A step forward towards developing an uncertainty analysis procedure for roll decay tests

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ABSTRACT

The general approach to estimate ship roll damping is to perform roll decay tests in calm water, as they represent the easiest and approach and the most efficient in terms of time. However, how to carry out roll decays is not simple, as many parameters may affect the results. By using proper mechanical devices to initially heel the ship, the results are more reliable, however, it is deemed necessary to estimate the uncertainty associated with the determined roll damping coefficients. This paper presents an approach of developing an uncertainty analysis procedure for roll decay tests.

Keywords: *Roll damping, decay roll tests, experimental techniques, nonlinear rolling, uncertainty analysis.*

1. INTRODUCTION

Roll damping represents the energy that a body loses when rolling. It is a representative parameter to characterize a ship's seakeeping behaviour. It may be derived experimentally performing roll decay tests. Roll decay tests are based on inducing an initial heel angle to the ship model, releasing it allowing to roll freely, and recording and analysing the oscillatory roll motion.

Roll decay tests are the most common approach to estimate roll damping, because they are low time-consuming, and the infrastructures required to carry them out are less sophisticated. Nevertheless, roll decay tests present some problems. The most typical is that roll damping estimations at large rolling angles are complex and, depending on the ship type, it may not be always feasible. Another aspect is that, even testing medium roll angles, a proper mechanical device should be used (Spounge et al., 1986; Bulian et al., 2009; Irvine et al, 2013; Oliva et al., 2018). Many alternatives have been presented, some of them account also with the fluid memory effects, which represent another aspect to consider (Oliveira, 2011; Söder et al., 2012; Zhao et al., 2016; Oliva et al, 2018). Lastly, there is no standard procedure not even to carry the tests but also to analyse them, which may lead to different roll damping estimations, as pointed out by Wassermann et al., 2016.

It is important to highlight that, although roll decay tests are the primary recommended technique in current and under development stability-related international regulations (IMO 2006, 2019), some research studies have shown that they may be non-conservative and present a different trend compared to the actual roll damping of the ship under regular beam waves (Oliva, 2018).

Despite all the above-mentioned aspects, as roll decay tests constitute an experimental technique, it is necessary to know the uncertainty associated with the roll damping estimations derived from them. This paper deals with it, in view of formulating in the future a procedure suitable to be implemented in an ITTC guideline.

2. METHODOLOGY OF ANALYSIS FOR ROLL DECAYS

The scope of roll decay tests is to determine the roll damping coefficients of a floating body. They consist on initially heeling the model up to a certain angle and then releasing it, recording the decaying oscillation curve.

To specify an uncertainty analysis procedure, it is necessary to understand how the test are carried out, how to model the roll motion and how-to post-process the experimental data to estimate the roll damping coefficients. In the following, these items are explained.

Experimental set-up

There are different experimental methodologies to carry out roll decay tests. They may be categorized depending on the forced induced to the ship model as (Oliva et al, 2018):

1. Only a roll moment is applied, without changing the ship model displacement;
2. A vertical force is applied, generating a roll moment, but changing the ship model displacement;
3. Pre-exciting the ship rolling a certain number of cycles and then releasing it. The ship model displacement is maintained.

In any case, a mechanical device should be used to create the initial heel angle or to pre-excite the ship.

The physical quantity measured, at least, should be the rolling amplitude as a function of time. The sample frequency of the measurements should be fixed taking into account the (undamped) natural roll period of the body tested. Considering existing computer capabilities, the author's recommendation is to use, at least, 100Hz.

Modelling of roll motion

Generally, the motion of the ship under the roll decay tests scenario may be modelled by a 1-DOF (Degree of Freedom) roll motion equation.

