Dynamic Instability of Taut Mooring Lines Subjected to Bi-frequency Parametric Excitation

Aijun Wang, *University of Strathclyde*

Hezhen Yang , *State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University*

Nigel Barltrop, *University of Strathclyde*

Shan Huang, *University of Strathclyde*

ABSTRACT

Parametric excitation or parametric resonance occurs when the offshore structure system parameter varies with time and meets a certain condition. Moored structure during the swell sea states causes the taut mooring line tension fluctuation which may induce very large dynamic motion of mooring lines. In this work, the taut mooring lines subjected to bi-frequency parametric excitation are studied. The parametric excitation equation of mooring lines is derived. The Bubnov-Galerkin approach is employed to obtain stability chart when consider the bi-frequency excitation. The responses of the mooring lines subjected to single- and bi-frequency excitation are discussed.

Keywords: *dynamic instability; mooring line; parametric excitation；Mathieu; bi-frequency*

1. INTRODUCTION

The dynamic instability is the oscillatory motion of dynamic system due to time-dependent variation of structure parameters e.g. inertia or stiffness due to the influence of externally applied force. Different the forced excitation, the parametric excitation is nonautonomous system. It can have catastrophic effects on the structures. The study indicated parametric excitation would cause very large increase in lateral dynamic motion of mooring lines due to the variation of axial tension. (Rönnquist et al., 2010). It is beneficial to avoid the mooring line design to locate in the unstable zone. Recently, some researchers have studied the dynamic instability due to parametric excitation such as parametric rolling of ships, spar and risers (Falzarano et. al 2003, Yang et al. 2015, and Zhang et. al 2010). However, previous work all focused on the single frequency excitation. Actually, the offshore structure is exposed to random waves which are multi-frequency excitation. So, it is necessary and beneficial to study the taut mooring lines subjected to bi-frequency parametric excitation.

2. THEORY AND MATHEMATICAL MODEL

The general dynamic equation of Bernoulli-Euler beam can be expressed as follow.

$$EI\frac{\partial^4 y}{\partial x^4} - T\frac{\partial^2 y}{\partial x^2} + m\frac{\partial^2 y}{\partial t^2} = f(x,t) \qquad (1)$$

Where EI is the bending stiffness, T is the axial tension, m is mass per unit length, $f(x,t)$ is external force on the beam.

For the mooring lines, the bending stiffness often can be neglected and the axial tension can be expressed by

$$T = T_0 + T_A\phi(t) \qquad (2)$$

Where $\phi(t) = \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \theta_n)$.

T_0 is the mean tension of the line.

T_A is the amplitude of lines dynamic tension.

ω_n is the tension variation frequency of the line.

θ_n is the random phase.

The hydrodynamic on the lines are calculated by Morison equation. It is nonlinear and can be linearized as the follows.

$$f(x,t) = -\frac{1}{2}\rho D C_D \left|\frac{\partial y}{\partial t}\right| \frac{\partial y}{\partial t} = -\sqrt{\frac{2}{\pi}} C_D \rho D \sigma \frac{\partial y}{\partial t} \quad (3)$$

Combing equations (1)-(3) leads to

$$m\frac{\partial^2 y}{\partial t^2} + \sqrt{\frac{2}{\pi}} C_D \rho D \sigma \frac{\partial y}{\partial t} - \left(T_0 + T_A \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \theta_n)\right)\frac{\partial^2 y}{\partial x^2} = 0 \quad (4)$$

Assumed that the ends are pinned, the lateral motion of lines can be written as

$$y(x,t) = \sum_{n=1}^{\infty} y_n(t) \sin\frac{n\pi x}{l} \quad (5)$$

Submit Eq. (2) into Eq. (1), it follows that

$$\sum_{n=1}^{\infty}[m\frac{d^2 y_n(t)}{dt^2} + \sqrt{\frac{2}{\pi}} C_D \rho D \sigma \frac{dy_n(t)}{dt} + \left(T_0 + T_A \sum_{n=0}^{\infty} A_n \cos(\omega_n t + \theta_n)\right)(\frac{n\pi}{l})^2 y_n(t)]\sin\frac{n\pi x}{l} = 0 \quad (6)$$

The above equation can be rewritten into the general form,

$$\frac{d^2 y_n(\tau)}{d\tau^2} + 2c_n \frac{dy_n(\tau)}{d\tau} + \left(a_n + 2q_n \sum_{n=0}^{\infty} A_n \cos(k\tau + \theta_n)\right) y_n(\tau) = 0 \quad (7)$$

where $\tau = \omega_0 t$, ω_0 is the basic frequency.

