SEA SPECTRA REVISITED

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ABSTRACT

The paper demonstrates that the sea spectra recommended by ITTC, based on the Bretschneider formulation, can be reduced to a nondimensional spectrum, the same for all the spectra, with unit area. In other words – these well-known spectra have geometrical affinity. This fact has been unknown in literature. Any ITTC sea spectrum, described by two parameters *A* and *B*, can be generated using the nondimensional spectrum. The same also applies to JONSWAP spectra. The latter requires in addition a third parameter, termed as the peak-shape parameter. The paper explores this possibility. The fact that all the sea spectra used by naval architects can be reduced to a common dimensionless spectrum of unit area opens up the possibility of approximating them by probability density functions of certain types. Such spectra, contrary to ITTC ones, are narrow-banded, with the bandwidth parameter less than 1, and have moments of any order.

Keywords: *sea spectra, approximations, and properties*

1. INTRODUCTION

The ITTC spectral formulation for fully developed seas, derives from Bretschneider, and are given by the following equation:

$$S(\omega) = (A/\omega^5)\,\mathrm{e}^{-B/\omega^4}, \tag{1}$$

where A and B are constants. It is convenient to apply a substitution $t = B/\omega^4$ for calculating the spectral moments. For the nth moment, we get:

$$\begin{aligned} m_n &= \tfrac{1}{4} A B^{\frac{1}{4}n-1} \int_0^\infty t^{-\frac{1}{4}n} \mathrm{e}^{-t}\, dt = \\ &= \tfrac{1}{4} A B^{\frac{1}{4}n-1} \Gamma(1-\tfrac{1}{4}n), \end{aligned} \tag{2}$$

where $\Gamma(x)$ is the gamma function, defined for positive x by the integral $\Gamma(x) = \int_0^\infty t^{x-1} \mathrm{e}^{-t}\, dt$. As the argument of the function gamma has to be positive, the moments exist only for $n < 4$. The 4th and higher moments are infinite. Therefore the bandwidth parameter ε is unity, which implies the ITTC spectrum is wide-banded.

Substituting $n = 0$, 1 and 2, and making use of the well-known feature of the gamma function: $\Gamma(x+1) = x\Gamma(x)$, the following results for the first moments:

$$\begin{aligned} m_0 &= \frac{A}{4B}, \\ m_1 &= \frac{\Gamma(1.75)}{3} \frac{A}{B^{3/4}}, \\ m_2 &= \frac{\sqrt{\pi}}{4} \frac{A}{\sqrt{B}}. \end{aligned} \tag{3}$$

For large ω, S decays as ω^{-5}. Due to this reason, the 4th and higher spectral moments do not exist, which is far from reality. For real seas all moments exist and the bandwidth parameter ε is less than 1, from the region $\langle 0.40,\ 0.80 \rangle$.

2. STANDARD SEA SPECTRA

ITTC spectra are defined by two parameters A and B, as seen in equation (1). In order to define these parameters, we have to use two characteristic values describing wave intensity. The most important is the significant wave height $h_s = 4\sigma$, where σ is the standard deviation of wave elevation. Hence, $h_s^2 = 16\sigma^2 = 16m_0 = 4A/B$. The constant A is then given by the equation:

$$A = \tfrac{1}{4} B h_s^2, \tag{4}$$

depending on the other constant B, related to one of the wave periods. Most frequently, the characteristic period is used $T_1 = 2\pi\, m_0/m_1$. Making use of equations (3), we get

$$B = \left[\frac{1.5\pi}{\Gamma(1.75)}\right]^4 \frac{1}{T_1^4} = \frac{691.18}{T_1^4} \tag{5}$$

A spectrum with the above constants A and B is called the ITTC spectrum. It is easy to show that all the characteristic frequencies, such as the modal frequency ω_m, the mean (average) frequency $\bar{\omega}$, and the average zero-crossing frequency ω_0 are in proportion to $B^{1/4}$, which means they are in the same proportions relative themselves. This in turn suggests that ITTC spectra have geometrical affinity, which can be proved rigorously. It is worth noting that the average peak frequency ω_2 does not exist for ITTC spectra.

