on parametric rolling and the effect of parametric rolling on heave and pitch motions in regular head seas. Secondly, the surge-roll coupled model with added resistance taken into account is used to predict parametric rolling in which the restoring variation is estimated with coupling from the vertical motion and diffraction effects, which are obtained with a strip theory. Thirdly, a coupled heave-roll-pitch mathematical model based on a nonlinear strip theory is used to calculate heave and pitch motions in regular head seas with parametric rolling taken into account. Finally, time-domain heave and pitch motions are analyzed in the frequency-domain by the Fourier transformation. The results of free running experiments, partially restrained free running experiments and simulations using the C11 containership show that the surge motion on parametric rolling is general small in regular head seas and heave and pitch motions are distinctly affected by parametric rolling and the pitch and heave motions in experiment include subharmonic component when parametric rolling occurs.

Key words: Parametric rolling, surge, heave, pitch, vulnerability criteria, IMO second generation intact stability criteria

1. Introduction

The methods to be used for direct stability assessment of parametric rolling are now under development by the International Maritime Organization (IMO) in the second generation intact stability criteria [1]. A predicting method for parametric rolling with quantitative accuracy is required in the criterion on parametric rolling. Parametric rolling in head seas as one of roll restoring variation problems is a nonlinear phenomenon with dynamic heave and pitch motions so that it is difficult to predict parametric rolling accurately in head seas. Therefore, it is urgent to develop a reliable method to predict parametric rolling in head seas.

In case of following waves, the encounter frequency is much lower than the natural frequencies of heave and pitch so that coupling with dynamic heave and pitch is not important. In addition, added resistance in

following waves is generally small. Thus several successful predictions of parametric rolling in following waves were reported [2]. In case of head seas, however, prediction of parametric rolling is not so easy because coupling with heave and pitch is significant and added resistance cannot be simply ignored. Effect of dynamic heave and pitch motions on parametric rolling was investigated so far by many researchers and is well established: restoring arm variation in head waves depends on dynamic heave and pitch motions [3]. Germany also pointed out that speed variation in wave could have large influence on the results of direct assessment for parametric rolling [4], but two of the authors present that the effect of surge on parametric rolling in regular head seas is rather limited by numerical simulations[5]. The effect of surge motion with added resistance taken into account on parametric rolling was investigated by some researchers [6, 7, 8, 9], but experimental study with and without surge was not conducted in the above researches. So the effect of surge motion on

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parametric rolling should be validated by experiments with and without surge motion.

Since in a seakeeping theory the effect of roll on heave and pitch motions is small, coupling from heave and pitch to parametric rolling is usually taken into account but not vice versa in the published papers [5, 6, 7, 8, 9]. The effects of parametric rolling on heave and pitch motions in head seas, however, are not always negligibly small. Rodriguez et al. [14] observed in their model experiment that heave and pitch motions could have subharmonic components when parametric rolling occurs in head waves but did not reproduce them in their numerical simulations. Then Neves et al. [15] using their nonlinear heave-pitch-roll mathematical model numerically revealed bifurcation structure of heave and pitch motions together with parametric roll. Later the authors [11] observed subharmonic pitch motion together with parametric roll in free-running model experiment at zero forward velocity using the optical 6-DOF motion measuring system but failed to quantitatively explain it with a coupled heave-roll-pitch mathematical model [10] based on a nonlinear strip theory.

For providing a reliable predicting method for direct assessment of parametric rolling, the authors conducted partially restrained experiments with a newly designed equipment and used existing free running experiment data to investigate predicting methods for parametric rolling of a post Panamax C11 class containership which is provided by an IMO's intercessional corresponding group as one of standard ships for developing second generation intact stability criteria.