$k = \frac{\omega_n}{\omega_0}$, $a_n = (\frac{\bar{\omega}_n}{\omega_0})^2$, $q_n = \frac{1}{2}\frac{T_A}{T_0}(\frac{\bar{\omega}_n}{\omega_0})^2$,

$c_n = \sqrt{\frac{1}{2\pi}} \frac{C_D \rho \mathrm{D} \sigma}{m\omega_0}$.

$\bar{\omega}_n = \frac{n\pi}{l}\sqrt{\frac{T_0}{m}}$ n=1, 2, $\cdots$

$\bar{\omega}_n$ is the natural frequency of the line.

This is the second order homogenous equation which named Hill equation.

3. SINGLE-FREQUENCY EXCITATION CASE

In this section, only single-frequency excitation is taken into account, and then the Eq. (7) can be expressed as follow. It is the well-known Mathieu equation.

$$\ddot{y} + 2c\dot{y} + \left(a + 2q\cos(2\tau)\right) y = 0 \quad (8)$$

For the single-frequency excitation, the top end motion of mooring lines corresponds to the floating structure during the regular sea.

3.1 Stability chart of single-frequency excitation

The stability chart is often used to identify the unstable and stable zones of dynamic instability due to parametric excitation. The stability chart can show the change instability as the parameters are varied and are very useful for the design guidance. It can be solved by Floquet theory or perturbation method. The stability chart of single-frequency excitation for the taut mooring lines is shown in the Fig.1. It can be seen that the unstable zone will shrink as the damping increases. The resonance will occur when excitation is integral or sub multiple of fundamental frequency.

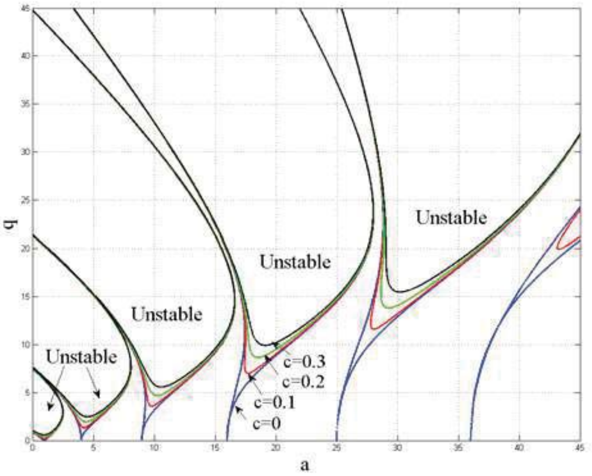
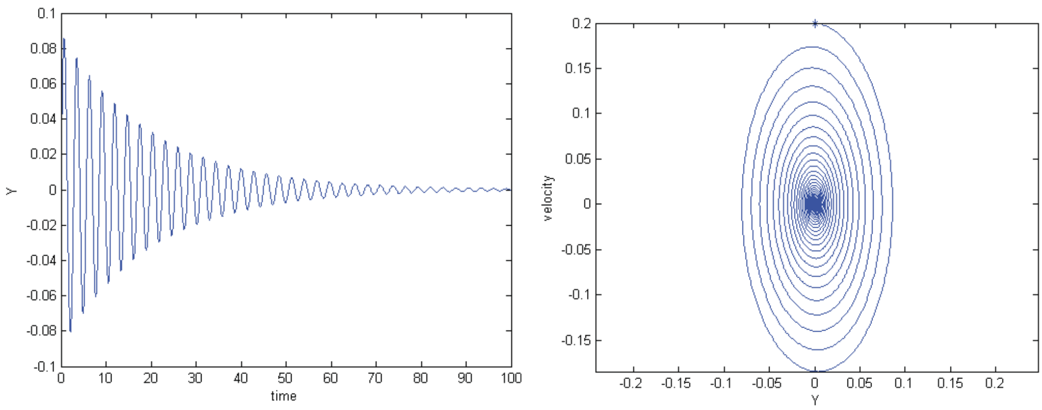


