

INTERNATIONAL CONFERENCE
on
STABILITY
OF
SHIPS AND OCEAN VEHICLES

25 - 27 MARCH 1975

AT

University of Strathclyde
Glasgow, Scotland

Organised by:

Department of Shipbuilding & Naval Architecture, University of Strathclyde

In collaboration with Department of Industry and Department of Trade.

INTERNATIONAL CONFERENCE

on

STABILITY OF SHIPS AND OCEAN VEHICLES

25 - 27th MARCH 1975

at

K325 of John Anderson Building,
University of Strathclyde,
Glasgow.

Organiser: PROFESSOR C. KUO
Secretary: MISS J. CHAMBERS

ABOUT THE CONFERENCE

Ship stability and avoidance of capsizing have always been of utmost importance to seafarers and to naval architects. In simple terms, stability determines the safety of life at sea. Impressive progress has been made recently towards rational understanding and formulation of the stability concept. It has become ever more necessary to study the stability of ships and ocean vehicles in actual operational conditions on the seaways. This requires not merely a profound knowledge of naval architecture, operational conditions, advanced mathematics, hydrodynamics and statistics, but a proper physical understanding of all the phenomena involved.

The field of interest is so broad that research effort, and the adaptation that follows, has been devoted only to limited aspects of the overall problem. But each study undertaken has attempted to produce a better understanding of specific aspects of the general stability problem. In the light of recent developments, it has become essential to establish a means of communication between those actively involved in research and those concerned with the application of research findings.

The aim of this Conference is therefore twofold: a) to provide an opportunity for those involved in stability work, whether for design, operation, research or regulatory purposes to discuss the available research findings at international level; and b) to see how these results can be applied in practice.

The Conference programme will be divided into five separate sections to facilitate comprehensive discussion. Section 1 will consider the Definitions of Stability which will strongly influence the future direction of research. Sections 2 - 5 will cover: Environmental Conditions and Experiments; Theoretical Studies; Correlation of Theory and Experiment; and Application of Research Findings. The final programme item will be a "Quiz Session on Stability" in which Conference participants will have an opportunity to question directly a panel of experts from shipbuilding, shipping, regulatory bodies, research and education.

As the emphasis of the Conference is to encourage wide exchange of ideas, it is intended that the presentations of technical papers will be brief to allow maximum time for general discussion. It is hoped that the direction of future research and development efforts will emerge from the Conference.

TECHNICAL PANEL

Professor C. Kuo

University of Strathclyde

Mr. J.A.H. Paffett

Ship Division, National Physical
Laboratory

Professor S. Motora

University of Tokyo

Professor K. Wendel

Hannover University

Professor F. Ursell

Manchester University

TIMETABLE AND PROGRAMME

MONDAY, 24th MARCH, 1975

1900 - 2100

INFORMAL RECEPTION AND REGISTRATION at Staff Club,
Livingstone Tower, University of Strathclyde,
Richmond Street.

TUESDAY, 25th MARCH, 1975

MORNING

0915 - 0930

Welcome Address by The Principal, Sir Samuel Curran,
F.R.S., University of Strathclyde.

0930 - 1300

SESSION 1: DEFINITION OF STABILITY

Chairman: Mr. N. Bell, (Dept. of Trade, London)

1.1 Bird, H. and Odabasi, A.Y. (U.K.)
"State of Art: Past, Present and
Future"

1.2 Krappinger, O. (W. Germany)
"Stability of Ships and Modern
Safety Concepts"

1100 - 1130

Coffee

1.3 Cleary, W.A. (U.S.A.)
"Marine Stability Criteria"

1.4 Kobylinski, L. (Poland)
"Rational Stability Criteria and
the Probability of Capsizing"

1300 - 1400

LUNCH

AFTERNOON

1400 - 1730

SESSION 2: ENVIRONMENTAL CONDITIONS AND EXPERIMENTAL STUDIES

Chairman: Mr. J. A. Paffett (Ship Division, N.P.L.)

2.1 Ewing, J.A. (U.K.)
"Environmental Conditions Relevant
to the Stability of Ships in Waves"

2.2 Fujii, H. and Takahashi, T.
"Experimental Study on Lateral
Motions of Ship in Waves"

1530 - 1600

Tea

2.3 Takaishi, Y. (Japan)
"Experimental Technique for Study-
ing of Ships Achieved in the Ship
Research Institute"

2.4 Miller, E.R. (U.S.A.)
"A Scale Model Investigation of the
Intact Stability of Towing and
Fishing Vessels"

2.5 Dudziak, J. (Poland)
"Safety of a Vessel in Beam Sea"

1930

RECEPTION AND BUFFET GIVEN BY BURMAH OIL COMPANY

WEDNESDAY, 26th MARCH, 1975

MORNING

0900 - 1300

SESSION 3: THEORETICAL STUDIES

Chairman: Professor F. Ursell (Univ. of Manchester)

- 3.1 Wellicome, J. (U.K.)
"An Analytical Study of the
Mechanism of Capsizing"
- 3.2 Haddara, M.R. (U.A.R.)
"Application of the Fokker-Planck
Equation to the study of the
Stability of the Mean and Variance"

1100 - 1130

Coffee

- 3.3 Abicht, W. (Germany)
"On Capsizing of Ships in Irregular
Seas"
- 3.4 Kastner, S. (Germany)
"On the Statistical Precision of
Determining the Probability of
Capsizing in Random Seas"
- 3.5 Boroday, I.K. and Nikolaev, E.P.
(U.S.S.R.)
"Methods for Estimating the Ship's
Stability in Irregular Seas"

1300 - 1400

LUNCH

AFTERNOON

1400 - 1730

SESSION 4: CORRELATION OF THEORY AND EXPERIMENT

Chairman: Mr. J. J. Jensen (Danish Shipowners' Assoc.)

- 4.1 Kure, K. and Bang, C.J. (Denmark)
"The Ultimate Half Roll"
- 4.2 Tamiya, S. (Japan)
"Capsize Experiment of Box Shaped
Vessels"

1530 - 1600

Tea

- 4.3 Paulling, J.R., Oakley, O.H. and
Wood, P.D. (U.S.A.)
"Ship Capsizing in Heavy Seas"
- 4.4 Kobayashi, M. (Japan)
"Hydrodynamic Forces and Moments
Acting on Two Dimensional
Asymmetrical Bodies"
- 4.5 Morrall, A. (U.K.)
"Simulation of Capsizing in Beam
Seas of a Side Trawler"

1930

CONFERENCE DINNER

THURSDAY, 27th MARCH, 1975

MORNING

0900 - 1245

SESSION 5: APPLICATION OF RESEARCH FINDINGS

Chairman: Mr. E. H. Middleton (U.S. Coast Guard)

- 5.1 Kastner, S. (Germany)
"Long term and Short term Stability
Criteria in a Random Seaway"
- 5.2 Kuo, C. and Odabasi, A.Y. (U.K.)
"Application of Dynamic Systems
Approach to Ship and Ocean Vehicle
Stability"

1100 - 1130

Coffee

- 5.3 Tsuchiya, T. (Japan)
"An Approach to Treating Stability
of Fishing Vessels"
- 5.4 Nielsen, G. (Denmark)
"A Practical Approach to Ship
Stability"
- 5.5 Dorin, V.S., Nikolaev, E.P. and
Rakhmanin, N.N.
"On Dangerous Situations Fraught
with Capsizing"

1315 - 1500

LUNCHEON RECEPTION: LORD PROVOST OF GLASGOW

AFTERNOON

1500 - 1700

SESSION 6: QUIZ SESSION ON STABILITY

Chairman: Professor M. Meek (Ocean Fleets Limited,
and Visiting Professor)

- Dr. B. Baxter (Yarrow Shipbuilders Ltd.)
- Dr. V. S. Dorin (Krylov Institute,
Leningrad)
- Mr. S. MacDonald (Royal National Lifeboat
Institution)
- Professor S. Motora (University of Tokyo)
- Mr. D. MacIver-Robinson (Department of
Trade, London)
- Professor K. Wendel (University of
Hannover)

1700

Tea

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This book is issued to Conference delegates only.

SESSION 1

DEFINITION OF STABILITY

<u>Paper No.</u>	<u>Author</u>	<u>Title</u>
1.1	Bird,H., Odabasi,A.Y. (U.K.)	"State of Art: Past, Present and Future"
1.2	Krappinger,O. (Germany)	"Stability of Ships and Modern Safety Concepts"
1.3	Cleary,W.A. (U.S.A.)	"Marine Stability Criteria"
1.4	Kobylinski,L. (Poland)	"Rational Stability Criteria and Probability of Capsizing"

STATE OF ART: PAST, PRESENT AND FUTURE

by

H. BIRD
Department of Trade,
London

and

A. YÜCEL ODABAŞI
University of Strathclyde,
Glasgow

1. INTRODUCTION

All types of floating structures, including ships, are designed, produced and operated to fulfil specific functions efficiently and safely. In that respect the stability of their motions can be of crucial importance and it was certainly because of this that so much interest has been shown by naval architects with assistance occasionally from applied mathematicians.