The 1-DOF roll motion nonlinear differential equation in calm water, at zero forward speed, and considering non-linearities in restoring and damping terms is as follows:

$$\ddot{\phi} + d\left(\dot{\phi}\right) + \omega_0^2 \cdot r\left(\phi\right) = 0 \qquad (1)$$

where:

- ϕ [rad]: is the roll angle (dots represent derivatives with respect to time);
- $d\left(\dot{\phi}\right)$ [$1/s$]: is the normalized damping function, assumed to be dependent only on the instantaneous roll velocity ($\dot{\phi}$). The roll damping term is generally defined by the linear-quadratic-cubic damping model (ITTC, 2011):

$$d\left(\dot{\phi}\right) = 2 \cdot \mu \cdot \dot{\phi} + \beta \cdot \dot{\phi} \cdot \left|\dot{\phi}\right| + \delta \cdot \dot{\phi}^3 \qquad (2)$$

where μ [*1/s*], β [*1/rad*] and δ [*s/rad²*] are the linear, quadratic and cubic damping coefficients, respectively. The δ and β coefficients may be fixed to zero, depending on the ship hull and on the presence of bilge keels, then using the so-called linear-quadratic or linear-cubic damping models;

- ω_0 [rad/s]: is the (undamped) natural roll frequency, defined as:

$$\omega_0^2 = \frac{\Delta \cdot \overline{GM}}{J_{xx}^v} \qquad (3)$$

where Δ [N] is the ship displacement, $\overline{GM}$ [m] is the metacentric height with respect to. the centre of gravity of the ship (G), considering the vessel freely floating with displacement Δ, and J_{xx}^v [$kg \cdot m^2$] is the total roll moment of inertia including the hydrodynamic added inertia;

- $r\left(\phi\right)$ [*nd*]: is the non-dimensional righting arm, which is equivalent to:

$$r\left(\phi\right) = \frac{\overline{GZ}\left(\phi\right)}{\overline{GM}} \qquad (4)$$

where $\overline{GZ}\left(\phi\right)$ [*m*] is the hydrostatic roll righting lever with respect to *G*.

Analysis of roll decays

Different methodologies to analyse roll decays exist, being themselves dependent on the mathematical model of the ship roll motion under the specific scenario of roll decays.

Some of them do not consider the non-linearities in the restoring and damping terms, some others only the non-linearities in one of the terms and the rest consider both.

The method used to analyse roll decays is relevant when considering an uncertainty analysis. In the present paper, the procedure considers the non-linearities in the restoring and damping terms, assuming the mathematical model described previously. The analytical procedure is described in detail in Appendix 1 of Bulian et al., 2009.

In the following, the linear-cubic damping model is considered, (see Eq. (5)) and the non-linear restoring is supposed calculated directly from the actual GZ curve, instead of obtaining the restoring coefficients from least square fitting.

The procedure is based on the logarithmic roll-decrement curve by approximating the nonlinear model of Eq. (1) by a linear equivalent model in a limited time window:

$$\ddot{\phi} + 2\cdot\mu_{eq}(A)\cdot\dot{\phi} + \omega_{0,eq}{}^{2}(A)\cdot\phi = 0\Big|_{\left[\tilde{t}-\frac{\Delta t}{2},\,\tilde{t}+\frac{\Delta t}{2}\right]} \qquad (5)$$

$$\begin{cases} \mu_{eq}(A) = \mu + \dfrac{4}{3\pi}\cdot\beta\cdot\left(\tilde{\omega}(A)\cdot A\right) + \dfrac{3}{8}\cdot\delta\cdot\left(\tilde{\omega}(A)\cdot A\right)^{2} \\[2ex] \tilde{\omega}(A) = \sqrt{\omega_{0,eq}^{2}(A) + \mu_{eq}{}^{2}(A)} \approx \omega_{0,eq}(A) \\[2ex] \omega_{0,eq}(A) = \omega_0 \cdot \\ \quad \cdot\sqrt{\dfrac{1}{\overline{GM}}\cdot\dfrac{\int_0^{2\pi}\overline{GZ}\left(\phi = A\cos(\alpha)\right)\cdot\cos(\alpha)\cdot d\alpha}{\pi\cdot A}} \end{cases}$$

where A $[rad]$ is the rolling amplitude, $\mu_{eq}(A)$ [*1/s*] is the equivalent linear damping coefficient and $\omega_{0,eq}(A)$ [$rad\cdot s^{-1}$] is the equivalent (undamped) roll natural frequency.