According to Pierson and Moskowitz the constants A and B are as follows:

$$A = c_1 g^2, \qquad B = c_2 (g/U)^4, \tag{6}$$

where $c_1 = 0{,}0081$, $c_2 = 0{,}74$, g is the acceleration due to gravity, and U is the mean wind speed at 19.5 m above the sea surface. A spectrum with such constants is called the *Pierson–Moskowitz* spectrum. As can be seen, it is a one-parameter spectrum, solely dependent on wind speed, which is not very convenient. In the applications, it is more convenient to utilise the significant wave height rather than the wind speed. To do so, the constant B has to be related to the significant wave height $h_s = 4\sigma$. Since $\sigma^2 = m_0$, therefore $(h_s/4)^2 = \tfrac{1}{4}A/B$. Hence, $B = 4A/h_s^2$.

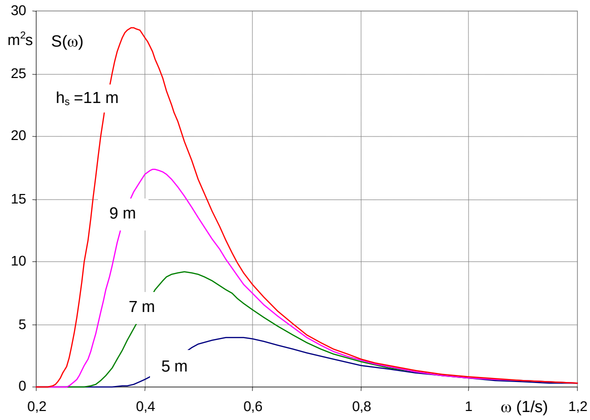

Figure 1. Pierson–Moskowitz spectra as a function of significant wave height h_s

The Pierson–Moskowitz spectra, depending on the significant wave height h_s, are shown in Figure 1. As can be seen, the modal frequency ω_m decreases with the significant wave height h_s, which can also be deduced from equation (7) for the modal frequency:

$$\omega_m = (0.8B)^{1/4}, \tag{7}$$

which results from: $S'(\omega) = 4B\omega^{-4} - 5 = 0$. Equation (7) yields $B = 1.25\,\omega_m^4$. Inserting it to equation (6) yields:

$$\omega_m = (0.8c_2)^{1/4}(g/U) = 0.877\,(g/U). \tag{8}$$

Equating $B = 4A/h_s^2$ to B given by equation (6) ψιελδσ a standard relation between wind speed and sea severity: $U^4 = (c_2/4c_1)(gh_s)^2$. Hence,

$$U = (c_2/4c_1)^{1/4}(gh_s)^{1/2} = 2.186\,(gh_s)^{1/2}. \tag{9}$$

For instance, for $h_s = 9$ m, the standard wind speed $U = 20.5$ m/s.

3. JONSWAP SPECTRUM

The JONSWP formulation is based on an extensive wave measurement programme known as the Joint North Sea Wave Project carried out in the years 1968–69. The spectrum represents wind generated seas with fetch limitation, and wind speed and fetch length are inputs to this formulation, which is as follows:

$$S(\omega)=\frac{\alpha}{c_1}S_{PM}(\omega)\gamma^{\exp\left[-(\omega-\omega_m)^2/2(\sigma\omega_m)^2\right]} \tag{10}$$

where γ = parameter, 3.3 as an average
$\alpha = 0.076\bar{x}^{-0.22}$ – scale parameter
σ = 0.07 for $\omega<\omega_m$, and 0.09 for $\omega>\omega_m$
$\omega_m = 2\pi\cdot 3.5(g/U)\bar{x}^{-0.33}$
$\bar{x} = gx/U^2$ – dimensionless fetch
x = fetch length (in m)

The scale parameter $\alpha = c_1$, if $\bar{x} = 26\,283$, which is quite large. Therefore, in most cases $\alpha > c_1$.

The parameter γ is called the *peak-shape parameter* and it represents the ratio of the maximum spectral energy density to the maximum of the corresponding Pierson–Moskowitz spectrum. The term associated with the exponential power of γ is called *the peak enhancement factor*, and the JONSWAP spectrum is the product of the Pierson–Moskowitz spectrum (with $B = {}^5\!/\!_4\,\omega_m^4$) and the peak enhancement factor. The effect of the peak-shape parameter on the JONSWAP spectrum for wind speed $U = 30$ m/s and fetch length $x = 280$ km is shown in Figure 2. The modal frequency in this case $\omega_m = 0.509$ 1/s and the ratio $\alpha/c_1 = 1.606$, which means that the area under the original Pierson–Moskowitz spectrum for $\gamma = 1$ is increased by 60.6%. γ-value increases area under the spectrum, hence – sea severity.

