# EFFECT OF ROLL RESTORING LEVER CALCULATION ON PARAMETRIC ROLL PREDICTION IN A STATIONARY SEA STATE

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**ABSTRACT**

Parametric roll in a containership in stochastic head sea has been studied in the paper. The time variation of restoring force with the wave, being the most important cause for parametric roll development, expressed through the roll restoring righting arm GZ, has been evaluated at each time instant. Probabilistic predictions for parametric roll using different calculation methods for GZ righting arm evaluation have been compared in the paper.

**Keywords:** *parametric roll, instantaneous GZ righting lever, stochastic sea*

## 1. INTRODUCTION

Important for parametric roll onset is the time-variation in roll restoring moment between the two adjacent wave conditions. The simplest way to represent moment righting arm is to assume sinusoidal variation of the GZ value over time as the wave propagates along the ship. This is usually done if Mathieu equation is used to describe the parametrically excited system. Another simple procedure is to fit a large number of GZ curves calculated for different wave positions along the ship with a non-linear function of roll angle and wave crest position relative to the hull (Jensen, 2007). In some studies GZ curves have been fitted by the non-linear expression also with regard to the non-linear wave elevation as the third fitting parameter (described e.g. in Bulian et al., 2006; Spyrou et al., 2008). In addition, Umeda et al., 2004 modeled restoring moment with the captive model experiment.

In Vidic-Perunovic and Jensen (2009), the ship has been statically balanced on the free surface, taking into account the roll angle and the variable wave encounter frequency due to surge. At each time instant sectional area and centre of buoyancy have been determined for the ship balanced on the new draught and trim angle during the wave passage and for the given roll angle. The hull geometry has been described for the bare hull, without containers and superstructure.

In a number of studies the so-called Grim effective wave (originally found in Grim, 1961) has been used for modelling the GZ righting arm. In this case the excitation wave has been modelled as critical for parametric roll development since the sea surface has been substituted by the wave of the length equal to the ship length. The wave crest position is restricted to amidships, representing that way the reduced stability condition for the ship.

Furthermore, efforts have been made to allow the critical excitation wave crest to travel along the hull, the methods can be found e.g. in Kroeger (1986) or lately in Bulian (2008). This has not been accounted for in the present paper.

In this study, the influence of GZ calculation on the simulated roll response in stochastic sea has been investigated. The presented roll model is well suited for roll predictions in longitudinal long-crested sea. In the present paper the container ship in head sea condition has been studied sailing with the constant speed. Different calculation methods for GZ righting arm (methods are described in the paper) have been included in the roll model. Probabilistic analysis has been conducted by use of each method in order to evaluate reliability index. For this purpose First Order Reliability Method (FORM) has been linked to hydrodynamic routine for roll calculation. Results have been compared and discussed.

## 2. METHODOLOGY

### 2.1 Roll Motion Model

The hydrodynamic routine for the extreme roll prediction is based on 1 DOF model. The procedure is simplified but similar to ROLLS (Krüger et al., 2004) and explained in Jensen (2007). The vertical motions are taken to be linear functions of the wave elevation The sway and yaw motions are ignored as the vertical motions have the largest influence on the instantaneous *GZ* curve.

The equilibrium equation for roll $\phi$ reads, with a dot signifying time derivative:

$$\ddot{\phi} = -2\beta_1\omega_\phi\dot{\phi} - \beta_2\dot{\phi}\left|\dot{\phi}\right| - \frac{\beta_3\dot{\phi}^3}{\omega_\phi} - \frac{\left(g-\ddot{w}\right)GZ(\phi)}{r_x^2} \quad (1)$$

where $r_x$ is the roll radius of gyration and $g$ the acceleration of gravity, $\ddot{w}$ is the vertical acceleration at c.o.g. due to heave and pitch. Roll motion is therefore dynamically uncoupled to vertical ship motions that have been only taken in evaluation of roll restoring. The roll frequency $\omega_\phi$ is given by the metacentric height $GM_{sw}$ in still water:

$$\omega_\phi = \frac{\sqrt{gGM_{sw}}}{r_x} \quad (2)$$

The damping is modeled by a standard combination of a linear, a quadratic and a cubic variation in roll velocity. Vertical motions are determined as the frequency dependent transfer functions by closed form expressions (Jensen et al., 2004). The head sea condition and the constant ship speed have been studied in the following. Stationary sea conditions are assumed and specified by a JONSWAP wave spectrum with significant wave height $H_s$ and zero-crossing period $T_z$.