Assuming that the (undamped) ship roll natural frequency ω_0, the metacentric height $\overline{GM}$ and the righting lever curve $\overline{GZ}(\phi)$ are known parameters, the step-by-step procedure is as follows:

1. Filter the raw measured data, if needed, and correct possible bias;
2. Determine the extremes C_i and corresponding time instants for each roll decay time history (see Fig. 1);
3. Determine the average amplitude A_i for each half cycle (also a complete cycle may be considered as well as other alternatives (Wassermann et al., 2016), however, care should be taken as there would be changes in the following equations), and calculate the equivalent linear roll damping coefficient $\mu_{eq}(A_i)$ and the equivalent linear frequency $\omega_{0,eq}(A_i)$ associated to A_i;

$$\begin{aligned} A_i &\approx \frac{|C_i| + |C_{i+1}|}{2} \\ \mu_{eq}(A_i) &= \frac{1}{t_{i+1}-t_i}\cdot\ln\left(\frac{|C_i|}{|C_{i+1}|}\right) \\ \omega_{0,eq}(A_i) &\approx \tilde{\omega}(A_i) = \frac{\pi}{t_{i+1}-t_i} \end{aligned} \qquad (6)$$

4. If different roll decay tests representing the same test case (same experimental set-up and initial heel angle and same ship and loading condition) have been carried out, data determined in the previous step can be aggregated. It allows a robust estimation of roll damping coefficients and a reduction of associate uncertainties when performing the step described in the following paragraph;
5. From the aggregated data, the analytical model of $\mu_{eq}(A)$, represented in Eq. 5, can be fitted through a least square fitting to determine the nonlinear roll damping coefficients (μ, β and δ). For the $\mu_{eq}(A)$ fitting, it should be considered μ_{eq} as a function of $\left(\tilde{\omega}(A)\cdot A\right)$. Moreover, as stated in Eq. 5, as for roll motion the system may be characterized as slightly damped, it may be assumed that $\tilde{\omega}(A) \approx \omega_{0,eq}(A)$. In Fig. 2 an example of experimental fitting is shown, however, in order to represent a readable X-axis, μ_{eq} is represented as a function of the roll amplitude, although in reality it has been considered as a function of $\left(\tilde{\omega}(A)\cdot A\right)$, as quoted.

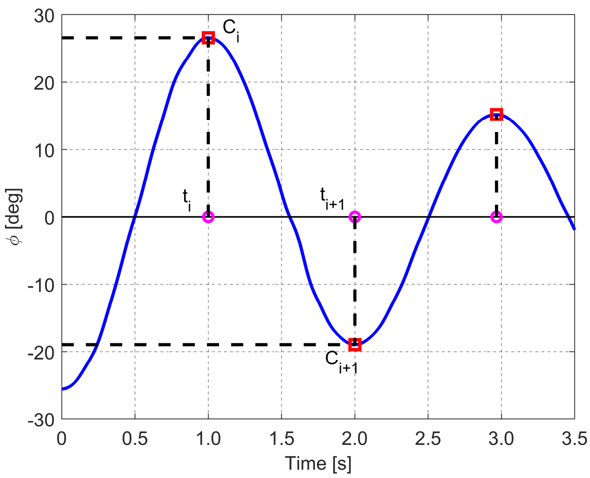

Figure 1: Example of roll decay curve.

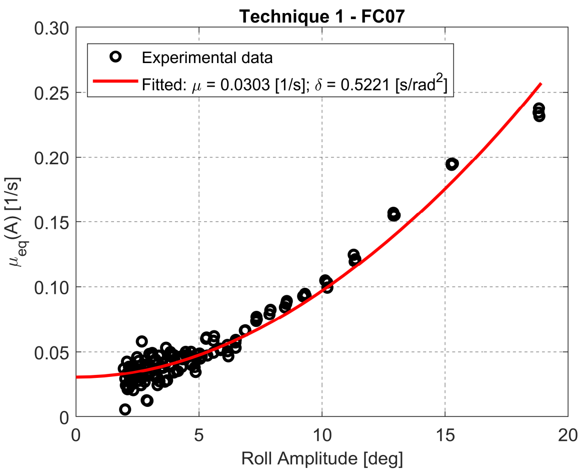

Figure 2: Example of equivalent linear roll damping fitting (quadratic damping coefficient has been fixed to zero).