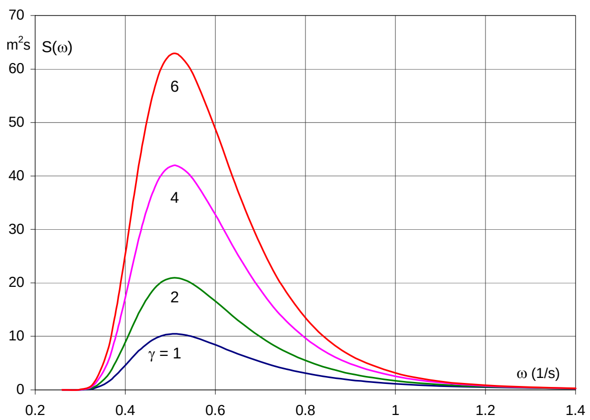

Figure 2. Effect of peak-shape parameter γ on JONSWAP spectra.

Assuming $A = \alpha g^2$, and $B = 1.25\,\omega_m^4$, the first equation in (3) yields for the area under the JONSWAP spectrum with $\gamma = 1$ the value:

$$m_0 = {}^1\!/\!_5\,\alpha g^2/\omega_m^4. \tag{11}$$

The parameter γ is actually a random variable, approximately normally distributed from 1 to 6, with mean 3.3 and variance 0.62, as shown by **Ochi** (1998). Similarly to the scale parameter α, also the peak-shape parameter γ can be presented as a function of dimensionless fetch:

$$\gamma = 7\bar{x}^{-0.142} \tag{12}$$

The JONSWAP spectral formulation, as given by equation (10), is a function of wind speed and fetch length, resulting in a spectrum of certain significant wave height, unknown beforehand, which is not very convenient. Ochi, in his book, provides a relationship between the resultant significant wave height, wind speed and fetch length, as follows:

$$U = kx^{-0.615}h_s^{1.08} \tag{13}$$

where U is in m/s, x in km, h_s in m, and k is a constant depending on γ-value. Its reasonable quadratic approximation is as follows:

$$k = 1.075\gamma^2 - 13.46\gamma + 116.41. \tag{14}$$

Ochi derived equation (13) using equation (10) for various combinations of fetch length and wind speed. With the help of equation (13) the JONSWAP spectrum can be presented now for a specified significant wave height h_s and fetch length x.

4. NONSTANDARD SPECTRA

ITTC spectra do not describe best real seas, as they are wide, with the bandwidth parameter $\varepsilon = 1$, whereas for real seas this parameter is from the range $\langle 0.40, 0.95\rangle$. This results from too slow decay of ITTC spectra for large ω; they should decay exponentially, whereas they decay as $1/\omega^5$. A question arises here, whether seas spectra could be approximated better to allow for an exponential decay, yielding all spectral moments. Before we answer this question, first we prove that ITTC spectra have geometrical affinity.

4.1 Nondimensional ITTC spectrum

Dividing the ITTC spectrum, given by equation (1), by the area $m_0 = ¼A/B$, yields a spectrum of unit area, as follows:

$$S_1(\omega) = (4B/\omega^5)\,e^{-B/\omega^4} \tag{15}$$

It is handy to introduce a new constant b in place of $B = (1/b)^4$, having the dimension of time. The above unit-area spectrum now takes the form: $S_1(\omega) = [4b/(b\omega)^5]e^{-1/(b\omega)^4}$, which can be presented shortly, as follows:

$$S_1(\omega) = bs(b\omega), \tag{16}$$

where $s(x = b\omega)$ is a unit-area nondimensional ITTC spectrum, given by:

$$s(x) = (4/x^5)\,e^{-1/x^4} \tag{17}$$

Equation (16) is a mathematical statement that between the unit-area spectrum $S_1(\omega)$ and the nondimensional spectrum there is an affinity. The scale of transformation along the ω axis is $1/b$. If $b > 1$ s the graph is diminished linearly b times along the ω axis, and increased b times along the vertical axis, to keep the area constant. When $b < 1$ s, it is the opposite – the graph is increased $1/b$ times along the ω axis, and reduced b times along the vertical axis.

The general ITTC formulation can be presented with the help of the nondimensional spectrum. Multiplying the unit-area spectrum, given by equation (16), by the area m_0, we get:

$$S(\omega) = ¼Ab^5 s(x), \tag{18}$$

where $b = B^{-1/4}$, $x = b\omega$, whereas $s(x)$ is the nondimensional (universal) ITTC spectrum, given by equation (17) and shown in Figure 3, common for all the spectra.