2. Mathematical Model

The mathematical model of the first approach for parametric rolling prediction in regular waves is expressed as (1).

$$\ddot{\phi} + 2\mu\dot{\phi} + \gamma\phi^3 + \frac{W}{I_{xx} + J_{xx}}GZ(t, X_G, \zeta_G, \theta, \phi) = 0 \quad (1)$$

where ϕ : roll angle, μ : linear roll damping coefficient, γ : cubic roll damping coefficient, W : ship

weight, I_{xx} : moment of inertia in roll, J_{xx} : added moment of inertia in roll, GZ : righting arm, t : time, ζ_G : heave displacement and θ : pitch angle, X_G : instantaneous ship longitudinal position. The dot denotes the differentiation with time.

In the first approach, heave and pitch motions obtained by a strip theory applied to an upright hull are used to estimate the restoring variation. In other words, coupling from heave and pitch to roll is taken into account but not vice versa. Coupling from parametric rolling to heave and pitch could also affect the prediction of parametric rolling. However, due to large roll amplitude and roll frequency of half the encounter frequency, coupling from parametric rolling to heave and pitch would be complicated, and here a coupled heave-roll-pitch mathematical model [10] based on a nonlinear strip theory as the third approach presented later. In the above two approaches, a constant speed is considered.

In the second approach, the added resistance in waves is calculated using Maruo's formula [16] for estimating speed loss and the surge motion, and then heave and pitch motions obtained by a strip theory applied to an upright hull are used to estimate the restoring variation.

The restoring variation consists of two components. One is the nonlinear Froude-Krylov component, which is calculated by integrating wave pressure up to wave surface with heave and pitch motions obtained by a strip theory. The other is the hydrodynamic effects which consist of radiation and diffraction components are extrapolated nonlinearly with regards to roll angle.

The first and second numerical approaches are based on the same principle, and here the formula on the second approach is shown as follows [5, 9]:

$$(M + M_x)\ddot{X}_G = T(\dot{X}_G, n) - R(\dot{X}_G) + F_x(X_G, t) - R_{AW}(\dot{X}_G, t) \quad (2)$$

$$T(\dot{X}_G, n) = (1 - t_p)\rho n^2 D_p^4 K_T \left\{ \frac{\dot{X}_G(1 - w_p)}{nD_p} \right\} \quad (3)$$

$$\ddot{X}_G = \frac{1}{T_e} \int_0^{T_e} \dot{X}_G(t) dt \quad (4)$$

$$X_G = \int_0^t \dot{X}_G(t) dt \quad (5)$$

$$\zeta_G(X_G, t) = \zeta_{Ga} \cos(\omega t - kX_G \cos \chi + \delta_H) \quad (6)$$

$$\theta(X_G, t) = \theta_a \cos(\omega t - kX_G \cos \chi + \delta_\theta) \quad (7)$$

$$F_X(X_G, t) = F_{Xa} \cos(\omega t - kX_G \cos \chi + \delta_X) \quad (8)$$

where M_X : added mass in surge, T : propeller thrust, R : ship resistance in calm water, F_X : wave-induced surge force and R_{AW} : added resistance in waves. Furthermore, ζ_{Ga} : amplitude of heaving, δ_H : initial phase of heaving; θ_a : amplitude of pitching, δ_θ : initial phase of pitching; F_{Xa} : amplitude of wave force of surging, δ_X : initial phase of wave force of surging; ω wave frequency. The dot denotes the differentiation with time.

Initial values for numerical integration with time are set as follows:

$$t = 0; X_G = 0, \dot{X}_G = 0, n = n^* \quad (9)$$

where, n^* : denotes the desired revolution number of propeller.

Furthermore, the calculation method of restoring variation in waves should consider non-uniform forward speed. Its Froude-Krylov component is calculated by integrating the incident wave pressure around the instantaneous wetted hull surface. As a result, the following formula is used.

$$W \cdot GZ = \rho g \int_L y'(x, X_G, t) \cdot A(x, X_G, t) dx + \rho g \sin \chi \cdot \quad (10)$$

$$\int_L z'(x, X_G, t) \cdot F(x) \cdot A(x, X_G, t) \cdot \sin(\zeta_{G0} + (X_G + x) \cos \chi - c \cdot t) dx$$

$$F(x) = \zeta_a k \frac{\sin(k \frac{B(x)}{2} \sin \chi)}{k \frac{B(x)}{2} \sin \chi} e^{-kd(x)} \quad (11)$$

where, $A(x, X_G, t)$: the submerged area of local section of the ship; $y'(x, X_G, t)$: the transverse position of buoyancy centre of local section, $z'(x, X_G, t)$: the vertical position of buoyancy centre of local section, $\zeta_{G0}=0$: the initial longitudinal position of a ship centre from a wave trough.