The fitting of $\omega_{0,eq}(A)$ may not be required if, as assumed previously, the variables in which it depends on (see Eq. (5)) are known.

3. GENERIC UNCERTAINTY ANALYSIS BACKGROUD

Uncertainty is the level of precision of a measurement or a parameter.

According to ITTC guideline 7.5-02-01-07 (ITTCa, 2017; ITTCb, 2017), uncertainties can be classified into three categories: standard uncertainty, combined uncertainty and expanded uncertainty.

The standard uncertainty of the result of a measurement can be categorized into two types:

- Type A: uncertainty components obtained using a method based on statistical analysis of a series of observations.
- Type B: uncertainty component obtained by other means (not statistical analysis).

The standard uncertainty is delimited to a result of a measurement. For quantities not measured directly, the uncertainties propagate to obtain the combined uncertainty.

The relationship between combined and individual uncertainties is given by the law of propagation. For uncorrelated and independent measurements, this law is as follows:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial y}{\partial x_i} \right)^2 \cdot u^2(x_i) \qquad (7)$$

where u is the standard uncertainty and the derivatives $(\partial y / \partial x_i)$ are the sensitivity coefficients, which represent the functional relationship of the measurement variables with the quantity.

The expanded uncertainty appears when the confidence limit is considered. The expanded uncertainty is related with the combined (or standard) uncertainty by a coverage factor κ such as:

$$U = \kappa \cdot u_c(y) \qquad (8)$$

The coverage factor equals to 2 assuming a Gaussian distribution and a confidence limit of 95%. However, for small number of samples, the coverage factor may be replaced by the inverse *Student t* at 95% confidence level.

As a result, the quantity of interest Y (measured or derived from measurements and other parameters) is represented as:

$$Y = y \pm U \qquad (9)$$

4. UNCERTAINTY ANALYSIS APPLIED TO ROLL DECAYS

Uncertainty analyses have been considered in most of the engineering fields and, to the naval field and towing tank experiments, they have been implemented in the main experimental techniques such as resistance towing tank tests or propulsion tests. In fact, there are many ITTC procedures or guidelines that deal with this topic and with how to implement uncertainty analysis in different tests.

However, uncertainty analysis applied in roll damping estimations is still not addressed by ITTC, as well as how to experimentally determine ship roll damping. The later aspect is being addressed currently by the ITTC Stability in Waves Committee (ITTCc, 2017), which has to update the recommended procedure of “Numerical Estimation of Roll Damping” (ITTC, 2011) to account also for experimental techniques to estimate roll damping, therefore, re-calling it as “Estimation of Roll Damping”. The former one, may be addressed as well when updating the recommended procedure, although it may need more development.

In the following, the process to perform uncertainty in roll decay tests is briefly introduced in order to gain some feedback from interested researchers or experimentalist to address the uncertainty issue of roll damping in conjunction with the ITTC.

Some related studies regarding this topic may be found in Irvine et al., 2013 and Park et al., 2016. According to Park et al., 2016, the sources of uncertainty in roll decay tests are:

- Curve fitting;
- Time measurement;
- Angle measurement.

In the following, the logarithmic decrement technique to analyse roll decays and the linear-quadratic-cubic damping model are considered.

As a result, the method produces an uncertainty for each rolling amplitude (A) and, furthermore, the determined rolling amplitude has also an uncertainty associated with its value.