Fig.1 Stability chart of single-frequency excitation
(Blue line--c=0; red line--c=0.1; green line--c=0.2; black line--c=0.3)

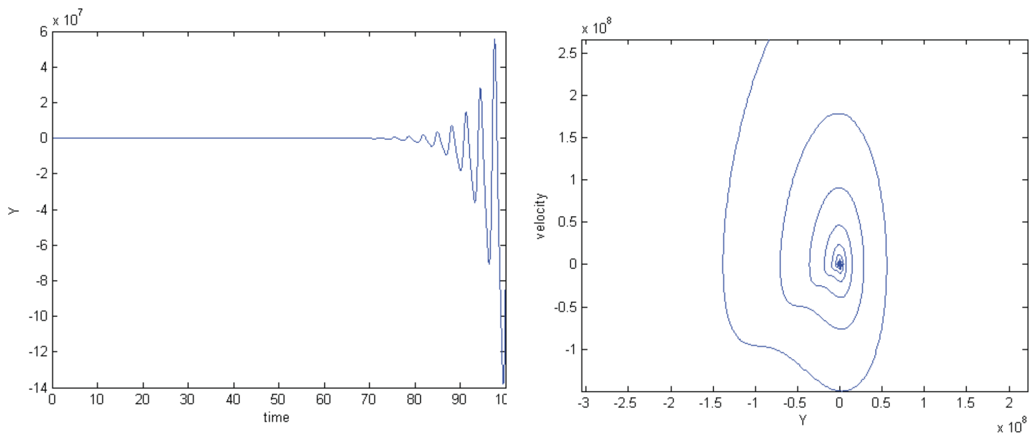
3.2 Dynamic response due to single-frequency excitation in time domain

Fig.2 present two case for the lateral dynamic response at the midpoint of the mooring line (Case I locate at stable zone and case II locate at unstable zone). The direction of response is orthogonal to the direction of the excitation. For the case II, It can be seen that the response will increase exponentially. This case should be removed for the design.



Response Phase plane trajectory

(Case I)



Response Phase plane trajectory

(Case II)

Fig.2 Dynamic response due to single-frequency excitation in time domain
(Case I: a=5; q=0.04; c=0.1; Case II: a=5; q=2.5; c=0.1)

4. BI-FREQUENCY EXCITATION CASE

For real sea conditions, the top end of mooring line is subjected to multi-frequency excitation from floating motions in the random waves. The parametric instability property of mooring line has great effects on the safety of the design case and it can cause the lateral motion exponential increase in the oscillation. Here, the parametric instability due to bi-frequency excitation is studied which is conducive to understand the mechanism of the dynamic instability.

$$\ddot{y} + 2c\dot{y} + \left(a + 2q(\cos(2\tau) + \mathrm{d}\cos(4\tau))\right) y = 0 \quad (9)$$

The dynamic instability for the bi-frequency excitation is obtained by the Bubnov-Galerkin approach (Perdesen, 1980). Fig.3 shows the stability chart for the bi-frequency excitation. It can be seen that the unstable zone for the bi-frequency excitation is obviously different from the single-frequency excitation.