### 2.2 Statistical Analysis – First Order Reliability Method

The linear wave elevation is taken to be normal distributed and can be written as function of space $X$ and time $t$

$$H(X,t) = \sum_{i=1}^{n}\left(u_i c_i(X,t) + \bar{u}_i \bar{c}_i(X,t)\right) \quad (3)$$

where $u_i, \bar{u}_i$ are standard normal distributed and uncorrelated variables to be determined by the stochastic procedure and with the deterministic coefficients given by

$$\begin{aligned} c_i(x,t) &= \sigma_i \cos(\omega_i t - k_i X) \\ \bar{c}_i(x,t) &= -\sigma_i \sin(\omega_i t - k_i X) \\ \sigma_i^2 &= S(\omega_i)d\omega_i \end{aligned} \quad (4)$$

where $\omega_i$, $k_i = \omega_i^2 / g$ are the $n$ discrete wave frequencies and wave numbers, $S(\omega)$ is the wave spectrum and $d\omega_i$ is the wave frequency increment. From the wave elevation and the associated wave kinematics, any non-linear wave-induced response $\phi(t)$ of a marine structure can in principle be determined by a time domain analysis using a proper hydrodynamic model:

$$\phi = \phi(t\left|u_1, \bar{u}_1, u_2, \bar{u}_2, ..., u_n, \bar{u}_n, \text{ initial conditions}\right.) \quad (5)$$

Each of these realisations represents the response for a possible wave scenario. The realisation which exceeds a given threshold $\phi_0$ at time $t{=}t_0$ with the highest probability is sought. This problem can be formulated as a limit state problem, well-known within timeinvariant reliability theory (Der Kiureghian, 2000):

$$
\begin{aligned}
&g(u_1, \bar{u}_1, u_2, \bar{u}_2, \dots, u_n, \bar{u}_n) \equiv \\
&\phi_0 - \phi(t_0 \,|u_1, \bar{u}_1, u_2, \bar{u}_2, \dots, u_n, \bar{u}_n) = 0
\end{aligned}
\tag{6}
$$

An approximate solution can be obtained by use of First Order Reliability Method (FORM). The determination of the design point $\{u_i^*, \bar{u}_i^*\}$ defined as the point on the failure surface $g=0$ with the shortest distance to the origin and the associated distance $\beta_{FORM}$ i.e. the reliability (or safety) index:

$$
\beta_{FORM} = \min \sqrt{\sum_{i=1}^{n} \left( u_i^2 + \bar{u}_i^2 \right)} \tag{7}
$$

can be performed by standard reliability codes (e.g. Det Norske Veritas, 2003). The critical wave episode is described as:

$$
H^*(X,t) = \sum_{i=1}^{n} \left( u_i^* c_i(X,t) + \bar{u}_i^* \bar{c}_i(X,t) \right) \tag{8}
$$

It is the wave scenario with the highest probability of occurrence that leads to the exceedance of the specified response level $\phi_0$.

## 3. NUMERICAL RESULTS

### 3.1 The Ship

An example containership has been analyzed with following main particulars:

*Lpp*=202m; *D*=10.1m; *B*=32.2m, where *Lpp* stands for the ship length between perpendiculars, *D* is the design draught and *B* is the breadth on the waterline. Sections used in the calculation are given in Fig.1.

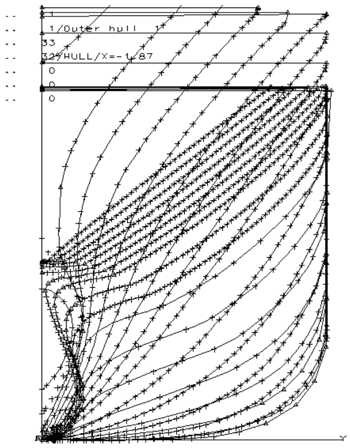


Figure 1. Containership body plan.