The uncertainties of the equivalent linear roll damping coefficient $\mu_{eq}(A_i)$ and the equivalent linear frequency $\omega_{0,eq}(A_i)$ are shown in Eq. 10 and 11, considering Eq. (6). Also, the uncertainty associated with the amplitude of rolling A_i is reported in Eq. 12.

$$\begin{aligned} &\mu_{eq}(A_i) = \mu_{eq,\det} \pm k\cdot u_{c,\mu_{eq}} \\ &u^2_{c,\mu_{eq}} = \left(\frac{\partial \mu_{eq}}{\partial t_{i+1}}\cdot u(t)\right)^2 + \left(\frac{\partial \mu_{eq}}{\partial t_i}\cdot u(t)\right)^2 + \\ &+\left(\frac{\partial \mu_{eq}}{\partial |C_i|}\cdot u(C)\right)^2 + \left(\frac{\partial \mu_{eq}}{\partial |C_{i+1}|}\cdot u(C)\right)^2 = \\ &= 2\cdot\left(\frac{1}{(t_{i+1}-t_i)^2}\cdot \ln\left(\frac{|C_i|}{|C_{i+1}|}\right)\cdot u(t)\right)^2 + \\ &+\left(\left(\frac{1}{C_i}\right)^2 + \left(\frac{-1}{C_{i+1}}\right)^2\right)\cdot\left(\frac{1}{(t_{i+1}-t_i)}\cdot u(C)\right)^2 \end{aligned} \tag{10}$$

$$\begin{aligned} &\omega_{0,eq}(A_i) = \omega_{0,eq,\det} \pm k\cdot u_{c,\omega_{0,eq}} \\ &u^2_{c,\omega_{0,eq}} = \left(\frac{\partial \omega_{0,eq}}{\partial t_{i+1}}\cdot u(t)\right)^2 + \left(\frac{\partial \omega_{0,eq}}{\partial t_i}\cdot u(t)\right)^2 = \\ &= 2\cdot\left(\frac{\pi}{(t_{i+1}-t_i)^2}\cdot u(t)\right)^2 \end{aligned} \tag{11}$$

$$\begin{aligned} &A_i = A_{i,\det} \pm k\cdot u_{c,A_i} \\ &u^2_{c,A_i} = \left(\frac{\partial A_i}{\partial C_{i+1}}\cdot u(C)\right)^2 + \left(\frac{\partial A_i}{\partial C_i}\cdot u(C)\right)^2 = \\ &= \frac{1}{2}\cdot u(C)^2 \end{aligned} \tag{12}$$

In Eq. 10 and 11, $u(t)$ is the standard uncertainty of the measured time, which may be determined from the sample frequency $f[Hz]$ as:

$$u(t) = \frac{1}{f} \tag{13}$$

and $u(C)$ is the standard uncertainty of the angle measurement. If the calibration of the instrument used to measure the rolling amplitudes is not available by the specifications of the system, this value should be obtained performing a calibration of the device, taking as a basis the ITTC procedure for the instrument calibration (ITTCd, 2017).

The uncertainty associated with the nonlinear damping coefficients should be determined considering Eq. (5), specifically the relationship between the equivalent linear roll damping coefficient and the nonlinear damping components, in which the mean amplitude and the equivalent linear frequency also appear. A simplified approach to derive the uncertainties may be considered. It is based on considering only the uncertainties coming from the curve fitting. In this situation, the confidence intervals $k\cdot u_{c,\mu}$, $k\cdot u_{c,\beta}$ and $k\cdot u_{c,\delta}$ may be derived, assuming a confidence level of 95% and that $k\cdot u_{c,\mu_{eq}}$ present a Gaussian distribution, thus, neglecting the fact that uncertainties at smaller rolling amplitudes are larger. This approach was firstly presented by Bulian et al. 2009 and used by the authors in Oliva et al, 2018 and Oliva, 2018.

5. PRACTICAL CASE

In the following, a practical application of the uncertainty analysis procedure for roll decays is given. It will be based on previous experimental data, whose detailed information may be found in Oliva et al, 2018 and Oliva, 2018. From these references, the decay test case selected for the present work is the FC07 and Technique 1, which corresponded to an initial heeling angle of 25.88 *deg* and the experimental set-up based on applying a roll moment, without changing the ship model displacement.