The radiation and diffraction components of the restoring variation are calculated as follows.

$$GZ_{R\&D} = -M_X / W \quad (12)$$

$$M_X = K - (KG - D)Y \quad (13)$$

$$M_X(X_G, t) = M_{Xa} \cos(\omega t - kX_G \cos \chi + \delta_{MX}) \quad (14)$$

$$Y = F_Y - (A_{23} \ddot{\zeta} + B_{23} \dot{\zeta} + C_{23} \zeta + A_{25} \ddot{\theta} + B_{25} \dot{\theta} + C_{25} \theta) \quad (15)$$

$$K = M_\phi - (A_{43} \ddot{\zeta} + B_{43} \dot{\zeta} + C_{43} \zeta + A_{45} \ddot{\theta} + B_{45} \dot{\theta} + C_{45} \theta) \quad (16)$$

where, KG : the distance from the keel to the gravity of ship; D : draft; M_{Xa} : amplitude of the restoring variation, δ_{MX} : the initial phase of the restoring variation.

Formulae of the wave exciting force, F_Y , and moment M_ϕ are available in the reference [12] as well as those for coupling coefficients in reference [13].

Due to large roll amplitude and roll frequency of half the encounter frequency, coupling from parametric rolling to heave and pitch would be complicated, and there is no theory can be used to investigate the effects of parametric rolling on heave and pitch motions in head seas, so the authors attempt to use a coupled heave-roll-pitch mathematical model [10] which is based on a nonlinear strip theory and based on same principle with the first and second approaches as the third approach. The mathematical model of the third approach for parametric rolling prediction in regular waves is expressed as (17), (18), (19).

$$(I_{xx} + A_{44}(\phi) \ddot{\phi} + N_1 \dot{\phi} + N_3 \phi^3 + A_{43}(\phi) \ddot{\zeta} + B_{43}(\phi) \dot{\zeta} + A_{45}(\phi) \ddot{\theta} + B_{45}(\phi) \dot{\theta}) = F_4^{FK+B}(\xi_G / \lambda, \zeta, \phi, \theta) + F_4^{DF}(\phi) \quad (17)$$

$$(I_{xx} + A_{44}(\phi) \ddot{\phi} + N_1 \dot{\phi} + N_3 \phi^3 + A_{43}(\phi) \ddot{\zeta} + B_{43}(\phi) \dot{\zeta} + A_{45}(\phi) \ddot{\theta} + B_{45}(\phi) \dot{\theta}) = F_4^{FK+B}(\xi_G / \lambda, \zeta, \phi, \theta) + F_4^{DF}(\phi) \quad (18)$$

$$(I_{yy} + A_{55}(\phi) \ddot{\theta} + B_{55}(\phi) \dot{\theta} + A_{53}(\phi) \ddot{\zeta} + B_{53}(\phi) \dot{\zeta} + A_{54}(\phi) \ddot{\phi} + B_{54}(\phi) \dot{\phi}) = F_5^{FK+B}(\xi_G / \lambda, \zeta, \phi, \theta) + F_5^{DF}(\phi) \quad (19)$$

Nonlinear Froude-Krylov forces are calculated by integrating the incident wave pressure around the instantaneous wetted hull surface. Radiation and

diffraction forces are calculated for the submerged hull considering time-dependant roll angle with the static balance of sinkage and trim. Two-dimensional hydrodynamic forces are calculated by strip theory. Hydrodynamic forces for the heave, pitch and diffraction models are calculated with the encounter frequency while those for roll mode are done with half the encounter frequency assuming parametric rolling. Linear and cubic roll damping coefficients are used in mathematic model which are obtained from roll decay test in experiment. Here in order to investigate the effect of parametric rolling on heave and pitch motions, roll damping coefficients is adjusted to tune amplitudes of parametric rolling. The model proposed by Neves et al. (2009) takes account of nonlinear hydrostatic coupling between roll and vertical motions as well as nonlinearity of both roll and vertical motions, while the model used here does also body-nonlinear hydrodynamic coupling between roll and vertical motions without nonlinearity of vertical motions.