### 3.2 GZ righting arm

Very important for the onset of parametric roll is the time variation of the roll restoring moment represented by the GZ righting arm variation with the wave. In the following three different types of calculations have been employed in the analysis and compared. They are denoted:

Method 1 – the ship underwater geometry has been calculated instantaneously taking care of the ship vertical motion and the wave position along the hull at each time step. The equations to determine instantaneous GZ righting arm in details can be found in Vidic-Perunovic and Jensen (2009).

Method 2 - the GZ curve has been approximated by the fifth order polynomial and the sin term, as a function of the heel angle and wave crest position along the ship measured from the aft perpendicular. The GZ curve in waves is then linearly scaled for different instantaneous wave heights. In a stochastic seaway the following approximation of the instantaneous value of the righting arm $GZ(t)$ is then applied

$$
\begin{aligned}
GZ(\phi,t) = {} & GZ_{sw}(\phi) + \\
& \frac{h(t)}{0.05L}\left( GZ(\phi, x_c(t)) - GZ_{sw}(\phi) \right)
\end{aligned}
\tag{9}
$$

where $GZ_{sw}$ signifies the righting arm in still water. More details and the full derivation can be found in the original paper by Jensen (2007).

Method 3 - The effective wave concept originally suggested by Grim (1961) has been used by Bulian et al. (2006) in GZ calculation, where the sea surface has been substituted by the wave of the length equal to the ship length. The wave crest position is restricted to amidships. The GZ curve has been given as a function of the non-linear heel angle and the non-linear effective wave elevation (expression given in Bulian et al., 2006), here, however, the GZ curve for the present will be fitted linearly with the wave elevation, in the same way as applied in Method 2.

### 3.3 Stochastic Sea Analysis

The zero-crossing period is chosen such that parametric roll can be expected due to occurrence of encounter frequencies in the range of twice the roll frequency. Vertical motion heave and pitch used in Eq. (1) have been calculated using closed form expressions (Jensen, 2004). Time domain simulations are carried out from t=0 to t=180s using 25 equidistantly distributed frequencies. In the following, results for the design point i.e. the most probable scenario, corresponding to a roll response of 0.5 rad (=28.7 deg) are shown (Fig. 2, Fig. 3, Fig. 4).

The basic physics of parametric roll of the ship is reflected in the simulated results. The critical wave episode amidships that excites the roll response of 0.5 rad obtained by use of method 1 (given in upper left Fig. 2 (a)), the corresponding GZ righting arm (given in upper right Fig. 2 (b)) calculated by use of exact GZ calculation and the most probable roll response yielding the conditioned value of roll at time t=180s (lower Fig. 2 (c)). The response has been zoomed in Fig. 3.

Just after the wave trough has been encountered by the midship position the ship rolls back to her initial stable position and roll angle is close to zero (at about t=163s). At the same time instant the GZ righting arm is close to zero. When the wave crest is encountered by the midship position (t=166s) the roll angle has almost reached its maximum to one side. As the time passes and the wave trough approaches the midship position, the ship starts rolling to another side and GZ righting arm increases. The maximum roll angle is reached in the time interval between the wave crest and wave trough positioned amidships (t=169.5s). For the wave trough positioned amidships (at about t=172s) the GZ value reaches its amplitude whereas the ship continues rolling towards another side. This complies very well to the development of parametric roll explained in Shin et al. (2004).