In this paper we have made an attempt to review the past and the present knowledge on stability theory and also expressed some views on possible future developments. In the review of past developments we have concentrated on concepts rather than mathematical details and hope that our account of this evolution will be informative. We have also tried to highlight the state of the art by distinguishing between different theoretical and practical approaches. Stability criteria have had a special place in our study since they provide us with the means for assessing adequacy of stability. We have come to the conclusion that the existing criteria are not adequate both from theoretical and from practical points of view. We have stated our criticisms on the existing criteria and pointed out the necessity for basic research. Finally, we have expressed our thoughts on the future and suggested the need for two different types of criteria suitable respectively for the ship master and for the approving authority.

In preparing this study we have used all the material available to us and even with such incomplete information it has not been possible in the

interest of conciseness to quote the names of all the contributors. We could not obtain or refer to all of the numerous contributions from USSR, East European countries, and Japan, either because they were not available, or were in a language that the authors could not understand. Thus, we will be delighted to hear more about the contributions from those parts of the world.

In this study we have avoided the temptation to digress into problems relating to damage stability as this would open up a much wider field. In avoiding discussion of those problems, however, we are fully aware that the damage stability criteria often dominate intact stability criteria, especially in passenger ships.

2. BRIEF REVIEW OF PAST DEVELOPMENTS

The development of ship stability theory has had a long period of evolution which still is far from complete. Throughout this period, numerous studies have been devoted to the various aspects of the problem and within the scope of this paper we shall classify them in three groups:

- 1) Studies on conventional ship stability that are based on stability in still water
- 2) Studies on large amplitude rolling and on the stability of ship motions

3) Studies on stability criteria.

This classification is perhaps arbitrary but nevertheless it helps us review the past developments systematically.

The first group of studies assumes that the stability of a ship can be determined from its geometry and weight distribution. The couple formed by weight and buoyancy in still water, when the ship is heeled, is a measure of stability, and the lever of the couple, GZ, is chosen as the representative quantity.

Certainly the understanding of this concept of ship stability is very old. The principal milestones in the development of stability theory may be stated as follows:

Piere Bouguer (1), in 1746, defined the metacentric radius, BM, as the ratio of waterplane moment of inertia, I, to the immersed volume, V, $BM = I/V$. Thus the metacentric height, GM, which was used as a measure of stability, was defined by

$$GM = KB + BM - KG$$

where KB is the vertical coordinate of the centre of the buoyancy and KG is the vertical coordinate of the centre of gravity. The righting lever was approximated by $GZ = GM \sin \vartheta$ or $GZ = GM \vartheta$, where ϑ is the angle of heel in radians.

In 1796 Atwood, (2), derived his celebrated formula for more accurate calculations of the righting levers which was given as

$$GZ = \frac{v \cdot h_1 h_2}{V} - BG \sin \vartheta$$

where v is the volume of immersed or emerged wedge, $h_1 h_2$ is the lever of transfer of volume, and BG is the vertical distance between the centre of buoyancy and the centre of gravity.

Canon Mosely, (3), in 1850 introduced the idea of "dynamic stability" to which we later refer as the quasi-dynamic stability. He derived the expression for the work done by the ship under the influence of some potential external forces and expressed this work as the area under the righting moment curve.

According to this definition, so long as the inequality $\int_0^{\vartheta_{max}} [M_r(\vartheta) - M_g(\vartheta)] d\vartheta > 0$

holds, the ship was assumed to be stable. Here $M_r(\vartheta)$ and $M_g(\vartheta)$ are the righting and heeling moments, respectively, and ϑ_{max} is the maximum angle of roll. The significance of that study was its attempt to relate the stability of ships to their rolling motion. Although this formula has been very widely taught to naval architects, his principle has not been developed further till the last decade.

In 1861, Barnes, (4), presented a numerical method for calculating the cross curves of stability and many other calculating procedures followed. This development opened up the possibility of applying the theoretical knowledge in practice.

Scribanti, in 1904, (5), found the expression for GZ values of wall-sided ships as

$$GZ = (GM + \frac{BM}{2} \tan^2 \vartheta) \sin \vartheta$$

which highlighted the problems arising from zero or negative initial metacentric height.

Later developments in this field aimed either at modifying or establishing techniques or for applying existing knowledge. In this respect we may mention the introduction of the concept of residuary stability by Prohaska in 1947, (6), and the influence of the variation of righting moment in a seaway by Grim, Wendel, Kerwin, Paulling and many others. This latter improvement is quite important and we shall discuss it in the following section.

Simultaneous with these developments, the second group of studies endeavored to define rolling motion of ships in a general sense but without considering the stability of the motion itself. Again we may summarize the important developments as follows:

Neglecting the damping effect, W. Froude (1861), (7), derived the expression for rolling motion in regular beam seas, by assuming that the beam and the draft of the ship are small in comparison with the wave length, and the presence of ship does not alter the wave form. In 1874, he also introduced the damping effect by using the best empirical damping as

$$-\frac{d\vartheta}{dn} = a\vartheta + b\vartheta^2$$

where n is the number of oscillations, and a and b are constants to be determined from experiments, (8).

In 1896 and 1898 Krylov (9,10) gave a more comprehensive representation of the theory of ship oscillations and the theory of rolling was further developed on the basis of what is known as the Froude-Krylov hypothesis.

Manning, (11), included the influence of ship's speed and wave direction by introducing the period of wave encounter while the various other aspects of rolling motion were extended by Scribanti (5), Johns (12), Lewis (13), Havelock (14), Serat (15), Watanabe (16), Urzell (17), and many others.

The relevance between ship motions and their stability was recognised a long time ago, and through the end of the 19th century A.M. Lyapunov (18) derived the conditions for stability of motion of a freely floating rigid body. This study, however, did not appear to have any influence in ship stability understanding. Though it was quite correct what Dahlman wrote in 1937, (19), stating that the whole capsizing event is a dynamical event because statical states of equilibrium do not arise among waves, it was Grim, (20), who first showed the possible influences of nonlinear resonance features and the subject has further been analysed by Vedeler (22), Baumann (23), Grim (24), Paulling and Rosenberg (25), and many others. We shall further discuss them in Chapter 3.

In 1953 St. Denis and Pierson (26), presented a statistical approach for analysing ship motions in confused seas. This development opened up a new field for further studies in ship motions to which efforts have been directed by many other authors such as Williams (27), Cartwright and Rydill (28), Voznessensky and Firsoff (29), made a statistical analysis of data on the rolling motion of ships and Grim (30) used the stochastic processes in the study of nonlinear ship rolling. Later statistical methods became more popular and we shall see the recent developments in the state of art.

The final group of studies were aimed at determining a safe minimum amount of stability for devising stability criteria. It is known that load line rules existed as early as 11th century, but the real efforts for establishing rules in ship stability came after 1870. In 1870 a British warship, "Captain", capsized and this accident brought forward the question of safe

minimum stability. At the beginning of the 20th century, Germanischer Lloyd made a study of stability because of a large number of foundered fishing boats. Thus, fatal capsizing events were the reason why so much attention was being paid to the question of minimum stability at that time. Throughout the evolution of ship stability criteria, various approaches were proposed and tried. Here we summarize them according to type rather than in chronological order.

One of the very first measures for judging the stability was the initial metacentric height. In the beginning of the 20th century, depending on the type and size of vessel, an initial metacentric height between 0.2 - 0.6 m. was considered sufficient. Foerster, (34), in that respect made his comment by stating that as displacement increases the minimum values for the stability factors may be reduced as the righting moment is a product of the displacement and the righting arm. Some of the proposed minimum GM values were as follows:

$$GM = \frac{0.243}{C_a} \frac{A_w}{A_L} \quad (m.)$$

$$GM = BM - \frac{\Delta \cdot B_0 R_m - k \sum M_e}{\Delta \cdot \sin \phi_m}$$

where A_w is the projected wind area, C_a is the block coefficient and Δ is the displacement of the ship, k is a constant whose value should be in the range 2.5 - 3.0 for good stability, ϕ_m is the angle for which the maximum righting arm GZ occurs, $\sum M_e$ is the sum of heeling moments, $B_0 R_m = GZ_m + BG \sin \phi_m$. As will be seen from its form, the second equation incorporates the righting arm curve as well as various heeling moments.

Parallel with the criterion of initial metacentric height, efforts were also made to establish principles based on the main dimensions of vessels for judging stability. This approach was adopted mainly for the convenience of designers, but it proved unsuccessful in practice due to various shortcomings.

The use of the righting arm curves for judging stability was first proposed by Reed, (32). Nevertheless the use of this curve followed Denny's paper in 1887, (33). This type of stability criterion was in frequent use in the design of vessels and

there were various standard curves suggested by different authors. Some of these standard curves are illustrated in Figure 1. Arguments about various standard curves, however, proved useful and the following features were considered to be significant:

- a) The initial part of the righting arm curve up to a heel of 10 degrees depends on the initial metacentric height.
- b) The angle ϑ_m at which the righting arm curve reaches its maximum value is very important.
- c) The vanishing angle ϑ_v where the righting arm becomes zero is also important.
- d) Magnitudes of the righting arms at 20, 30 and 40 degrees have a strong influence on the vessel's stability.