3. Experiments

Both the free running experiment and the partially restrained experiment with a 1/65.5 scaled model of the post Panamax C11 class containership were conducted at the seakeeping basin (length: 69m, breadth: 46m, height: 4m) of China Ship Scientific Research Center, which is equipped a flap wave maker at the two adjacent sides of the basin.

The ship model was drove by a propeller in regular head seas in the free running experiment. Pitch and roll amplitude are measured by the MEMS (Micro Electro-Mechanical System)-based gyroscope placed on the ship model and wave elevation was measured by a servo-needle wave height sensor attached to the towing carriage. In order to directly measure the heave motion, an optical 6-DOF motion measuring system attached to the towing carriage is also used to measure ship motions. Here the optical system is only used to measure ship motion at zero speed because the towing carriage has mechanical vibrations with forward speed

which affects the precision measure of the optical tracker.

The ship model was towed by the towing carriage in regular head seas in partially restrained experiment and a newly designed equipment was used to measure ship motions including roll, pitch and heave motions and excited wave moment/force including roll moment, yaw moment ,sway force and surge force. Roll and pitch motions are measure by potentiometer sensor. Heave motion are measured by displacement sensor. Roll moment, yaw moment, sway force and surge force are measured by four sensors based on electromotive strain gauge.

The principal particulars and body plan of the C11 class containership are shown in Table 1 and Fig.1, respectively. The ship model in free running experiment and partially restrained experiment are shown Fig.2 and Fig.3, respectively.

Table 1 Principal particulars of the C11 containership

Items	Ship	Model
Length:L	262.0m	4.000m
Draft:T	11.5m	0.176m
Breadth:B	40.0m	0.611m
Depth:D	24.45m	0.373m
Displ.:W	67508ton	240.2kg
C_B	0.560	0.560
GM	1.928m	0.029m
T_ϕ	24.68s	3.05s
K_{YY}	0.24L	0.24L