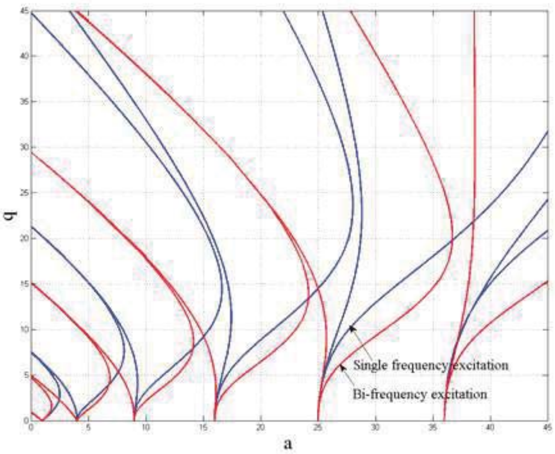


Fig.3 Stability chart for single- and bi-frequency excitation (d=-0.5)
(Blue line- single-frequency excitation; red line- bi-frequency excitation)

4.1 Effects of different d

Fig.4 and 5 give the stability charts for different d =0.5 and -0.5 respectively. The sign of d mean the different phase between the two excitations. The unstable zone changes when d turns into negative and the shape is also different. It is interesting to find that the closed zone exists.

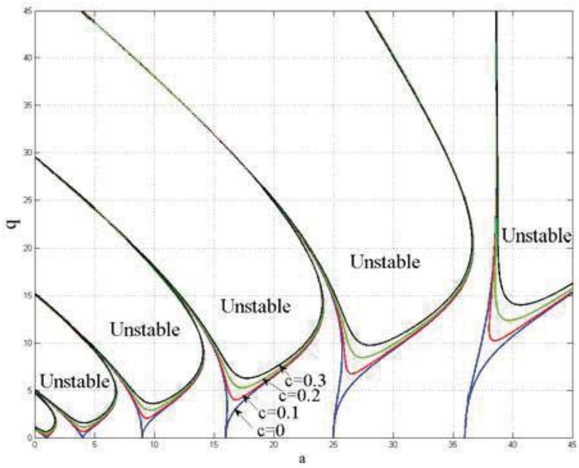


Fig.4 Stability chart for bi-frequency excitation for different damping (d=-0.5)
(Blue line--c=0; red line--c=0.1; green line--c=0.2; black line--c=0.3)

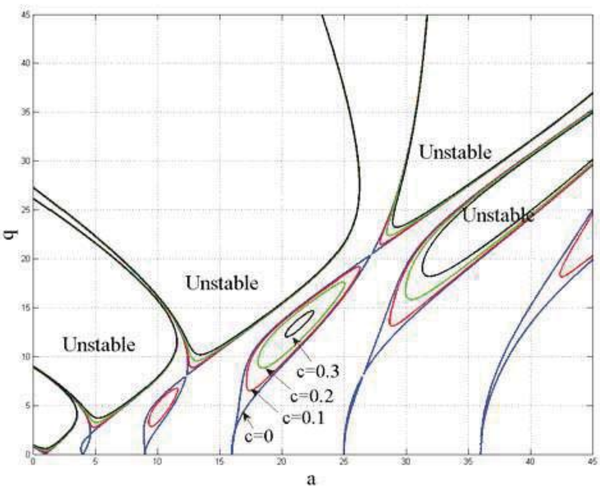


Fig.5 Stability chart for bi-frequency excitation for different damping (d=0.5)
(Blue line--c=0; red line--c=0.1; green line--c=0.2; black line--c=0.3)

4.2 Effects of different damping for positive d

The effects of different damping for positive d are compared in the Fig.6-8. The damping varies from 0 to 0.3. It can be seen that the unstable zone of bi-frequency excitation is more sensitive than the single-frequency excitation. The unstable zone changes more as the damping varies.