The dynamical lever curve which is the integral of the righting arm curve was also used as a stability criterion. In 1913, Benjamin, (35), made his proposal after comparing a large number of vessels which operated successfully. He came to the conclusion that the minimum dynamical levers were

- 1) $e_{60} \geq 0.2 \text{ m.rad for } \vartheta = 60^\circ$
- 2) $e_{30} \geq 0.05 \text{ m.rad for } \vartheta = 30^\circ$

His suggestions were strongly opposed because of the choice of the limiting angles. Later Benjamin, himself, modified his proposal and offered a standard curve instead. Pierrottet, (36), made a proposal of complete draft criteria in 1935. In this study he proposed a limit angle and required that the dynamic lever at this angle must be equivalent to or greater than the sum of the total amounts of work done by wind, waves, centrifugal force and the movement of the passengers on board. He also gave a procedure for numerical computations. The proposal was not well received because the criterion was too severe and the limiting angle was too large.

In 1939, Rahola, in his Ph.D. thesis, (37), made a significant contribution towards achieving workable stability criteria. The study was based on the results of official inquiries into some 30 capsizes and, by making effective use of the state of knowledge at the time, he proposed the following combined criteria:

$GZ \geq 0.14 \text{ m for } 20 \text{ degrees}$

$GZ \geq 0.20 \text{ m for } 30 \text{ degrees}$

$GZ \geq 0.20 \text{ m for } 40 \text{ degrees}$

$\vartheta_m \geq 35 \text{ degrees}$

and $e = 0.08 \text{ m radian for } \vartheta_v$

where the limit angle ϑ_v was defined as the smallest of ϑ_m , angle of heel for immersion of non-watertight openings, angle of heel for shifting of cargo, or 40 degrees. Rahola's work has been of great value and formed the basis of several national criteria, including the IMCO criteria published in 1968.

Skinner, (38), in his paper considered stability of small ships under the influence of simultaneous action of wind, waves, and shipping water. He came to the conclusion that the magnitude of maximum GZ value and the corresponding angle were sufficient to define the righting arm curve.

Operational factors which affect the stability of ships were considered by Steel, (39), in 1956. He analysed several casualties of specific ship types and stated that the minimum standards should not be accepted automatically but due consideration must be given to the type of ship, service and the nature of the cargo.

Yamagata, in 1959, (40), presented the stability criteria adapted in Japan. The approach used was similar to Pierrottet's proposal and a procedure for calculation was also given.

Norrby, in his studies, (41-42), offered some modification on Rahola's dynamic stability criterion for coastal vessels. He proposed the dynamic lever as $e = 0.08 \frac{\Delta_s}{\Delta} \text{ m.rad}$ at the limiting angle which was the lowest of ϑ_m , the flooding angle or 35 degrees, where Δ_s was the summer load displacement and Δ was the displacement considered. He concluded that once the stability criterion was decided, the minimum required GM could be estimated by ship masters after measuring the rolling period and using Kato's formula for the radius of gyration.

In 1962, IMCO (Inter-governmental Maritime Consultative Organisation) established the Sub-committee on Subdivision and Stability which was to deal with the stability of all types of ships, including fishing

vessels. The progress made by IMCO was reviewed by Nadeinski and Jens, (43), for fishing vessels and by Thomson and Tope, (44); more generally. The outcome of the studies were certain recommendations for judging the stability of ships which had very little difference from Rahola's results. We shall discuss them later.

3. PRESENT STATE OF ART

During the last decade, research in ship stability has taken a new direction and Grim, (20), and Wendel, (45), were the pioneers of these new developments. They both introduced the effects of the variation with time of ship's restoring moment in a sea-way, but used this variation for different purposes. In fact, the basic idea was not new. Pollard and Dudebout, (46), and Kempf, (47), had mentioned the importance of the subject, but the application came after 1952.

Here we shall summarize each approach separately and for the sake of convenience we shall first consider Wendel's approach.

Statistical analysis of casualty records indicates that an important part of capsized ships, especially between 30 and 60 metres in length, were under the action of following or quartering seas with 5-7 Beaufort wind forces. Inspired by this fact, Wendel and a group of naval architects led by him concluded that the most critical stability condition arises when the ship is acted upon by a wave whose length and velocity are the same as those of the ship. The amount of total loss in restoring moment was found to be a function of wave height, wave geometry, wave steepness and the location of the wave crest relative to the ship's length. It was pointed out that the worst condition arises when the crest is at amidships. The method of calculation was first to superimpose a one-dimensional wave form on the ship's profile drawing, allowing sinkage and trim, and to compute the righting arms with one of the known methods. The results of these computations yielded very severe losses in restoring moments which were generally unrealistic. In order to modify the results, Arndt and Roden, (48), proposed the introduction of Smith effect, e.g. the effect of orbital motion of water particles, into the pressure computations. Their results are illustrated in Figure 2. Studies in this direction were further developed by the contributions of Paulling, (49), Upahl, (50), and many others.

Later contributions were on the probability of capsizing and the results were presented in (51).

Research on this subject still continues and a Sub-committee at IMCO has considered some proposals based on the stability losses in following seas, see for example, (52), (53).

In 1952, Grim dealt with the variation of restoring moment in waves for a different purpose. He considered the equation of rolling as

$$I \ddot{\theta} + \Delta (GM + \delta GM \cos \sigma t) \theta = 0$$

where I is the virtual mass moment of inertia about the rolling axis, Δ is the displacement, GM is the metacentric height, δGM is the maximum variation in metacentric height, σ is the wave frequency, t is the time and θ is the angle of roll. By making use of the known results on the stability of Mathieu's equation, he pointed out the possible instability regions. In 1954, (24), he considered more general rolling motion as

$$I \ddot{\theta} + \Delta \cdot GZ(\theta) = M$$

where M is the excitation.

The importance of his approach lies in the attempt to relate the stability of a ship to its motions and this idea forms the basis of the majority of today's research activities. Following Grim's study, Vedeler, (22), Bauman, (23), and Kerwin, (54), extended the understanding by considering more general equations to represent non-linear rolling motion. Paulling and Rosenberg, (25), indicated possible instabilities due to nonlinear coupling effects. Haddara, (55), in his work tried to discover some unstable regions from the results of Hill's equation.

While the deterministic case was being studied extensively, the behaviour of a ship in random sea conditions was also examined by many research workers. Following the Pierson and St. Denis's introduction of the subject, Cartwright and Rydill, (28), paid more attention to rolling motion. Kato et. al., (56), presented an experimental study for irregular wind and wave conditions. The study carried out in Japan by a group of

scientists, (57), gave not only the existing methods of assessing stability, but also a summary of the computational procedures for calculating the important factors necessary for the evaluation of stability. Hasselman, (58), and Vassilopoulos, (59), showed how to treat the nonlinear systems under random excitations. Kastner, (60), extended this application and studied the behaviour of phase trajectories of rolling motion (see Figure 3). De Jong, (61), in his Ph.D. dissertation tried to solve the problem with the aid of Fokker-Planck-Kolgomorov equation and defined the stability with the probability of threshold crossing. Similar studies have been carried out both theoretically and experimentally by many research centres (see for example, (62), (63), (64)).

One of the most important features of all this work was the tendency towards solving the single or coupled nonlinear rolling equations and then searching for stability with the aid of the determined solution. When the equations of motion are severely nonlinear, the approximation methods which linearise the equations, in one way or another, may not yield reliable solutions. To overcome this difficulty Odabasi, (65), introduced the Lyapunov's Direct Method into the stability computation of ships. Later the general definition of stability, (66), and the method of assessing the stability, (67), have been further studied by Kuo and Odabasi.

Here we have only summarised the contributions which were available to the authors and we sincerely hope that the participants of this conference will cover the grounds which were left open by us and moreover will contribute towards a more rational understanding with their papers and discussions.

4. STABILITY CRITERIA

Stability criteria are the only guidance available both to the designer and to the ship master for judging the stability of ships and in that respect the necessity of having such regulations or recommendations is fairly obvious. Establishing the stability criteria is however a very difficult task since one ought to find a compromise amongst very many diverse and conflicting demands and factors influencing the optimum design requirements.

A glance at casualty records indicates that the sizes, types, loading conditions, environmental conditions and

totally the causes of the incidents are diverse. Table 1 which has been prepared from a study of a recent sample of UK casualties during the last fifteen years illustrates this fact very clearly. While this is the case one would be optimistic to believe that a standard GZ- δ curve or an oversimplified graphical approach is a satisfactory solution to the problem.

From an examination of the remarks column in Table 1, it will be apparent that difficulties are often experienced in establishing primary and secondary causes of loss, or of distinguishing the former, which is the more important, from the latter. Furthermore, there is invariably a good deal of speculation about cases where ships have disappeared without trace as in 6 of the above sample.

In relatively few cases is the issue a clear-cut case of inadequate stability and there are usually complicating circumstances. This will be illustrated by the summary of this sample, which is given at the end of the paper. (Table 2) Other samples would no doubt reveal different distributions or additional primary and secondary causes.

Even in cases where stability is deemed inadequate (e.g. by IMCO standards), absence of a "weather criterion" makes it impossible to state what sea conditions might have been survived. The same applies to those ships which had stability greater than IMCO recommendation but still capsized, and consequently little is learned from what is often a very protracted and expensive investigation of the case.

In practice many alternative approaches have been proposed to overcome the difficulties. Some of these preferred simplicity and devised purely empirical criteria. Some others used idealized theoretical computations as their basis for stability criteria. A common feature of all the proposals was their dependence on practical experience.