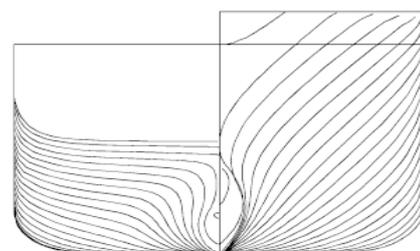


Fig. 1 Lines of C11 containership



Fig.2 The ship model in free running experiment



Fig.3 The ship model in partially restrained experiment

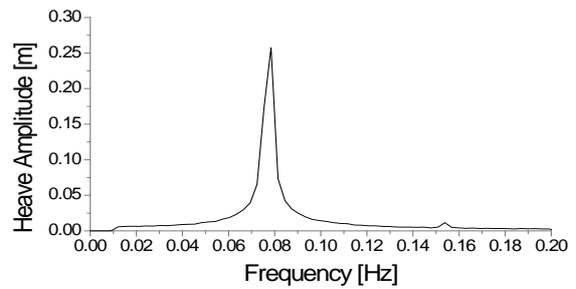
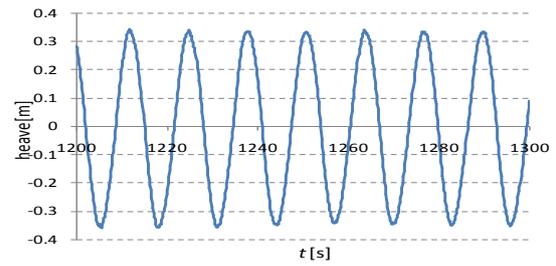
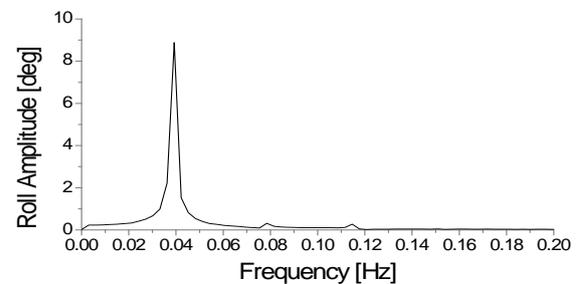
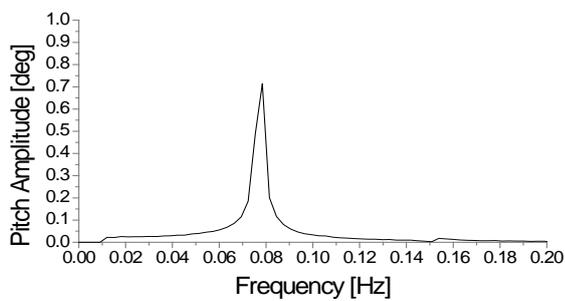
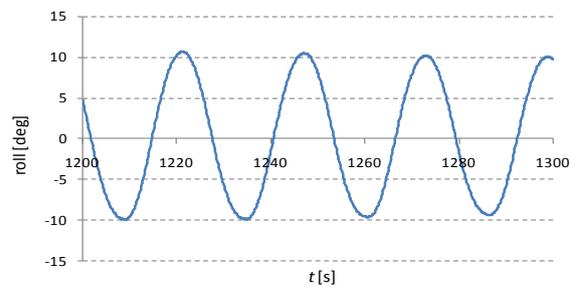
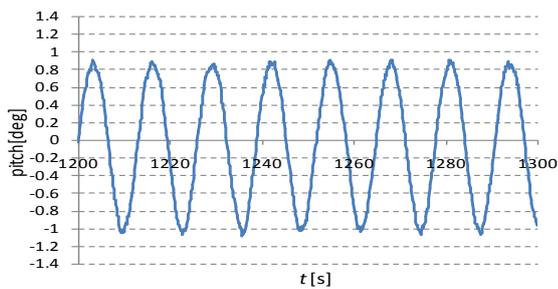
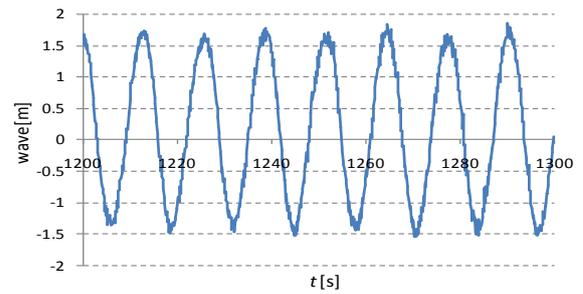
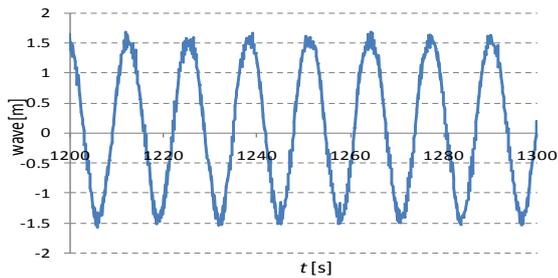


Fig.4 Wave, pitch and heave motions in time and frequency domains while heeling is restrained with $\lambda/L_{pp}=1.0$, $H/\lambda=0.01$, $\chi=180^\circ$, $Fn=0.0$, $1/(Te)=0.0772\text{HZ.}(\text{Exp}2)$