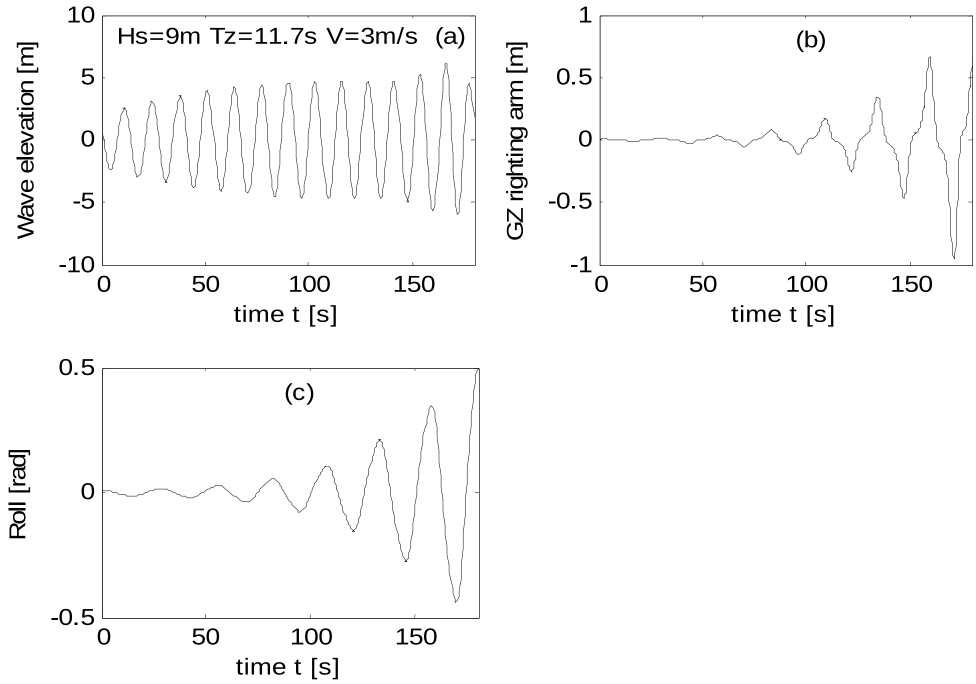


Figure 2. Critical wave episode amidships (a). Corresponding GZ righting arm (b). Most probable roll when $\phi_0$=0.5rad, head sea condition. Results have been obtained by use of method 1.

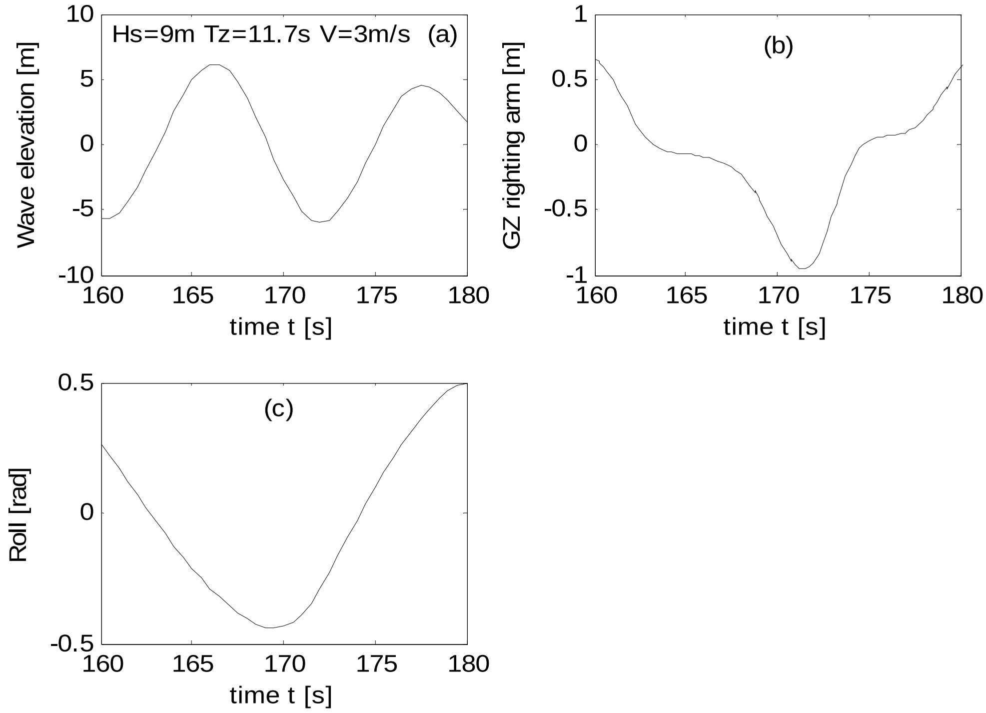


Figure 3. A Fig. 2, zoomed in the last 20 sec.