Though different in numerical values, today we may meet three main groups of criteria in assessing the stability of ships, (43), (68), (69):

- 1) Criteria based on certain formulae for GM and freeboard values. These types of criteria are mainly used for small vessels and in some countries for tugs and fishing vessels.

- 2) Criteria based on minimum stability requirements for, GM, statical and dynamical righting levers. This is the approach adopted by IMCO and the numerical evaluation of minimum values are achieved by the pillar diagram analysis of casualty records. Following the IMCO recommendations, their results were approved and applied by many nations.
- 3) Criteria based on the computation of heeling levers for comparison with righting levers. This is the only approach which explicitly uses a theoretical approach, though some oversimplifications are involved. The users of this form of stability criteria are mainly USSR, Eastern European countries and Japan.

Because of their frequent use here we shall summarise the last two methods of assessing the stability.

IMCO requirements on statical and dynamical stability are as follows:

- a) $GM \geq 0.15$ metre for passenger and cargo ships, $GM \geq 0.35$ metre for fishing vessels, and $GM \geq 0.05$ metre for ships carrying a timber cargo on deck.
- b) $\vartheta_m > 25^\circ$, preferably $\vartheta_m > 30^\circ$, where ϑ_m is the angle GZ attains its maximum value.
- c) $GZ \geq 0.2$ metre at $\vartheta \geq 30^\circ$
- d) Dynamical lever ≥ 0.055 metre radians at $\vartheta = 30^\circ$.
- e) Dynamical lever ≥ 0.09 metre radians at $\vartheta = 40^\circ$ or $\vartheta = \vartheta_f, \vartheta_p$ being the angle of flooding.
- f) Dynamical lever difference between $\vartheta = 40^\circ$ or $\vartheta = \vartheta_f$ and $\vartheta = 30^\circ$ must be greater than 0.03 metre radians.

Additionally, for passenger ships:

- g) Angle of heel must be less than 10° due to movement of passengers.
- h) During the turning of ship the angle of heel must be less than 10° under the influence of the moment

$$M_a = 0.02 \frac{V_o^2}{L} \Delta \left(KG - \frac{T}{2} \right)$$

(tonne metre)

where M_a is the heeling moment, V_o is the service speed, L is the waterline length, T is the draft, and KG is the vertical coordinate of the centre of gravity.

As one can easily see, the IMCO recommendations are very similar to those obtained by Rahola (see Figure 1).

To illustrate the important features of the third group of stability criteria, we shall give the summary of a proposal submitted to IMCO by the USSR delegation, (70). According to this criterion a ship is assumed to be stable if the weather criterion K fulfils the following inequality:

$$K = \frac{M_c}{M_v} \geq 1.0$$

where M_c is the capsizing moment and M_v is the "dynamical" heeling moment. M_v is calculated by the formula

$$M = 0.001 P_v A_v Z \quad (\text{tonne metre})$$

where P_v is the wind pressure, A_v is the projected lateral area above the effective waterline, and Z is the distance between the centre of the projected lateral area and the effective waterline. The values of P_v are given in Figure 4.

Capsizing moment is determined by means of a graphical procedure. Before commencing the procedure, it is first necessary to calculate the amplitude of rolling ϑ_r by the formula

$$\vartheta_r = x_1 x_2 y$$

where x_1, x_2 and y are coefficients and they are given in Figures 5, 6 and 7.

If we use the statical stability diagram, the capsizing moment is

$$M_c = \Delta \cdot \overline{OM}$$

and if we use the dynamical lever curve, capsizing moment is

$$M_c = \Delta \cdot \overline{BE}$$

The determination of OM and BE can easily be followed from Figure 8.

As, for the time being, most of the nations are using the IMCO recommendations as their stability criteria, we concentrate our attention on the activities of this Organisation. When the IMCO made the recommendations on minimum stability in 1968, it was generally understood that those recommendations were only a first approach. Although some alternative proposals have been made by certain delegations, notably USSR, they have been somewhat inconclusive. IMCO has recently set up a new Working Group to formulate improved stability criteria. Further details can be seen in Appendix I.

Here we present certain characteristics of two ships in order to show the desirability of improved criteria. Those ships more than fulfilled the minimum stability requirements of IMCO but yet capsized, (71), (72), (73). (See Tables 3 and 4) These examples show that IMCO recommendations, by themselves, are not sufficient to provide acceptable safety of ships, and as in both the cases the weather conditions were not too severe, we must look for some other basic reasons causing the capsizes. In the United Kingdom the Department of Trade is conscious of this situation and issues additional guidance where considered necessary (see Appendix II) and takes an active part in supporting relevant research.

5. CRITICAL REVIEW OF EXISTING KNOWLEDGE

As has been seen from the preceding chapters, the stability of a ship is judged by means of the righting arm curve which is determined from the geometry of the ship and the vertical location of the centre of gravity, and by hypothetical wind and wave forces which are assumed to be potential. Thus, the problem is reduced to the stability of a conservative system. As a natural consequence of the theory or ordinary differential equations, if there is no additional disturbance, the ship will settle in her new equilibrium position. Thus, this is only a quasi-dynamic approach.

While appreciating the value of practical experience and accepting the desirability of basing regulations on a rational analysis of such experience, we should like to draw attention to the following aspect.

Considering that most of the criteria

are based on Rahola's results with minor modifications and additions, the following aspects appear rather weak even in a semi-empirical approach:

1) It seems that all the results are concluded from so-called histograms, in fact only pillar diagrams of righting levers and dynamic levers of vessels which suffered stability casualties were used, (43). The statistical procedures adopted seem questionable and are not familiar to the authors. A closer examination of these graphs and the real records, however, shows that

a) The types and sizes of these vessels are very much different and the range considered is considerable.

b) Their loading conditions at the time of the incidents are different, the assumption apparently made that the loaded arrival condition can be taken as safe does not seem justified.

c) The environmental conditions for each casualty are different, the significance of this fact is ignored.

d) The ages especially of so-called "existing" ships are different.

e) The fact that some of the so-called existing ships (assumed to be safe) might become casualties in the future was neglected.

For these reasons the authors feel that if any similar study is carried out in future it should be based on a weighted statistical analysis, weight coefficients being derived from the aforementioned factors. Furthermore, it should be recognised that the results so obtained are not valid for new unconventional ships.

2) A ship often capsizes while executing certain oscillatory motions under the influence of varying actions of winds and waves. The action of wind and waves are not potential, that is, they cannot be idealized in order to introduce them into GZ- ϕ diagram. While this is the case, a quasi-dynamic

approach under over-simplified assumptions cannot be expected to be powerful enough to cover the whole range of ships. Our examples in Section 4 are not thought to be exceptional cases.

- 3) There is a dangerous tendency to extending the existing criteria to other parts of the marine world. The current application to floating offshore structures is a typical example. They, in reality, have very little in common with ships so far as the stability of motion is concerned. They certainly make their maximum rotational motions about one of the principal axes for which the virtual mass moment of inertia is minimum, and considering the large asymmetric superstructures this axis may not coincide with one of the symmetry axes for the underwater part. Furthermore, the influence of wind and waves on these structures are also quite different.
- 4) Although many attempts have been made to define the ship stability mathematically, see for example (74), (75), no stability criteria have hitherto provided a rational definition of stability. This, in the authors' opinion, is the first task to be achieved uniquely. For various stability concepts, see (76).
- 5) The last and most important comment on current criteria is on their absolute nature. When a ship fulfils the criteria requirements it is announced to be stable without any reference to environmental conditions in spite of the knowledge that there is no absolutely stable dynamic system. It is, therefore, necessary to incorporate the stability criteria with environmental conditions by specifying, at least, some operational zones and seasons. In practice, when analysing casualties involving stability, the trend is to state whether the ship complies with the IMCO recommendations and this is the final word to be said. Thus, in that respect, no lesson is learnt and no improvement is achieved from those analyses.

Today, efforts are increasingly being directed towards a more explicit evaluation of the stability properties of ships and floating offshore structures. According to the general trend of these activities, we may examine them in

certain groups.

The first group of studies continues to utilise classical stability theory by means of some manipulations or corrections. In the majority of these research activities, a modified stability criteria is the aim and attempts are made to achieve this by introducing some deterministic or stochastic corrections. The authors think that the classical theory is insufficient to provide safety measures for all the diverse floating structures and the studies carried out may yield trivial results, if the motion dynamics are not considered.

The second group examines the influence of following and quartering seas, mostly from a static point of view. The aim of these studies is to devise formulae for calculating the reduction in righting arms of a ship under the action of assumed deterministic or stochastic wave conditions, and then to formulate the stability criteria. Although a study on this basis may yield some useful results, it will be wrong to interpret and to use the results as static quantities. The difference between the static and the dynamic uses are best illustrated in Figure 9. As is seen from the time histories in that case, even 25% loss in restoring moment does not give a significant increase to the amplitude of rolling motion. This fact has been observed in many experiments, especially for vessels over 100 m in length, (77). It should, however, be emphasised that the results may be used effectively provided that they are incorporated with the equations of motion.