4. Results and Discussions



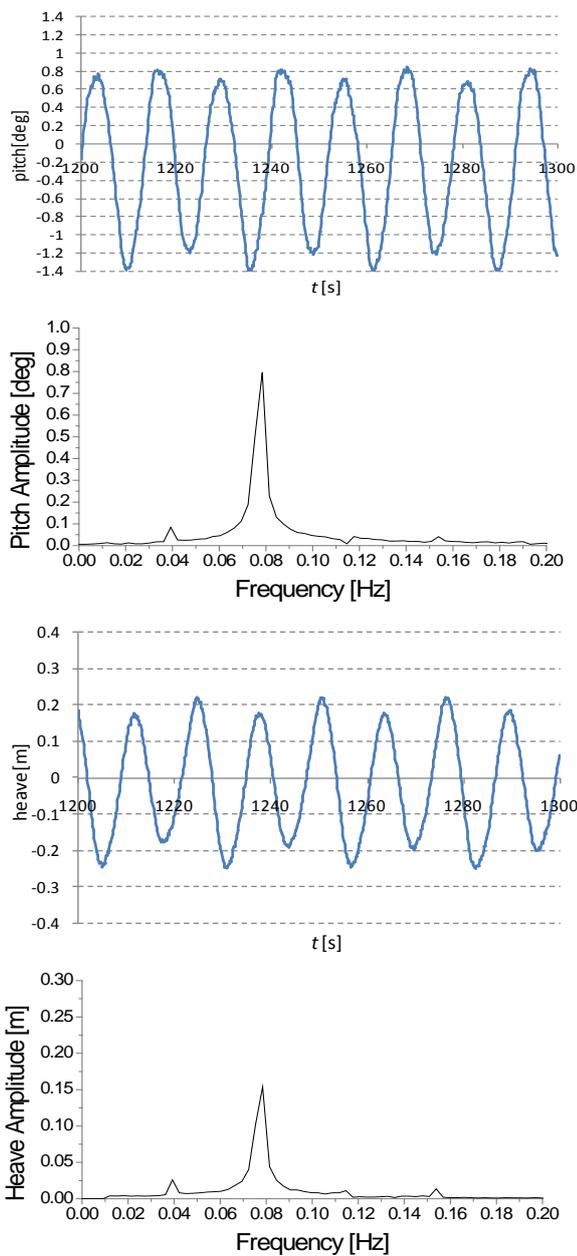


Fig.5 Wave, pitch and heave motions in time and frequency domains while parametric rolling occurs with $\lambda/L_{pp}=1.0$, $H/\lambda=0.01$, $\chi=180^\circ$, $F_n=0.0$, $1/(T_e)=0.0772\text{HZ}$.(Exp2)

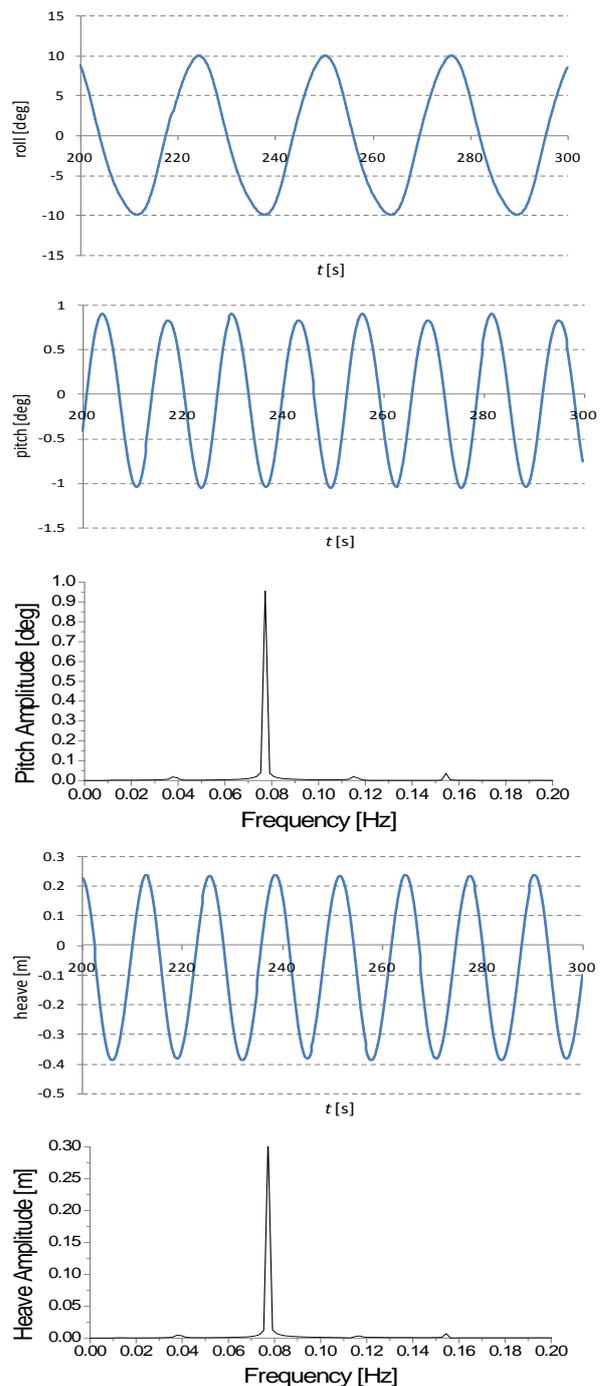
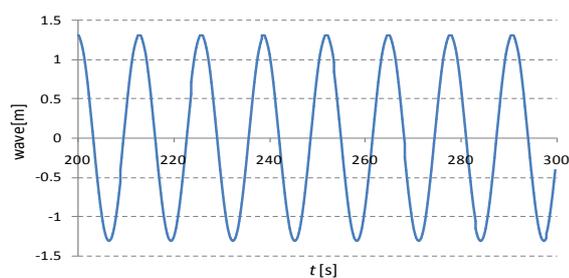