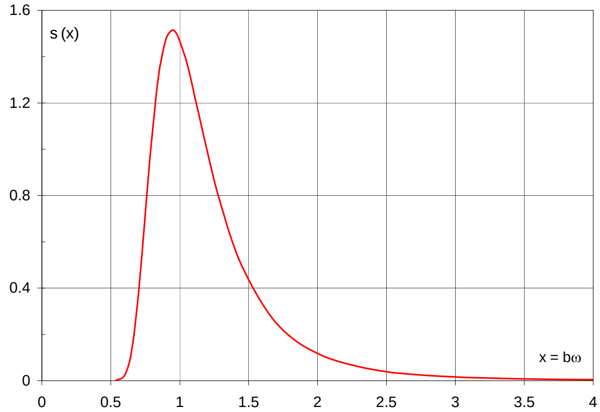

Figure 3. Nondimensional ITTC spectrum.

The largest energy density equals $s_m = 5\cdot 1.25^{1/4}e^{-5/4} \approx 1{,}515$ and occurs at the nondimensional modal frequency $x_m = 0{,}8^{1/4} \approx 0{,}946$. The spectrum begins practically at $x \approx 0.52$. The nondimensional spectrum is a generic spectrum – to obtain any ITTC spectrum, the abscissa axis is divided by $b = B^{-1/4}$, and the ordinate axis is multiplied by $¼Ab^5$. In particular, maximum of spectrum occurs at $\omega_m = (0.8B)^{1/4}$ and equals $S_m = 1{,}515\cdot ¼Ab^5 \approx 0{,}379Ab^5$.

Spectral moments of the nondimensional spectrum, denoted by s_n, are given by equations (3), in which $A = 4$, and $B = 1$. Applying substitution $x = b\omega$ in the integral $m_n = \int_0^\infty \omega^n S(\omega)\,d\omega$, for $n = 0, 1, 2$ and using for spectrum equation (18), it is easy to express spectral moments by moments of the nondimensional spectrum:

$$m_n = ¼Ab^{4-n}s_n, \qquad \text{for } n = 0, 1, 2, \ldots \tag{19}$$

It follows from equation (18) that in order to carry out calculations, it is sufficient to approximate the nondimensional spectrum $s(x)$, given by equation (17). Note that this function can be considered as if it were a probability density function since it satisfies all the conditions required for the probability density function. Hence, for the approximation any probability density function that diminishes exponentially can be used. There are many possibilities. Four of them will be discussed here: the log-normal distribution, a generalised gamma distribution, the gamma distribution and the Weibull one.

The Log-normal Distribution. Probability density is given by the equation (**DNV**, 1996):

$$f(x) = \frac{1}{\sqrt{2\pi}\,\beta(x-a)}\exp\left\{-\tfrac{1}{2}\left[\frac{\ln(x-a)-\mu}{\beta}\right]^2\right\}, \qquad (20)$$

where a, μ and β are constants to be fixed; a is a lower bound. The modal value occurs at a point $x = a + \exp(\mu - \beta^2)$. Moments of any order $n = 0, 1, 2, \ldots$ with respect to the lower bound are as follows:

$$s_n' = \exp[\tfrac{1}{2}(n\beta)^2 + n\mu]. \qquad (21)$$

To get moments with respect to the origin, they have to be suitably transformed, which is not difficult. We get these:

$$\begin{aligned} s_1 &= s_1' + a, \\ s_2 &= s_2' + 2as_1 - a^2, \\ s_3 &= s_3' + 3as_2 - 3a^2 s_1 + a^3, \\ s_4 &= s_4' + 4as_3 - 6a^2 s_2 + 4a^3 s_1 - a^4, \text{ etc.} \end{aligned} \qquad (22)$$

Knowing the nondimensional moments s_n, moments of the spectrum itself can be obtained with the help of equation (19). The coefficients in equation (22) follow the pattern of the Pascal triangle.

The best method of finding the constants a, μ and β, describing the log-normal distribution (20) is the least squares method. Minimising the sum of squared deviations between the functions $s(x)$ and $f(x)$ at the range $x < 1{,}7$ yields: $a = 0.545$, $\mu = -0.595$, $\beta = 0.566$. The differences between the two curves are at the second decimal place, i.e., around the thickness of a line, as can be seen in Figure 4.