The standard uncertainty of the measured time corresponds to:

$$u(t)=\frac{1}{f}=\frac{1}{120}=0.0083\ [s] \tag{14}$$

The standard uncertainty of the angle measurement will be estimated, because the actual value is not known due to the usage of an optical trackable system. It will be estimated to be 0.1 *deg*, therefore:

$$u(C)=0.0017\ [\text{rad}] \tag{15}$$

The uncertainty associated with the amplitude of rolling is, consequently:

$$u(A_i)=\sqrt{\frac{1}{2}\cdot u(C)^2}=0.0012\ [rad] \tag{16}$$

The uncertainties of the equivalent linear roll damping coefficient $\mu_{eq}(A)$ and the equivalent linear frequency $\omega_{0,eq}(A)$ are represented in Fig. 3 and 4, respectively.

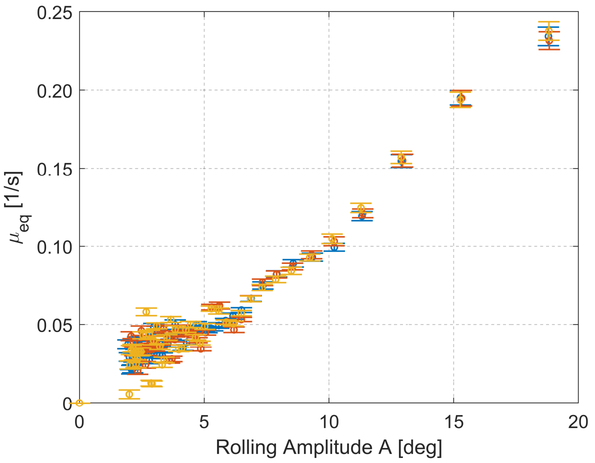

Figure 3: Uncertainty analysis of the equivalent linear roll damping coefficient.

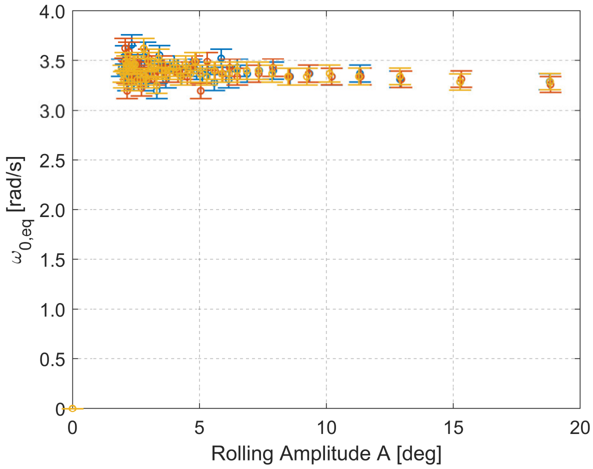

Figure 4: Uncertainty analysis of the equivalent undamped roll natural frequency.

In Fig. 5 and 6, the percentual difference of the uncertainties of $\mu_{eq}(A)$ and $\omega_{0,eq}(A)$ are represented, which have been calculated following Eq. 17:

$$diff[\%]=\frac{\kappa\cdot u(y)}{y} \tag{17}$$

From these results, it may be seen that the uncertainties at smaller rolling amplitudes are larger than uncertainties at medium and large rolling amplitudes, which is coherent, because the angle measurement precision is constant throughout the whole tests and at smaller amplitudes, the difference between the measured value and the amplitude measurement uncertainty is smaller, therefore, the relative difference is much larger.

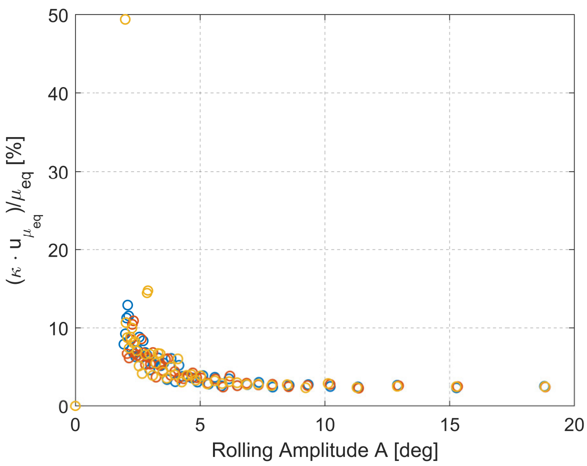

Figure 5: Percentual uncertainty of the equivalent linear roll damping coefficient.