The last group of studies tries to relate the stability concept to the equations of ship motions. The ultimate goal of this approach is to devise a procedure for assessing the ship's stability from its motions under deterministic or stochastic environmental conditions. In the authors' understanding this is the best possible way of defining and evaluating the stability of ships. There are nevertheless, certain points associated with this dynamic systems approach needing to be clarified, for the treatment involves a broader spectrum of knowledge than is to be found in classical naval architecture, and it is therefore important to make the following remarks:

- 1) Modelling, i.e. mathematical

formulation, is undoubtedly one of the most important parts of the problem and deserves much more attention in future. Thus, one should be very careful in making assumptions and should also know the limits of validity of his model. Although it is generally possible to find the solution of a formulated problem, it may not correspond to the real physical events if the formulation is not realistic.

- 2) During the mathematical treatment, efforts should be spent on relating the mathematical symbols to physical quantities. This certainly helps to understand and to interpret the mathematical outcomes and their relations with the other quantities involved.
- 3) Final results should be put in a form that can be easily applicable to practical problems by those not having specialised knowledge.

6. REFLECTIONS FOR FUTURE DEVELOPMENTS

Today we have clearer and more rational ideas on the stability of ship motions than ever before. Even a glance at the programme for this Conference will show how the interpretation of ship stability has changed in recent years. There are, however, still some gaps in knowledge and we need to develop these ideas in order that they can be applied in practice. In that respect we would like to suggest concentration of efforts on the following aspects:

1) Theory

There is a very big gap between theory and practice and although theoretical development seems to be going in the right direction there appears to be some important aspects which need to be considered first;

a) There exists no precise definition of stability in terms of mathematics and the motion characteristics. This should be one of the first priorities.

b) Most of the studies so far aim to achieve a quick result and directly relate to specific casualties. The results of such a study yield only a point on a multi-dimensional surface and are far from being conclusive. Thus there is an urgent need to spend more effort on basic features of the stability analysis in order

to gain more insight into the relations between the ship-wave system parameters and the stability of ship motions.

c) Some of the recent studies seem to be of academic interest and the mathematics tend to dominate the real concepts. If we wish to achieve progress in the practical application of such knowledge, we must make our message clear and precise by emphasising the physical meanings of the formulae involved. In the authors' opinion, the use of somewhat more sophisticated formulae need not make life more difficult provided that they are understood and properly interpreted. Application in control engineering gives a very good example of this kind of philosophy.

d) Some important factors, such as shipping water, influence of cargo movement, and types of cargo appear to be ignored in some of the theoretical studies, although in many casualties they have been of vital importance. Here we may quote the Code of Safe Practice for Bulk Cargoes introduced by IMCO (78), as a positive step.

e) Every effort should be made to exploit known theory and this may be achieved by making some modifications and simplifications without sacrificing from safety.

2) Application

Experience in applying current criteria has revealed certain important defects which should be recognised when establishing any improved stability criteria. Examples of these are:

a) From casualty statistics it is apparent that the majority of lost ships are under 70 m in length and the predominance of fishing vessel casualties is obvious. The reason for the latter is no doubt the greater exposure to risk in that type of vessel which has to spend a large amount of time at sea in all weather conditions, often far from a place of refuge and obliged at times to undertake manoeuvres, in order to maintain station which can be dangerous. Thus it is necessary to allow for this in regulations for fishing vessels.

b) Inquiry reports indicate that, especially in small vessels, ship masters are not always able to ascertain the vessel's stability accurately, partly because of their lack of knowledge both on stability theory and on the cargo characteristics, and partly because of shortage of time. Consequently, some alternative form of stability information should be provided which can be easily and quickly used by ship masters. This has already been recommended by the UK Department of Trade, (79). Since it is very difficult, if not impossible, to establish very simple stability regulations which will ensure safety of ships under all conditions, it is desirable to develop two different forms of criteria - one for ship masters and one for designers and approving authorities, the second being more elaborate.

c) Casualty statistics will continue to have an important role to play. It should, however, be kept in mind that the so-called histograms, referred to in Part 5, should not be confused with any meaningful statistical analysis.

d) Increasing demand for marine products and consequently sea transportation has created some new and unconventional types of vessels such as offshore supply ships and pipe laying barges. They may also require special treatment and this aspect has recently been under discussion by the UK and other delegations at IMCO, (80).

3) Education

Another important practical aspect is the necessity for educating ship masters. This need is urgent especially for masters of small ships.

Finally, we should like to emphasise the role of maintenance and survey in order to keep the standard of stability and seaworthiness at its intended level.

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APPENDIX ISTAB XVI/8 - ANNEX IIIPROPOSED TERMS OF REFERENCE OF THE
AD HOC WORKING GROUP ON IMPROVED
INTACT STABILITY CRITERIA

1. Examination of information concerning the casualty experience gained so far in the application of the Recommendations on Intact Stability for Passenger and Cargo Ships under 100 metres in length (Resolution A.167 (ES.IV)). Such examination need not exclude ships of greater length.
2. Review of the present state of theoretical knowledge and experimental research concerning capsizing phenomena and environmental conditions.
3. On the basis of 1 and 2 to identify and summarize typical dangerous situations and to establish theoretical models for those situations.
4. Identify the relevant ship and environmental parameters which mutually influence safety relevant to rolling behaviour.
5. Formulate suggested criteria taking account of the parameters in 4.
6. Establish a practical calculation procedure for these criteria.
7. Undertake comparative calculations for a sample of ships using both the newly developed criteria and existing criteria (including Resolution A.167 (ES.IV)) and their relevance to casualty experience.
8. Submit to the Sub-Committee the Ad Hoc Working Group's recommended criteria together with any explanatory material and other basic information necessary for their understanding and application.

APPENDIX II

REGULATIONS AND RECOMMENDATIONS
CONCERNING SHIPS STABILITY WHICH ARE
CURRENTLY PUBLISHED IN THE UNITED
KINGDOM

1. The Merchant Shipping (Load Line)
Rules 1968 (HMSO 1968)

Schedule 4

This reproduces the standards recommended by IMCO as adopted in the Assembly Resolution A 167 (ES IV), promulgated in 1968.

Rule 30 and Schedule 7

Stability information to be provided for the guidance of the Master.

2. Survey of Load Line Ships -
Instructions for the Guidance of
Surveyors (HMSO 1972)

Appendix 5

This provides detailed guidance to surveyors, as well as to ship owners and shipbuilders, regarding the application of the 1968 stability regulations to ships of various types and the extent to which they apply to existing ships, hopper barges and other small ships.

Amplification of the damage stability requirements relevant to non-passenger ships under the 1966 Load Lines Convention is also included (that aspect is not considered in this paper).

Guidance is given on the special stability problems affecting certain types of ships, eg. container ships, dredgers, Floating cranes, hydrofoils, tugs, also bulk cargoes such as coal, grain, ore concentrates (guidance on the IMCO "Code of Safe Practice"), timber and other deck cargoes. Additionally allowances to be made for ice accretion are given dependent on the area of operation.

Advice is also given to surveyors (but not yet published) regarding special stability standards for ships whose mode of operation makes compliance with Schedule 4 impracticable. These include drilling rig supply ships which have additional problems (now under review by IMCO) concerning low freeboard aft and pipe cargo on deck which tends to trap sea water.

3. Merchant Shipping Notices (HMSO)

Numerous Notices to Ship Owners, Masters and Shipbuilders are issued from time to time. Several of these relevant to stability which are currently in force are Nos M466, 567, 590, 599, 604, 608, 613, 618, 627, 666, 670, 687 and 699.

4. Guidance on the Design and
Construction of Offshore
Installations (HMSO 1974)

Section 4 Primary Structure

4.3.1 - Floating Intact Stability

Stability is to be adequate to withstand an overturning moment due to wind of maximum velocity 100 knots. Area under righting moment curve should not be less than 40% in excess of the area under the wind heeling moment curve. Range of stability to be not less than 35°.

(See also the Offshore Installations (Construction and Survey) Regulations 1974, Part IV, paragraph 3)

Authors' Note

The fragmentation of rules and recommendations on such an important subject is undesirable and we would recommend publication of a comprehensive manual which could be developed from item 2 above.

TABLE 1

No.	Type	Year Built	Dimensions (m)	Gross Tons	Crew	Cargo	Date of Casualty	Location of Casualty	Wind Beau fort No.	Sea	GM (m)	Free-board (m)	Lives Lost	Remarks
1	Wood MFV	1935	14.7 x 4.5 x 2.5	25	3	Fish	13.10.63	55°29'N; 5°51'E	9/10	Heavy 8m waves	0.55	0.3	3	Reason for loss not certain but Court concluded probably capsized in extremely heavy weather
2	Landing craft ferry	1966	24.4 x 5.8 x 1.52	35	4	Trailers & lorries on deck	11.11.66	off Islay, Scotland	4/5	moderate	1.22	0.6	2	Deck cargo shifted and she capsized. Court blamed master for not securing deck cargo.
3	Fishing Trawler	1956	30.5 x 6.7 x 3.3	166	9	Fish	14. 2.65	Dogger Bank	10	Very Rough	0.61	0.55	9	Vessel disappeared while fishing near Western edge of Dogger bank. Court concluded that she was probably overwhelmed by a wave or succession of waves.
4	Cargo Coaster	1959	30.7 x 6.8 x 3.1	195	6	Coal	29. 6.73	Scottish Coast	4	moderate	0.3	0.15	5	The coal was trimmed and hatch boards fitted. Tarpaulins were not fitted because some deck cargo was to be added. Spray penetrated covers, causing list to 35°. Hold filled with sea water and she sank. Court blamed master for not fitting tarpaulins and criticised stability information.
5	Cargo Coaster	1963	31.0 x 7.6 x 3.1	199	3	Soya Beans	21.11.71	English Channel	10/11	Heavy	0.94	0.37	None	Master decided to abandon ship as he was afraid she was going to be overwhelmed and because of difficulties with defective steering gear controls. Court criticised master's lack of knowledge of stability data and of the ship's ability to survive.
6	Fishing Trawler	1958	36.1 x 7.8 x 3.9	274	13	-	13. 1.65	off Orkneys	9	Severe	0.54	0.9	13	Court concluded she was overwhelmed by a sea or seas.
7	Bucket Dredger	1906	41.5 x 9.2 x 3.5	336	-	-	26. 6.67	Sunderland harbour	calm	calm		0.6	None	Bucket ladder had been raised to move vessel, thus reducing stability. Vessel heeled slightly and water entered open sidescuttles, she flooded and capsized.
8	Sand Suction Dredger	1955	43.4 x 7.9 x 2.6	278	6	Gravel (from sea bed)	13. 3.67	Bristol Channel	7	Heavy swell	1.39	0.24	None	Operating with open hatchways. She took 2 heavy seas on o.s.beam. 1st heeled her to 30°/40°, 2nd capsized her.