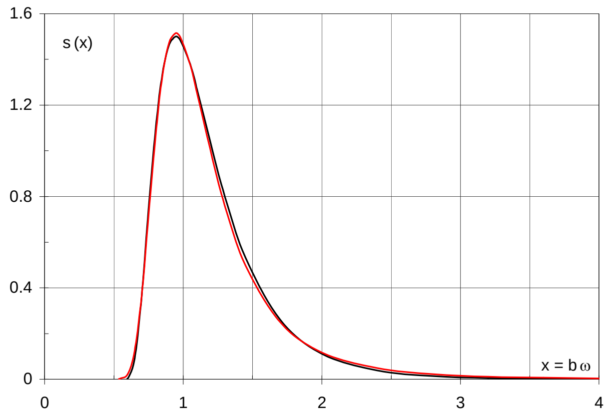

Figure 4. Nondimensional ITTC spectrum and log-normal distribution.

Generalised Gamma Distribution. Probability density function is given by (**DNV**, 1996):

$$f(x) = [c\lambda/\Gamma(\beta)][\lambda(x-a)]^{c\beta-1}e^{-[\lambda(x-a)]^c}, \qquad (23)$$

where 4 constants a, c, β and λ; are to be fixed; a is the lower bound, whereas the inverse of λ is the scale of distribution $\alpha = 1/\lambda$. The constant c governs the speed of tail decay. The modal frequency occurs at a point: $x = a + \alpha[(c\beta - 1)/c]^{1/c}$.

The spectral moments with respect to the lower bound are these:

$$s_n' = \alpha^n \Gamma(\beta + n/c)/\Gamma(\beta). \qquad (24)$$

The moments with respect to the origin are given by equations (22). The four constants, obtained with the help of the least squares method are as follows: $a = 0.6$, $c = 0.5$, $\beta = 10.6$ and $\lambda = 213$. As can be seen in Figure 5, the two curves differ insignificantly but the fit is slightly worse than in the case of the log-normal distribution.

It can be shown that, depending on the selection of the constants a, c, β and λ the generalised gamma distribution becomes a variety of different distributions, such as chi square, gamma, exponential, Rayleigh, Maxwell and Weibull, a fact which is not very well known. In other words, the above mentioned distributions are particular cases of the generalised gamma distribution, therefore they cannot get better results, which will be clearly seen below.

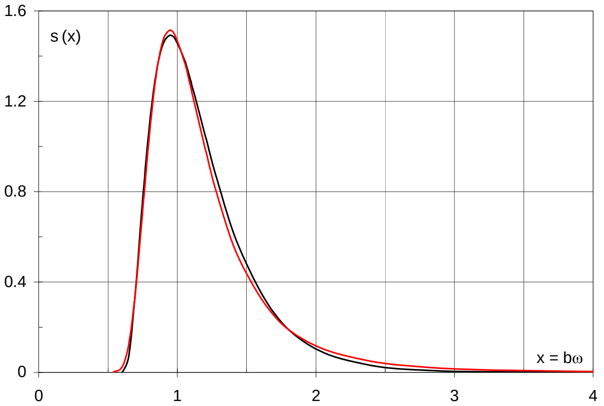

Figure 5. Nondimensional ITTC spectrum and generalised gamma distribution.

Gamma Distribution. Probability density function is given by (**DNV**, 1996):

$$f(x) = [\lambda/\Gamma(\beta)][\lambda(x-a)]^{\beta-1}\mathrm{e}^{-\lambda(x-a)}, \tag{25}$$

where three constants a, β and λ; are to be fixed; as before a is the lower bound, the inverse of λ is the scale of distribution $\alpha = 1/\lambda$, and β is called the shape parameter. The modal frequency occurs at a point $x = a + \alpha(\beta - 1)$.

The spectral moments with respect to the lower bound are as follows:

$$s_n' = \alpha^n \Gamma(\beta+n)/\Gamma(\beta). \tag{26}$$

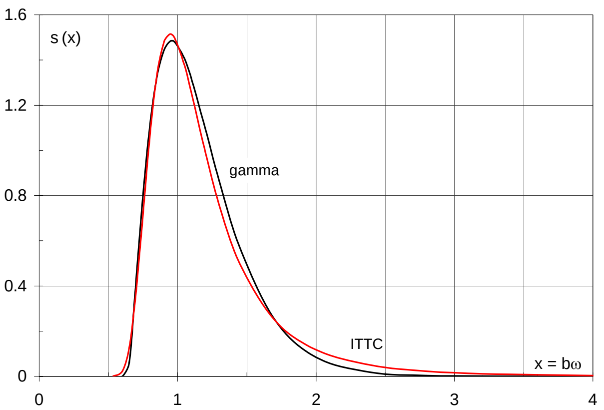

Figure 6. Nondimensional ITTC spectrum and gamma distribution.