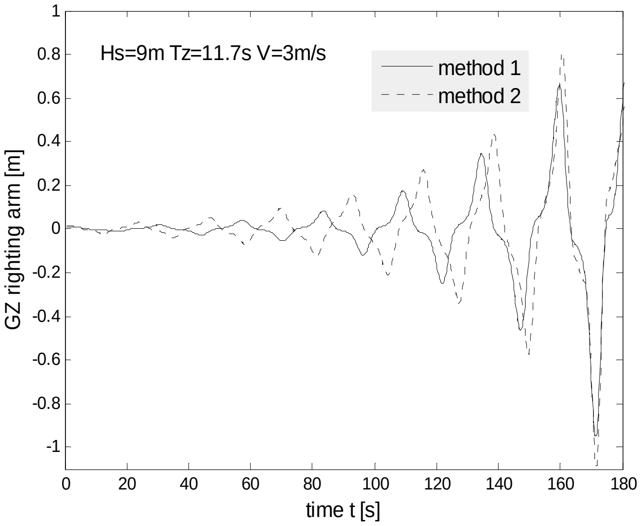


Figure 4. GZ righting arm yielding max roll $\phi_0$=0.5 rad by exact calculation (full line, method 1) and fitted by 5$^{th}$ order polynomial plus the sinus term (dashed line, method 2).

The GZ righting arm follows the roll angle in time. The excitation wave period is about twice smaller than the roll period of the hull.

In Fig. 4 the two responses have been compared: The most probable GZ righting arm for the conditioned roll angle calculated by use of exact GZ calculation (full line, method 1) and GZ fitted to the calculation by the fifth order polynomial plus the sinus term (dashed line, method 2), for the excitation sea state $H_s$=9m $T_z$=11.7s V=3m/s. The amplitude of the simulated GZ curve is larger when using method 2, which implies that the probability for parametric roll occurrence increases in this case. Namely, the safety index has been calculated smaller in the probabilistic analysis using method 2 (Fig. 5).

As the result of the probability analysis, safety index by the three different methods for GZ calculation has been compared in Fig.5 for maximum conditioned roll angle taking values from 0.3-0.5 rad. Method 1 (given by circles) stands for the exact calculation of GZ righting arm evaluated at each time step (Vidic-Perunovic and Jensen, 2009). Method 2 (given by stars) stands for GZ fitted to the calculation by 5$^{th}$ order nonlinear polynomial (Jensen, 2007). Method 3 (given by diamonds) accounts for GZ evaluated at the effective Grim wave with the crest fixed amidships at all times (Grim, 1961; Bulian et al., 2006). Results have been generated for four different significant wave heights ($H_s$=7m, 9m, 11m and 14m), zero up-crossing period $T_z$=11.7s and constant ship speed V=3m/s.

Expectedly, the shortest distance to the failure surface has been obtained by the Method 3 by which the smallest values for reliability index have been calculated, whereas predictions by use of Method 1 and Method 2 are very close for different roll angles. The accurate GZ calculation represents the least conservative method of the three, but the calculation time is much longer than utilizing Method 2. Further, results for the reliability index by use of the Method 2 have been compared to the predictions by Monte Carlo analysis in Table 1, for the sea state $H_s$=14m, $T_z$=11.7s and the ship speed V=3m/s. Good agreement is clearly seen from the results.

## 4 CONCLUSION

Susceptibility of a containership to parametric roll has been evaluated in stochastic sea. The present analysis has been limited to constant ship speed and head sea operational condition. Different methods for GZ righting arm calculation have been employed in order to obtain roll response. The value of reliability index has been calculated for the several significant wave heights and the zero up-crossing period and the ship speed relevant for parametric roll occurrence. First order reliability method (FORM) has been used to determine the design point for different maximum conditional roll angles. Following conclusion can be drawn from the presented study:

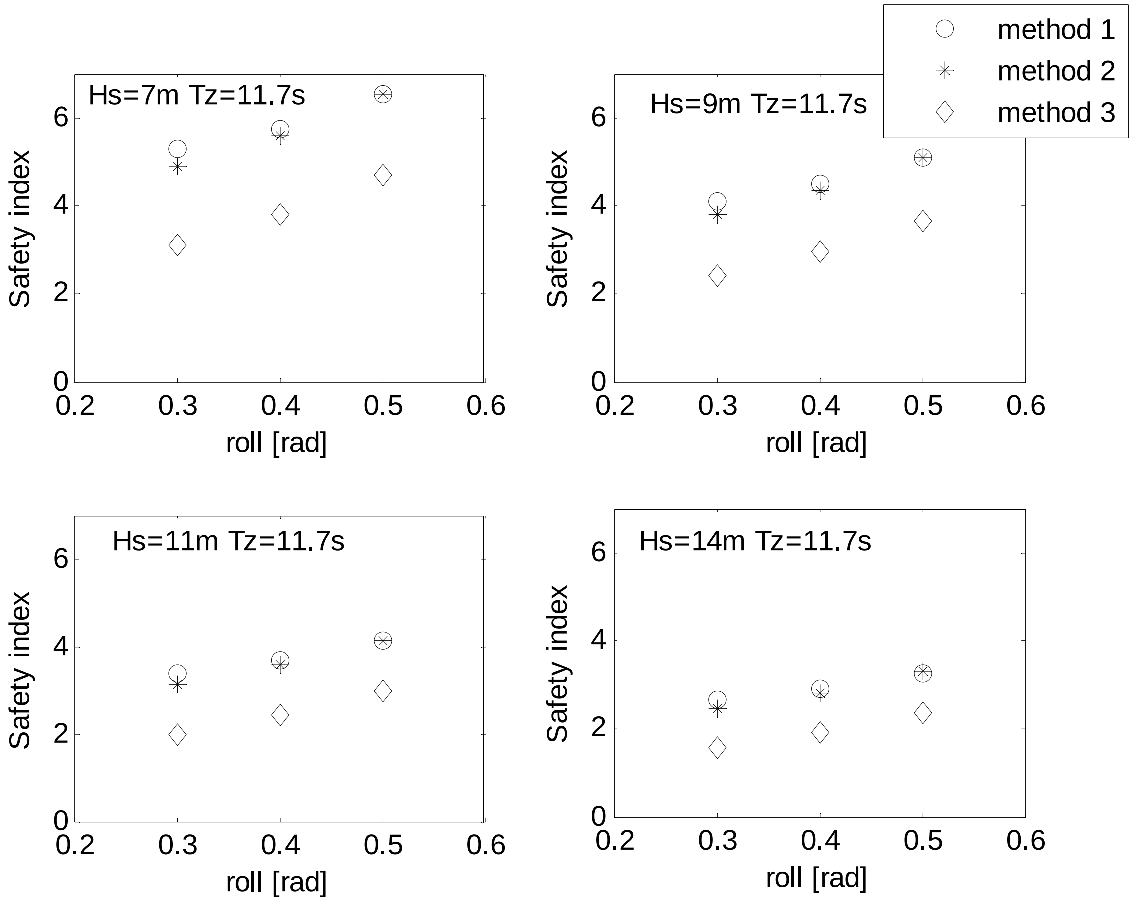


Figure 5. Safety index calculated for head sea in different sea states by exact GZ calculation (circles, method 1), GZ fitted by 5th order polynomial plus the sinus term (stars, method 2), GZ by use of the fixed Grim effective wave (diamonds, method 3).

Table 1. Reliability index by Method 2 and Monte Carlo analysis for the sea state $H_s$=14m $T_z$=11.7s and the ship speed V=3m/s

| Roll [rad] | Reliability index $\beta$ by Method 2 | Reliability index $\beta$ by Monte Carlo analysis 90% confidence interval |
|---|---|---|
| 0.3 | 2.4673 | 2.4099 - 2.5743 |
| 0.4 | 2.8285 | 2.7194 - 2.9719 |
| 0.5 | 3.3041 | 3.2247 - 3.6202 |

Reliability index against parametric roll has been predicted the smallest by Method 3 that makes use of the Grim effective wave with the wave crest fixed amidships for the GZ calculation.

Predictions by Method 2, in which the GZ arm has been fitted to the calculation by the 5th order polynomial plus the sinus term, are close to the predictions by Method 1 where the GZ has been evaluated at each time step using exact underwater hull geometry.

It is clear that Method 1 is the most time consuming due to the detailed ship hydrostatics and GZ calculation during the wave passage. This can be the only drawback when running probabilistic analysis.

## 5 ACKNOWLEDGEMENTS

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