No.	Type	Year Built	Dimensions (m)	Gross Tons	Crew	Cargo	Date of Casualty	Location of Casualty	Wind Beaufort No.	Sea	GM (m)	Free-board (m)	Lives lost	Remarks
9	Offshore Supply Vessel	1967	45.0 x 11.5 x 4.8	666	12	Drill Pipes and Stores	7. 1.70	North Sea	8/9	High Seas and swell	1.49	0.77	None	vessel was lying off rig waiting for weather to improve when sea water entered engine room and hold through improperly closed openings. Vessel capsized.
10	Cargo Coaster	1953	47.8 x 8.5 x 4.0	522	10	Steel plates Steel piles	6. 2.70	River Tees Estuary	4/5	Slight sea, swell 1.8	0.23	0.49	10	The steel plates had to be discharged first and were therefore loaded on top of the piles. Master had difficulties calculating stability which he over-estimated. She capsized in beam swell shortly after leaving port.
11	Cargo Coaster	1958	48.7 x 8.7 x 3.5	498	8	Lead Concentrate	30.11.72	off Suffolk Coast, England 52°23'N; 1°55'E	7/8	Rough sea and swell	1.28	0.48	4	During voyage vessel shipped one or two seas which cleared. She then took a heavy sea which heeled her to 5°-10°. This gradually increased to 15° and eventually she capsized. Court concluded that list caused by shift of cargo which became fluid due to vibration and ship motion. Court criticised Owners for not properly trimming cargo. Recommended research into procedure for testing concentrates and margin of safety relative to IMCO Code of Safe Practice.
12	Coastal tanker	1940	49.7 x 8.5 x 4.3	574	10	Liquid Ammonia	26.12.72	Irish Sea	5/6	Moderate sea and swell		0.45	None	List of 30° developed during voyage due to flooding of P side buoyancy tank via 10 cms dia air pipe which had broken. Almost capsized, saved by flooding opposite side tank.
13	Fishing Trawler	1937	50.4 x 8.4 x 4.5	533	19	Fish	18.10.61	54°15'N; 0°10'E	7/8	Rough confused swell	0.33		5	Vessel in following sea took one or two heavy rolls to port and recovered. Later a large sea flooded the port side of deck and she capsized. (Most of crew saved by life raft.)
14	Fishing Trawler	1950	52.0 x 8.9 x 4.4	600	20	-	11. 1.68	Not known	7/8	3.6 m	0.65		20	Vessel disappeared after leaving Hull on way to Norwegian fishing grounds. Court unable to establish cause of loss.

No.	Type	Year Built	Dimensions (m)	Gross Tons	Crew	Cargo	Date of Casualty	Location of Casualty	Wind Beau fort No.	Sea	GM (p)	Free-board (m)	Lives Lost	Remarks
15	Cargo Coaster	1956	53.4 x 8.4 x 3.5 RQD 4.4	565	9	Fish meal in bags	14.11.67	Antwerp Harbour	calm	calm	0.09		None	Vessel was being loaded and holds were filled. Remainder of cargo was being added on deck to a height of 1.5 m. Master had incorrectly estimated GM at 0.50m. List developed and she capsized at berth before any deck cargo could be taken off.
16	Fishing Trawler	1949	54.4 x 9.2 x 4.6	659	19	Fish	4. 2.68	Isafjord, Iceland	12	very rough	0.31	0.70	18	Capsized in hurricane when master was turning to starboard into wind. Vessel was heavily iced. Large quantity of water came on board port side and she lay over to Port. Stability reduced by ice, consumption of fuel and little fish cargo. Abnormally severe weather.
17	Fishing Trawler	1948	55.4 x 9.4 x 4.6	658	20	Fish	26. 1.68	North of Iceland	11/12	Very Rough		0.74	20	Court could not establish cause of loss with certainty but probably capsized in exceptionally severe weather. Hurricane force winds and severe icing.
18	Stern Trawler	1972	56.8 x 12.2 x 5.3 7.8	1106	36	Fish	8. 2.74	North Cape Bank	9	Very rough, waves up to about 15 m			36	Disappeared while fishing off the North Cape of Norway. Like other trawlers in the area she was "laying and dodging" to maintain station in very severe weather. Appears to have broached to when turning and swamped by a succession of large waves.
19	Cargo Coaster	1971	58.0 x 9.9 x 4.4	696	7	Packaged timber in holds and on Deck	4. 7.71	Baltic Sea 56°21'N; 16°48'E	5	Low sea and swell	0.24	0.55	None	Voyage Västervik to Boston UK. Rolling easily in moderate swell then listed heavily to port, 50° reported. This due to shift of deck cargo and entry of water through open sidescuttles. Quartering sea, possible synchronous rolling.
20	Cargo Coaster	1957	63.0 x 10.4 x 4.3 RQD 5.5	1074	12	Coal (anthracite)	29.12.62	Off Lizard English Channel	9/10	Very Rough			12	Dutch coaster observed this vessel pass her then turn rapidly 90° to port. SOS by Aldis then observed. Immediately after, her masthead lights disappeared. Court concluded that her steering gear had broken down and she broached to and capsized.

No.	Type	Year Built	Dimensions (m)	Gross Tons	Crew	Cargo	Date of Casualty	Location of Casualty	Wind Beau fort No.	Sea	GM (m)	Free-board (m)	Lives Lost	Remarks
21	Sand Suction Dredger	1963	73.1 x 11.9 x 5.3	1317	11	Pebble Ballast (from sea bed)	8. 9.65	Thames Estuary 51°46'N; 01°24' 30"E	8	Confused High sea	1.50	0.28	4	At start of dredging operation weather was Force 4, sea smooth but deteriorated during loading. Cargo distribution uneven and S list developed. Master continued to load in spite of worsening weather. Shipped heavy seas. S. Buoyancy spaces flooded through air pipes and access hatch and she capsized. Court criticised Master.
22	Cargo Vessel	1945	128 x 17.1 x 8.8	7307	35	Wheat and corn in bulk	21. 2.64	37°22'N; 48°51'W	9/10	Very Rough	0.71		15	Left Philadelphia for London with slight P list which tended to increase. Locking bars were fitted to all cargo hatches except No 3 which had cargo on top. List to P increased to 20° due to entry of water in No 3. Capsized while under tow. Court blamed master for not fitting locking bars, not correcting initial list and not plugging air pipes.

TABLE 2

	Stability considered inadequate for relevant sea conditions	Shift of cargo	Entry of water	Steering gear failure	Master's insufficient knowledge of stability and/or bad loading
Primary causes of casualty	36%	14%	23%	9%	18%
Secondary causes of casualty	9%	14%	9%	-	5%

TABLE 3

Ship	M/T AYGAZ	M/T EDITH TERKOL
Date of incident	24.3.1969	27.7.1972
LPP (m)	55.50	58.60
Breadth (m)	9.00	58.60
Depth (m)	4.25	4.15
Draft (m)	1.96	1.75
Trim (m)	1.25 (aft)	1.52 (aft)
Displacement (tonne)	627.77	661.63
Wind Force	7	6

Bird and Odabasi

TABLE 4

Comparison with IMCO recommendations

Item	M/T AYGAZ	M/T TERKOL	I.M.C.O.
GM (m)	0.73	0.64	0.15
GZ _{30°} (m)	0.292	0.350	0.20
ϕ_{\max} (degrees)	29	45	25
Dynamic lever up to 30° (m rad)	0.1016	0.071	0.055
Dynamic lever up to 40° (m rad)	0.1480	0.132	0.090
Dynamic lever between 40° - 30° (m rad)	0.0464	0.061	0.030