The moments with respect to the origin are given by equations (22). The three constants, obtained with the help of the least squares method are as follows: $a = 0.632$, $\beta = 2{,}633$ and $\lambda = 5$. As expected, the gamma distribution fits somewhat worse than in the previous case, which is well seen in Figure 6.

Weibull Distribution. Probability density function is given by (**DNV**, 1996):

$$f(x) = \lambda\beta(x-a)^{\beta-1}\mathrm{e}^{-\lambda(x-a)\beta}, \tag{27}$$

where three constants a, β and λ; are to be fixed; as before a is the lower bound. The modal frequency occurs at a point $x = a + \alpha[(\beta-1)/\beta]^{1/\beta}$, where $\alpha = \lambda^{-1/\beta}$ is the scale of distribution, and β is the shape parameter.

The spectral moments with respect to the lower bound are as follows:

$$s_n' = \alpha^n \Gamma(1+n/\beta), \tag{28}$$

The moments with respect to the origin are given by equations (22). The three constants, obtained with the help of the least squares method are as follows: $a = 0{,}689$, $\beta = 1{,}567$ and $\lambda = 2{,}8$; the scale $\alpha = 0{,}518$. Both curves differ yet more than in the two previous cases, which can be seen in Figure 7. This spectrum has the smallest bandwidth parameter $\varepsilon = 0.492$, as its tail has the fastest decay.

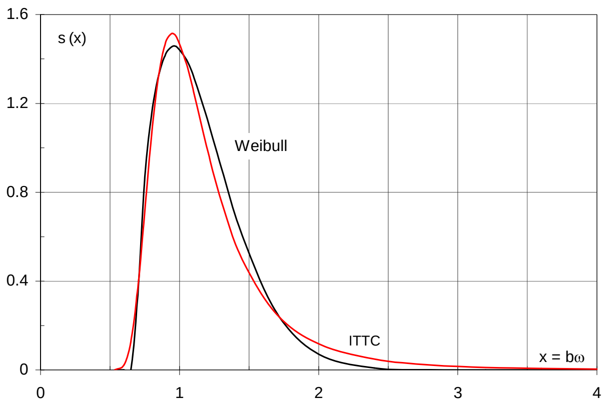

Figure 7. Nondimensional ITTC spectrum and Weibull distribution.

Statistical parameters of the discussed spectra are compiled in Table 1 according to the bandwidth parameter ε. It helps to select a spectrum according to this parameter. The constant c at the generalised gamma distribution

(23), governing the tail decay, can always be chosen in such a way to get a spectrum with a given bandwidth ε.

As can be seen from Table 1, all the approximations of ITTC spectrum have practically the same modal frequency x_m. The best approximation is the log-normal one. The mean frequency x_1 is identical with the centre of gravity of the area under the spectrum, whereas the zero-crossing frequency is the same as radius of inertia of the spectrum area. Therefore, for any spectrum the following holds $x_0 > x_1$, supported by Table 1. The nondimensional frequency is understood as $x = b\omega$, where $b = B^{-1/4}$.

Table 1. Statistical parameters for nondimensional spectra of various approximations.

	ITTC	log-normal	general. gamma	gamma	Weibull
x_m	0.946	0.945	0.947	0.959	0.960
x_0	1.331	1.257	1.232	1.203	1.194
x_1	1.225	1.192	1.177	1.159	1.155
x_2	∞	1.646	1.520	1.416	1.371
ε	1	0.646	0.586	0.528	0.492

where x_m is the nondimensional modal frequency, x_0 is the zero-crossing frequency, x_1 is the mean (characteristic, visual) frequency, and x_2 is the peak frequency.

4.2 Nondimensional JONSWAP Spectrum

The nondimensional ITTC spectrum $s(x)$, shown in Figure 3, refers to fully developed seas at open sea, whereas the JONSWAP spectrum represents wind-generated seas with fetch limitation. Contrary to ITTC spectra, for given wind speed and fetch length the resulting sea severity (in terms of the significant wave height h_s) is random, having, however, a determinate modal frequency. In literature there is no explanation provided for this randomness, which probably results from the time elapsed from the previous storm. The random sea severity is governed by the peak-shape parameter γ, of random nature, whose mean value equals 3.3.