Fig.6 Wave, pitch and heave motions in time and frequency domains while parametric rolling occurs with $\lambda/L_{pp}=1.0$, $H/\lambda=0.01$, $\chi=180^\circ$, $F_n=0.0$, $1/(T_e)=0.0772\text{HZ}$.(simulation of approach 3 with adjusted roll damping coefficients)



The results of experiments indicate the frequency of heave and pitch motions is equal to the encounter wave frequency in case without parametric rolling as shown in Figs.4 which coincide with a linear seakeeping theory. When parametric rolling occurs

with amplitudes of 10 degrees as shown Figs.5 in restrained experiments, heave and pitch motions are affected by parametric rolling and their large and small amplitudes alternatively appear. This phenomenon seems like “subharmonic pitch” and “subharmonic heave” [15]. The heave and pitch motions are analyzed in the frequency-domain by the Fourier transformation. One distinct phenomenon was observed that pitch and heave motions in the experiments has both half the encounter wave frequency and the encounter wave frequency components when parametric rolling occurs while this phenomenon is not obvious for heave motions in the reference[11] by the authors. This phenomenon in the simulation is not as distinct as that in the experiment as show in fig.6.

Although pitch and heave motions are lightly affected by parametric rolling in numerical simulations, the distinct phenomenon cannot reproduced in numerical simulations as show in fig.6. Therefore, in order to provide a reliable numerical method for predicting parametric rolling, the simulation model should be updated and the effects of parametric rolling on heave and pitch motions in head seas should be precisely taken into account.