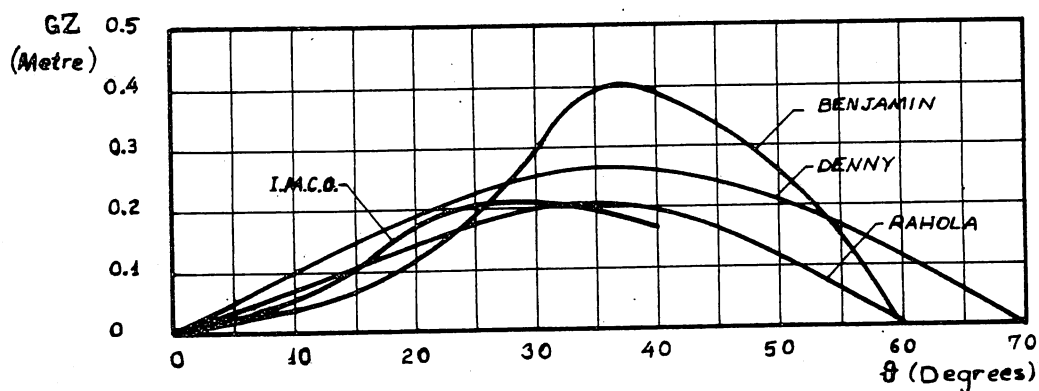


Figure 1 Standard Stability Curves

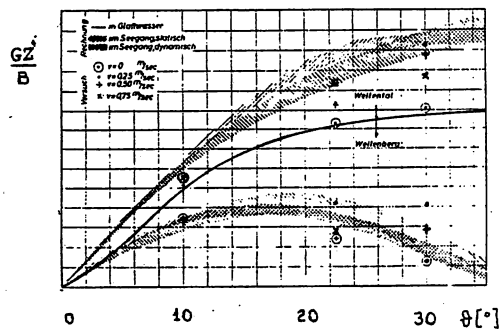
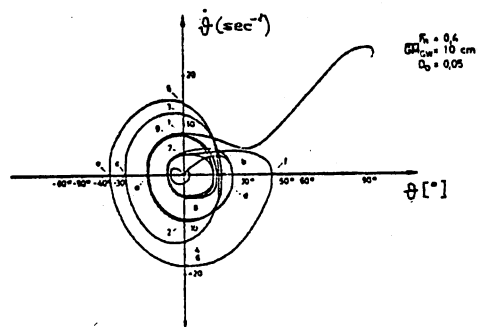


Figure 2 Variation of Righting Arm Curve in Waves

Figure 3 Phase Trajectory of a Capsizing Model, $F_n = 0.4$

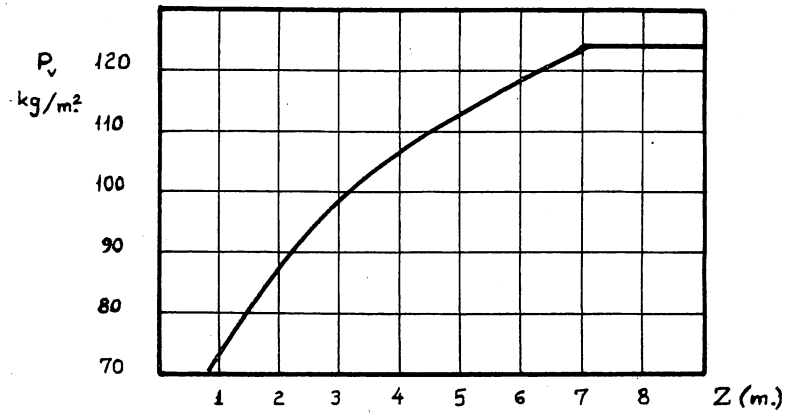


Figure 4

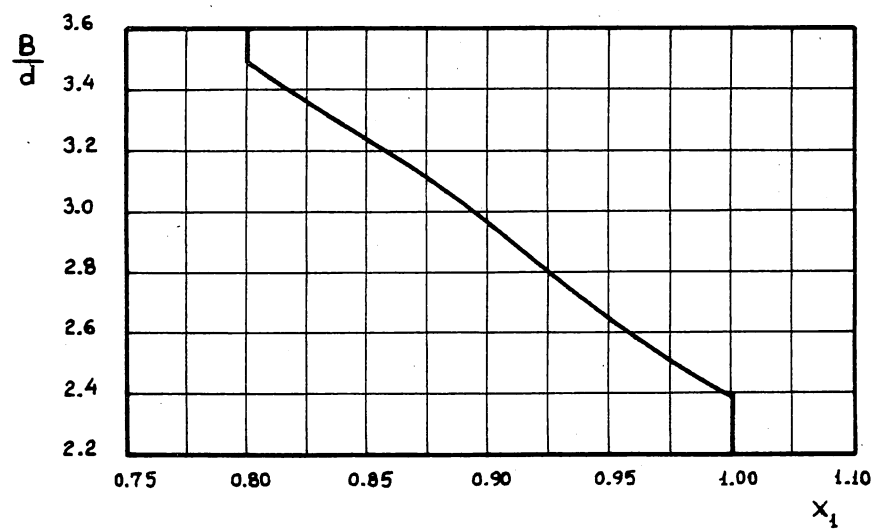


Figure 5

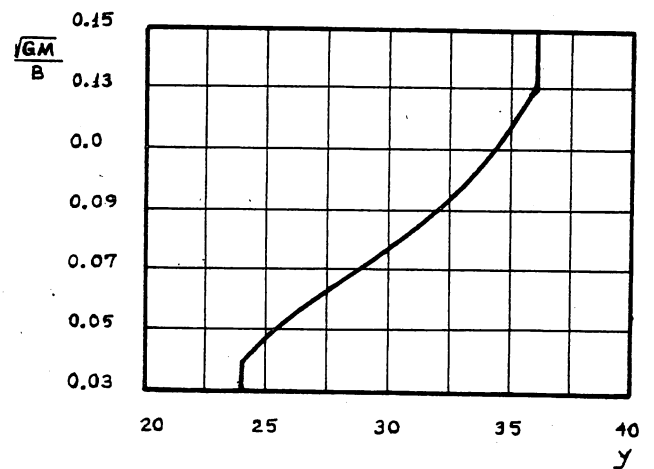
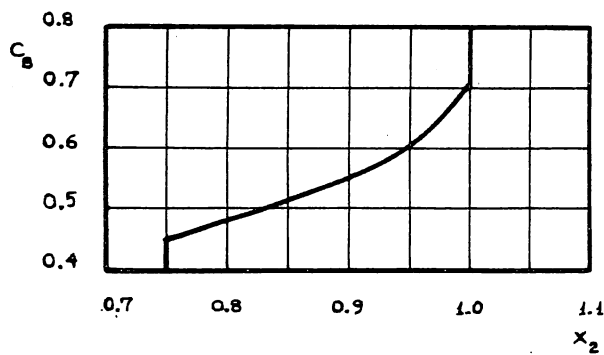