Dividing JONSWAP spectrum, given by equation (10), by the area $m_0 = {}^1\!/_5\,\alpha g^2/\omega_m^4$ for spectrum with $\gamma = 1$, and introducing, as before, the nondimensional frequency $x = b\omega$, leads to a nondimensional JONSWAP spectrum $s^*(x)$:

$$s^*(x) = s(x)\,\gamma^{\exp\left[-(x-x_m)^2/2(\sigma x_m)^2\right]}, \tag{29}$$

where $s(x)$ is the nondimensional ITTC spectrum, given by equation (17) and shown in Figure 3, whereas x_m is the nondimensional modal frequency $x_m = 0{,}8^{1/4} \approx 0{,}946$. Graphs of $s^*(x)$, shown in Figure 8, illustrate the effect of γ-parameter on the nondimensional JONSWAP spectra. Since spectrum with $\gamma = 1$ has a unit area, other spectra, with $\gamma > 1$, have areas clearly greater than 1 (see Table 2).

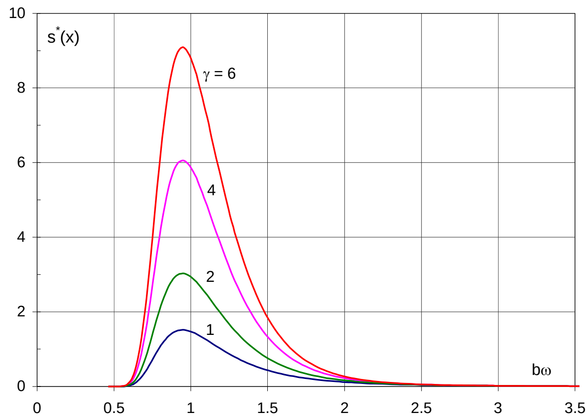

Figure 8. Nondimensional JONSWAP spectra $s^*(x)$ as a function of γ-parameter

Table 2. Area under the nondimensional spectrum $s^*(x)$ for specified γ-value.

γ	1	2	3	4	5	6
m_0^*	1	1.862	2.697	3.515	4.323	5.122

m_0^* varies almost linearly with γ-parameter. Its linear and quadratic approximations are these:

$$m_0^* = 0.198 + 0.825\gamma, \\ m_0^* = 0.1366 + 0.8755\gamma - 0.0075\gamma^2, \tag{30}$$

For the mean value of $\gamma = 3.3$, $m_0^* = 2.944$. With the help of the above quantity, the area under the JONSWAP spectrum can now be easily calculated as the product of the area for $\gamma = 1$, given by equation (11), and the quantity m_0^*:

$$m_0 = m_0^* \alpha g^2 / 5\,\omega_m{}^4. \tag{31}$$

The γ-related m_0^* factor has the meaning of a coefficient of amplification for the JONSWAP spectrum. Since $m_0 = h_s^2/16$, and substituting for α and ω_m the expressions given at equation (10), the following is obtained for sea severity:

$$h_s = 0.00102\,x \cdot \bar{x}^{-0.45} \sqrt{m_0^*}. \tag{32}$$

For fetch length $x = 280\,000$ m and wind speed $U = 30$ m/s, $\bar{x} = 3052$. Assuming $\gamma = 3.3$, the above yields for $h_s = 7.72 \cdot 2.944^{1/2}$ m $= 13.25$ m.

Solving equation (32) with respect to wind speed, yields:

$$U = 96.92\,x^{-11/18}\,m_0^{*-5/9}\,h_s^{10/9}, \tag{33}$$

where U is in m/s, x in km, h_s in m, and m_0^* is a constant depending on γ-value. Contrary to equation (13), showing some degree of approximation, equation (33) is strict.

Normalising the nondimensional spectra $s^*(x)$ with respect to the coefficient of amplification m_0^*, a unit-area nondimensional JONSWAP spectra $s_{JP}(x)$ is obtained, shown in Figure 9.

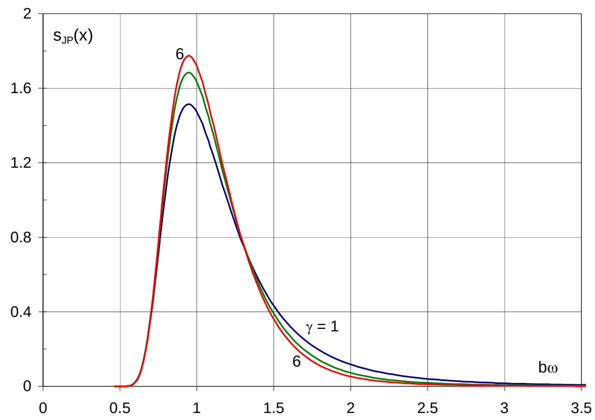

Figure 9. Normalised JONSWAP spectra as a function of γ-parameter.