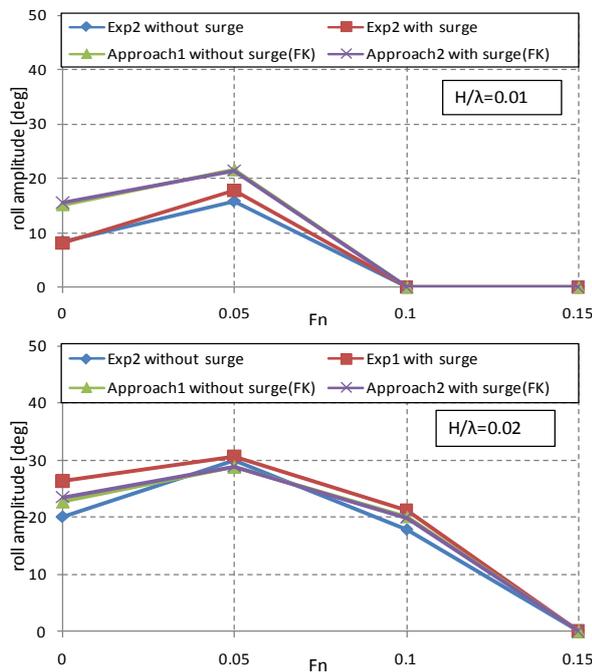


Fig.7 The effect of surge motion on parametric rolling as the function of the Froude number in experiments and simulations with $\lambda/L_{pp}=1.0$, $\chi=180^\circ$ (FK:only Froude-Krylov components of restoring variation are considered).

The surge motion is free in the free running experiment noted as Exp1 while the surge motion is restrained in the partially restrained experiment noted as Exp2. The effect of the surge motion on parametric rolling is generally small by comparing the results between the two experiments as show in figs.7. The results of simulations also indicate that the effect of the surge motion on the parametric rolling is generally small as show in figs.7 and 8. This is because the difference of XG is very small between with and without surge motion although ship forward speed is periodically varied while surge motion is considered in regular head seas, and then that results in the difference of wave profile as well as the change of GZ is very small [5].

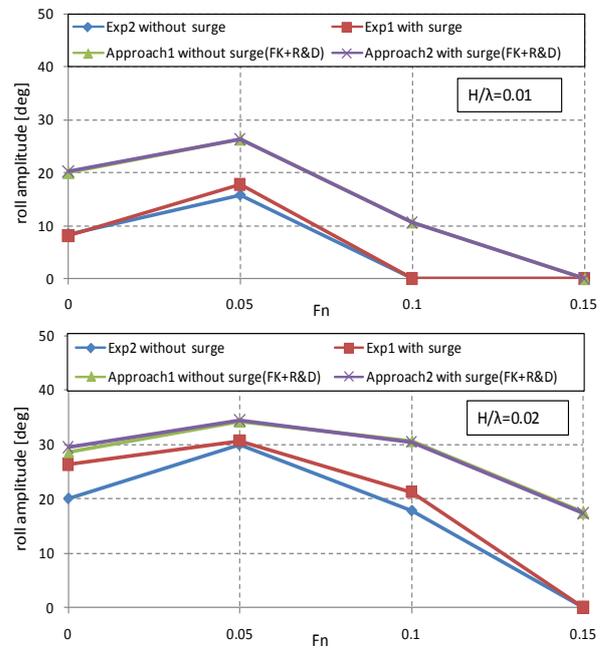


Fig.8 The effect of surge motion on parametric rolling as the function of the Froude number in experiments and simulations with $\lambda/L_{pp}=1.0$, $\chi=180^\circ$ (FK+R&D:the radiation and diffraction components of restoring variation are also considered).

The calculations in the restoring variation are executed both with and without the radiation and diffraction components. The prediction with Froude-Krylov, radiation and diffraction components is larger than that with the Froude-Krylov on its own. This is because the amplitude of GZ variation with Froude-Krylov, radiation and diffraction components

is larger than that with the Froude-Krylov on its own in regular head seas [5]. Therefore, for conservatively predicting parametric rolling, the dynamic effect of radiation and diffraction force should be taken into account.

5. Conclusions

As a result of experimental and numerical studies on predicting methods of parametric rolling in regular head seas, the following remarks and recommendations are noted:

- 1) The pitch and heave motions in the experiments consist of both half the encounter wave frequency and one the encounter wave frequency components when parametric rolling occurs, and the large and small amplitudes alternatively appear.
- 2) The effect of surge motion on parametric rolling in regular head seas is generally small in experiments and simulations and the surge motion could be ignored for providing a simple predicting method with quantitative accuracy on parametric rolling in direct stability assessment.
- 3) The dynamic effect of radiation and diffraction force should be taken into account for conservatively predicting parametric rolling in direct stability assessment.
- 4) The effects of parametric rolling on heave and pitch motions in head seas should be precisely taken into account for providing a reliable numerical method for direct stability assessment of parametric rolling.

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