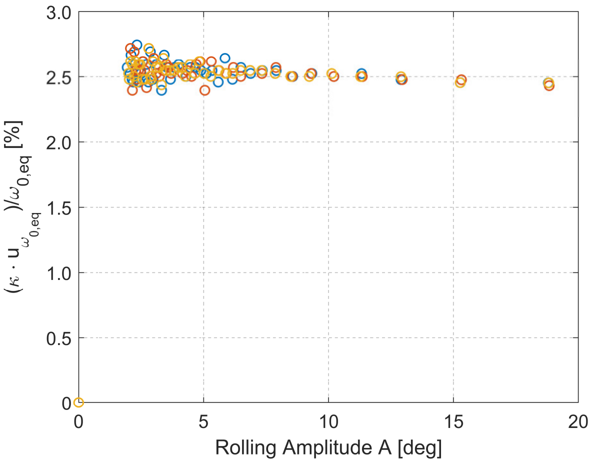

Figure 6: Percentual uncertainty of the equivalent undamped roll natural frequency.

Lastly, the uncertainties associated with the nonlinear roll damping coefficients, only considering uncertainties coming from the curve fitting, are equal to:

$$
\begin{aligned}
u(\mu) &= 0.00095\ [1/s] \\
u(\delta) &= 0.00950\ [s/rad^2]
\end{aligned}
\qquad (18)
$$

In this practical case, the linear-cubic damping model has been considered, because the linear-quadratic-cubic damping model gave negative nonlinear coefficients. The values reported in Eq. 18 constitute a percentual uncertainty of the linear damping coefficient (μ) of 6.3% and a percentual uncertainty of the cubic damping coefficient (δ) of 3.7%, calculating the percentages following Eq. 17.

These last results also present the expected outcomes. The linear damping coefficient presents a larger percentual uncertainty because it is mostly related to small rolling amplitudes, which as reported in Fig. 5 and 6 present the largest experimental uncertainties. Despite of the posted results, it should be emphasized that, for the linear and cubic damping coefficients, uncertainties associated with the equivalent roll damping, the equivalent undamped rolling frequency and the rolling amplitude have not been considered. If considered, the uncertainties of nonlinear damping coefficients would be larger.

6. CONCLUSIONS

Uncertainty analysis when determining roll damping parameters should be performed, due to the importance of roll damping in the seakeeping behaviour of a ship or platform but also because it is informally accepted the existence of large uncertainties associated with this parameter and it could be interesting to demonstrate if this common assumption is true (or not).

In the paper, the procedure to determine the uncertainties associated with the equivalent linear roll damping and the equivalent undamped roll frequency uncertainties are presented. Both of them require to know the uncertainty associated with the time measurement, which may be easily determined from the sample frequency, and the uncertainty associated with the angle measurement, which, depending on the device used, may be easy to determined or may be more complex, such as when using optical trackable systems. Also, a simplified approach to determine uncertainties associated with the nonlinear damping coefficients is presented. This approach consists on only considering the uncertainties coming from the curve fitting procedure, which may represent a significant simplification.

This paper represents a first step forward towards developing an uncertainty analysis procedure for roll decay tests. However, further work needs to be carried out to improve the uncertainty assessment and to consider all the uncertainties when determining the nonlinear damping coefficients uncertainties.

7. ARISING QUESTIONS

During the development of the present work, some questions have emerged:

- Nowadays, how important is the roll damping uncertainty analysis.? How often uncertainty analyses are included when determining roll damping experimentally? When carrying out CFD validations, are experimental values including uncertainties used?
- How can we determine the standard uncertainty of the angle measurement when using an optical trackable system to measure it?
- Is it necessary to use a more complex approach to determine the uncertainties of the nonlinear roll damping coefficients?

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