The ratio between the maximum density for specified γ-value and γ = 1, denoted by λ, has the meaning of a real peak-shape parameter. As is seen in Figure 9, the greater the γ-value the greater the λ-value. Values of λ are shown in Table 3.

Table 3. λ-value for specified γ-value for the JONSWAP spectrum

γ	lg γ	λ
1	0	1
1.2	0.079	1.021
1.5	0.176	1.045
2	0.301	1.074
3	0.477	1.112
4	0.602	1.138
5	0.699	1.157
6	0.778	1.171

The above λ-value can be very well approximated relative to the logarithm of γ:

$$\lambda = 1 + 0.261\,\lg\gamma - 0.0525\,\lg^2\gamma, \tag{34}$$

Assuming that the modal density is λ times greater relative to the nondimensional ITTC spectrum and applying affinity transformation we get that the latter spectrum has to be λ times reduced along the abscissa axis with the centre of transformation at the modal frequency. The area and the modal frequency are then unchanged. The following results in such a case for the normalised JONSWAP spectra:

$$s_{JP}(x') = \lambda s[x = x_m + \lambda(x' - x_m)], \tag{35}$$

where $s(x)$ is the generic nondimensional ITTC spectrum, given by equation (17), $x = b\omega$ is the nondimensional frequency, the constant b equals $B^{-1/4} = x_m/\omega_m$, $x_m = 0{,}8^{1/4} \approx 0{,}946$, and x' is a new x after transformation. The peak-shape parameter λ governs the concentration of the spectrum around the modal value. Comparison between the normalised JONSWAP spectrum for the extreme value of $\gamma = 6$ (curve 1) and a spectrum obtained through the affinity transformation (curve 2) is shown in Figure 10. As can be seen, there are some modest differences between the two curves, particularly in regions away from the modal frequency.

A perfect approximation for the normalised JONSWAP spectra $s_{JP}(x)$ can be achieved by applying the log-normal distribution, given by equation (20). Minimising the sum of squared deviations between the functions $s_{JP}(x)$ and $f(x)$ at the range $x < 2$ yields: $a = 0.4926$, $\mu = -0.5877$, $\beta = 0.4487$. As can be seen in Figure 11, computed for the extreme value of $\gamma = 6$, differences between the two curves are invisible in the scale of this figure.

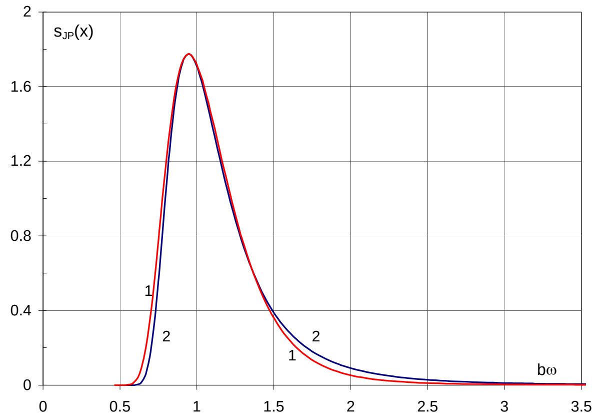

Figure 10.

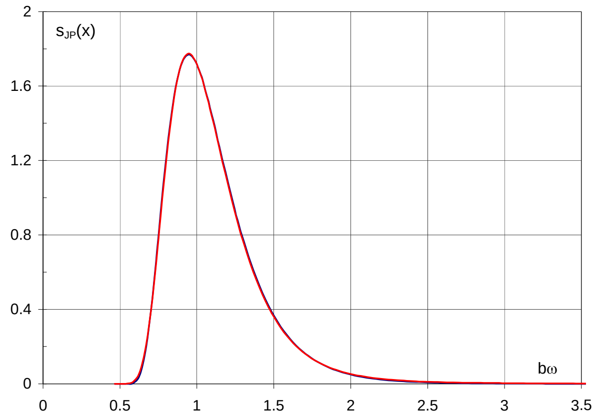

Figure 11.

5. CONCLUSIONS

The paper demonstrated that the ITTC and JONSWAP spectra can be reduced through the affinity transformation to a common unit-area nondimensional spectrum that can be precisely approximated by a log-normal distribution. Such approximated spectra, contrary to the original, are narrow-banded, with the bandwidth parameter less than 1, and have moments of any order.

6. REFERENCES

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