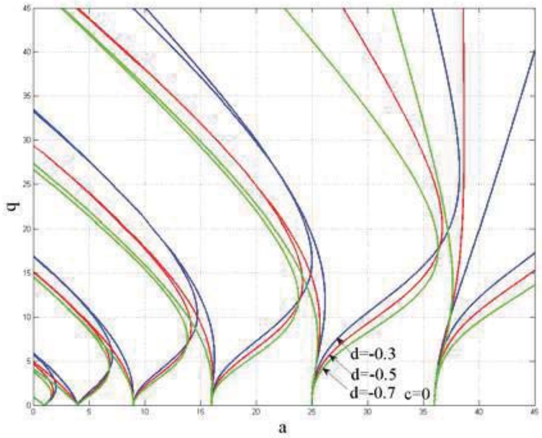


Fig.6 Stability chart for bi-frequency excitation for different damping (c=0.0)
(Blue line--d=-0.3; red line--d=-0.5; green line--d=-0.7)

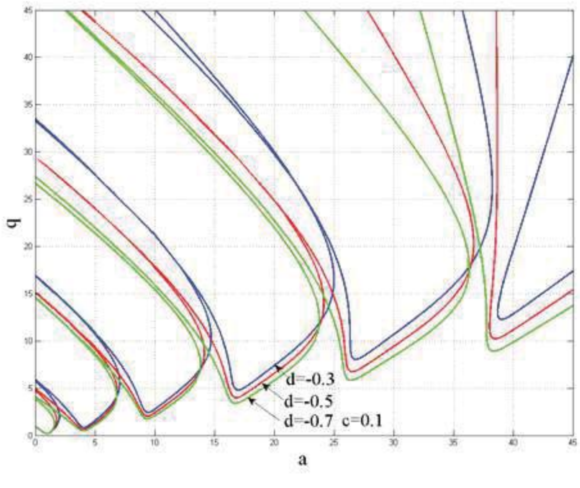


Fig.7 Stability chart for bi-frequency excitation for different d (c=0.1)
(Blue line--d=-0.3; red line--d=-0.5; green line--d=-0.7)

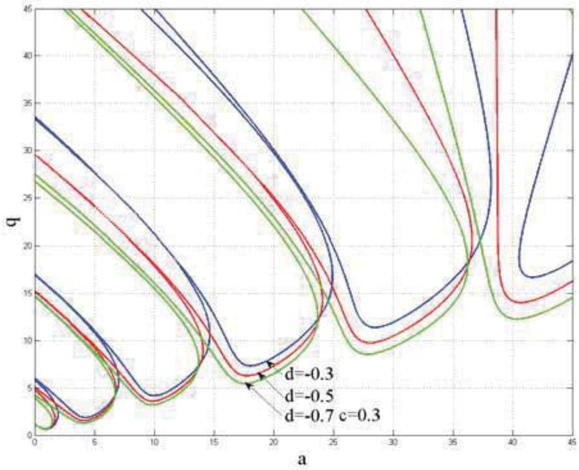


Fig.8 Stability chart for bi-frequency excitation for different d (c=0.3)
(Blue line--d=-0.3; red line--d=-0.5; green line--d=-0.7)

4.3 Effects of different damping for negative d

Fig.9-11 present the effects of different damping for negative d on the dynamic instability zone. The damping varies from 0 to 0.3. It can be seen that the unstable zone of bi-frequency excitation is completely different from the single-frequency excitation. The unstable zone changes more as the damping varies. The safety case will turn into unsafely when damping varies or the sign of d changes.

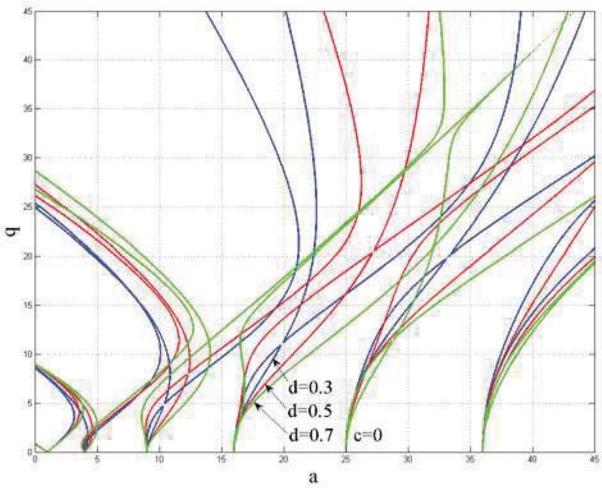


Fig.9 Stability chart for bi-frequency excitation for different d (c=0.0)

(Blue line--d=0.3; red line--d=0.5; green line--d=0.7)

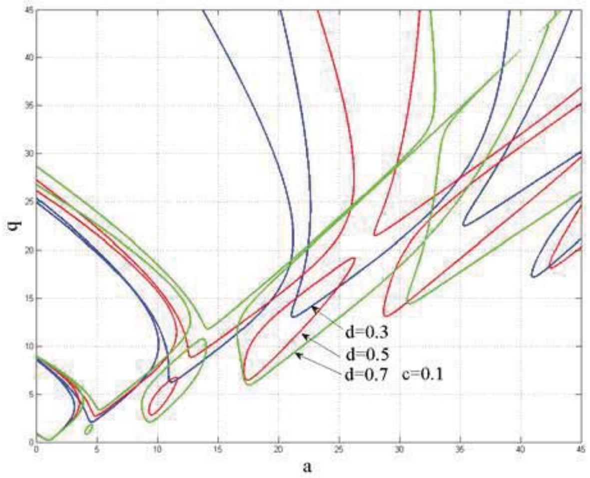


Fig.10 Stability chart for bi-frequency excitation for different d (c=0.1)
(Blue line--d=0.3; red line--d=0.5; green line--d=0.7)

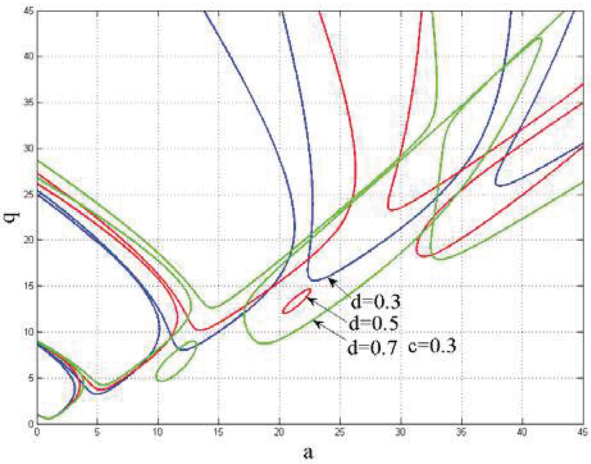


Fig.11 Stability chart for bi-frequency excitation for different d (c=0.3)
(Blue line--d=0.3; red line--d=0.5; green line--d=0.7)

5. CONCLUSIONS

In this work, the taut mooring lines subjected to single and bi-frequency parametric excitation were studied. The responses of the mooring lines subjected to single- and bi-frequency excitation were discussed. The unstable zone of bi-frequency excitation is obviously different from the single-frequency excitation. The safety case in the single-frequency excitation may become unsafely in the bi-frequency excitation. The effects of different parameters on the stability chart were discussed. The results indicate that multi-frequency should be given consideration in a more accurate prediction of parametric excitation.

ACKNOWLEDGEMENTS

This work was financially supported by the National Natural Science Foundation of China (Grant No. 51379005).

6. REFERENCES

Rönnquist, A. Remseth, S. and Udahl, G. 2010. Unstable Non-Linear Dynamic Response Investigation of Submerged Tunnel Taut Mooring Elements Due to Parametric Excitation. 23rd Nordic Seminar on Computational Mechanics.

Jeffrey Falzarano, Jun Cheng, and Samrat Das. 2003. Parametric excitation of floating offshore platforms. 8th International Conference on the Stability of Ships and Ocean Vehicles.

Yang H.Z., Xu P. , 2015. Effect of hull geometry on parametric resonances of spar in irregular waves. Ocean Engineering. 99:14-22.

Libang Zhang, Ayman Eltaher, Paul Jukes. 2010. Mathieu instability of TTR due to VIV. Proceedings of the 5th International Offshore Pipeline Forum.

Pedersen, P. 1980. Stability of the solutions to Mathieu-Hill equations with damping. Ingenieur-Archiv, 149 (1),15-29.