Figure 6

Figure 7

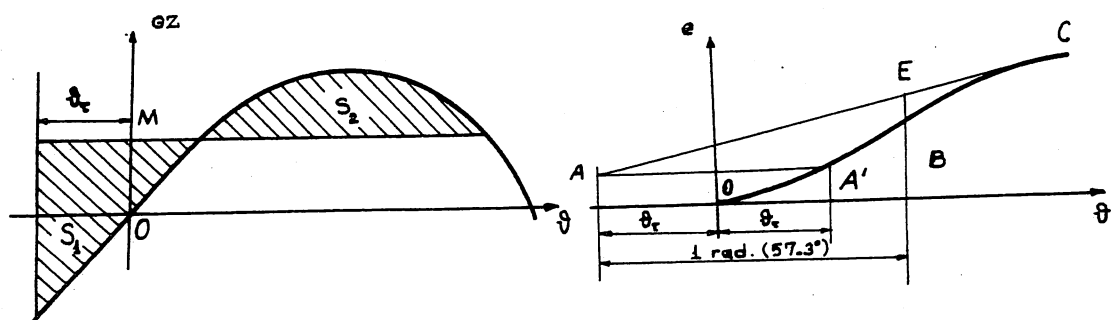


Figure 8

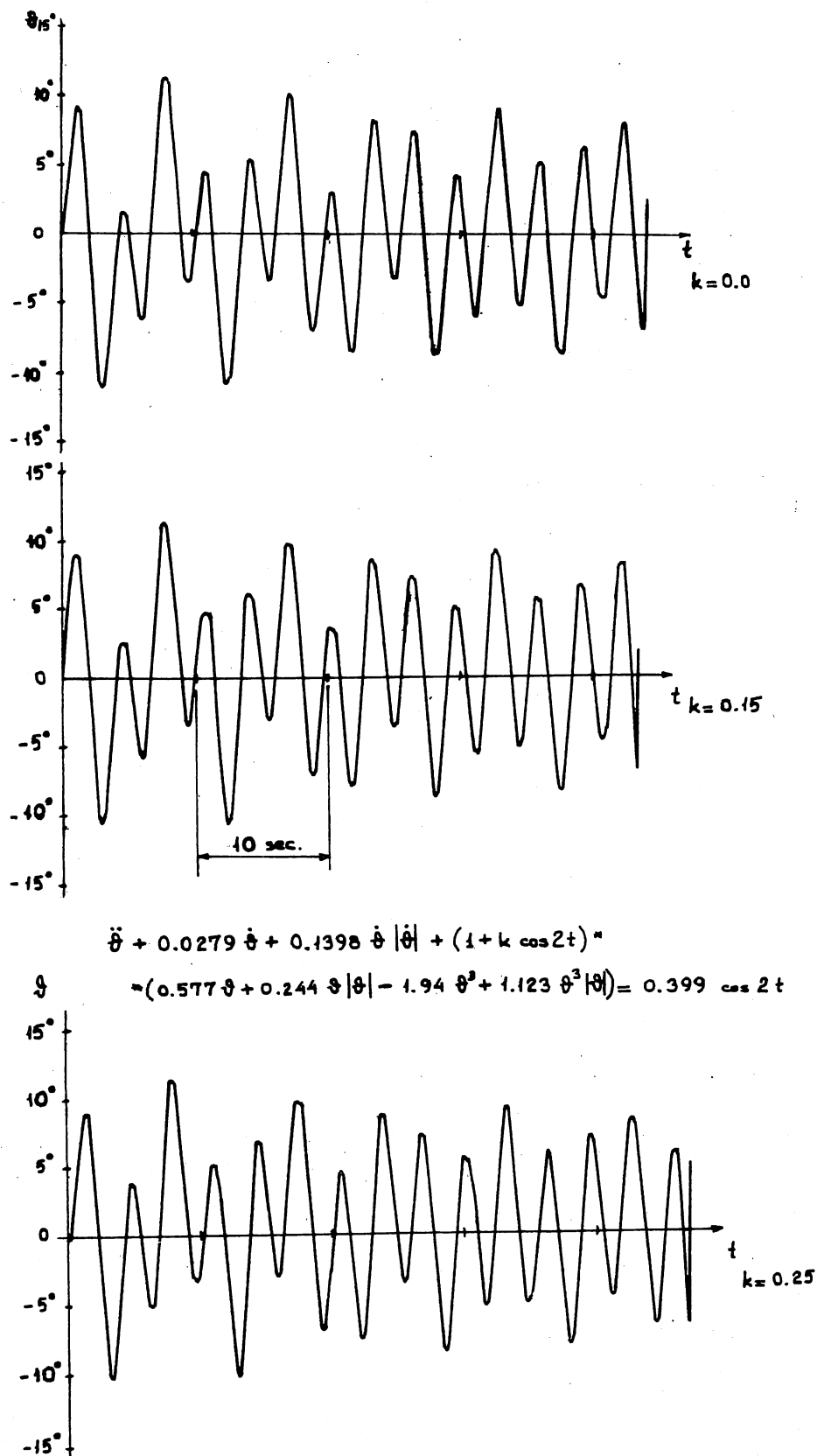


Figure 9 Analog Computer Results

STABILITY OF SHIPS AND
MODERN SAFETY CONCEPTS

by

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1. INTRODUCTION

The application of science to naval architecture and shipbuilding commencing in the 18th century has initiated big changes in this field: Sails and oars, for thousands of years the only means of propulsion were gradually replaced by propellers driven by steam engines, and wood - hitherto solely used for the construction of ships - had to give way to iron and steel. The possibilities to overcome the confinements which navigators experienced since the beginning of human history seemed to be unlimited.

Of course there were still some troubles left; but they were considered as teething troubles and their conquest was thought to be only a matter of time. The euphoria that progress of science and technology will solve all problems was characteristic of that time. This was especially true with respect to safety problems. The goal was not less than perfect safety, and perfect safety was thought to be the consequence of the perfect understanding of the relevant physical relationships.

The attitude of many scientists working now in this field does not seem to have much changed since those days. Working on safety problems still means to them to advance mathematical models of certain physical situations. Obviously they do not realise that the creation of such models is not sufficient in order to design safe ships. With respect to ship stability it seems that recent research work is rather rising an additional problem: Which one

or which combination of the methods proposed to judge sufficient stability should be adopted? There is quite a choice today. In the following a few examples are given as illustration.

2. SOME APPROACHES TO STABILITY

Wendel and his school advocate to investigate the balance of righting and heeling moments (1), (2), (3). The righting moment is calculated for the ship in following waves as mean of the moments attained for the midship section on the crest and in the trough. No dynamic effects are considered.

In Japan, USSR and other countries the well known weather criterion is used. It takes account of the rolling motion and the dynamic effect of gusts. The alteration of righting arms caused by following waves as well as heeling moments (beside that from wind) are not taken into account.

Abicht considers the combined effect of the rolling motion of the ship and the oscillation of the righting arms caused by waves overtaking the ship (5). His result is a probability of capsizing. In spite of many simplifying assumptions the method is onerous to apply. Therefore, it seems scarcely possible to include heeling moments etc. in a proper (i.e. in a probabilistic) manner.

Last but not least the IMCO criterion (the principle of which is a very old one) shall be mentioned.

If a naval architect who tries to find

out what to do in order to make ships safe against capsizing or if an administrator who looks for some advice how to improve stability regulations decides to study the literature on the forementioned and other methods they will soon make a rather bewildering experience: There are people who are quite satisfied with the IMCO criterion. Other specialists promote the weather criterion and consider it physically sound and sufficient to avoid capsizing. On the other hand we find, e.g. in (1)*, that the method on which the IMCO criterion is based is considered obsolete, and that the author does not think too much of the physical principles underlying the weather criterion. The paper of Abicht (and many others of similar kind) gives the impression that only much more sophisticated methods than those general at the disposal of naval architects would be adequate.

To me the real problem is the lack of a comprehensive safety concept which allows us to explain explicitly the significance of physical facts in the context of safety. In order to explain what I mean by "safety concept" and to demonstrate that it comprises more than even perfect knowledge of all physical relationships which might be of relevance in connection with capsizing I will indicate some relevant ideas.

3. SAFETY CONCEPT

Safety problems accompanying the development of air and spacecrafts as well as lacking reliability of large electronic systems induced efforts to bring more rationalism into this field. It became generally accepted that perfect safety is not more than a fiction and that all attempts to reach it must lead into a dead end. As a more realistic approach the probability that a certain event (an accident or a disaster etc.) does not occur during a given time was introduced. This probability is a measure of merit which makes it possible to judge different safety provisions. As such it aids much to improve safety. Additionally it is a necessary tool to optimize the safety level with respect to some economic object of the overall mortality rate.

In order to determine the probability that a ship is safe against capsizing, the following steps would be necessary :

1. Determination of the multi-dimensional joint probability distribution of possible wave formations, wind and gust data, icing, loading conditions etc.
2. Limit beyond that the ship will capsize given as function of the data mentioned under 1.
3. Integration over the probability distribution mentioned under 1. up to the limit as defined under 2. The result of the integral is the probability that a ship is safe against capsizing.

Although the principle of this procedure is very simple, I think that neither step one nor step two will be done during our lifetime; it is rather likely that a complete solution will never be achieved.

But if it would be possible, there would be still another problem: It would not make much sense to go through all the trouble necessary to determine the probability and then to choose that probability which represents "sufficient safety" arbitrarily. An optimization (which would have to account for mortality rates, economic aspects etc.) would be in order.

It is obvious that this task would not be much easier than the determination of the probability itself.

The case indicated above is not unique. It happens quite often that the optimum solution of complex problems is known in general but cannot be actually reached because information and relationships in particular are lacking. In such cases it has proved expedient to abandon the goal of an overall optimum solution and to attempt to make relevant details "good enough". In order to apply this idea to our problems we have to define the meaning of "to make relevant details good enough" in our context.

4. SUGGESTED TREATMENT

Capsizing may occur as consequence of each of an infinite number of possible situations.* (See following page) As examples I would like to mention reports of ships having capsized during a turning manoeuvre, or under wind action in calm water, (e.g. the Swedish man of war "WASA"), or under the action of following

waves either after a monotonic increase of heel or after increasing rolling motions.

It may be considered as a "good enough detail" of the safety against capsizing if, for example, a ship can withstand static heeling moments which are likely to occur during its life time. Other "good enough details" might be based on the fact that a ship should not capsize under the simultaneous action of waves and wind. Depending on the relative direction of waves and wind we may have the conditions on which the weather criterion is based, or a similar case where the rolling motion is excited by following waves, or a fast ship heeled by wind and overtaken by a number of high waves.**

Two remarks are to be made with regard to the forementioned examples of "safety details".

1. The physical (or mathematical) models on which the safety details are based are necessarily more or less extensive simplifications of real situations. This does not matter too much. It would be practically impossible to establish a model for each possible realistic situation. Therefore, each model would have to cover anyhow a certain set of real situations, some of them, of course, only approximately.
2. The degree of safety with respect to a certain set of situations depends respectively on the assumption of the wind force, wave height etc. or a combination thereof. Because the optimum degree of safety (i.e. the conditional probability that a ship will not capsize under the assumed set of situations)

is not known explicitly it seems good enough to assume such values of wind forces, wave heights etc. or combinations thereof, which do not capsize ships which are considered sufficiently safe.***

The joint result of both the mentioned circumstances would be a random scatter of the real, but unknown conditional probability that a ship does not capsize under the assumed set of situations. It is not possible to make an estimate of this scatter. But I do not think it of much practical relevance.

For each ship one of the safety details mentioned above will dominate. In general it will be the same detail for all ships of a certain type. Therefore, it could be useful to try to define the range - expressed by suitable ship characteristics - in which a distinct detail dominates. But it would be completely wrong to infer from the existence of such a dominating safety detail that it provides a stability criterion which is generally valid.

It is obvious from the literature that experts more often discuss the advantages of a distinct criterion than the range of its applicability. Because thereby each expert favours his own criterion it is so hard to reach agreement how to formulate safety requirements. It is hoped that the philosophy indicated in this paper might be of some help in the future.

*** In connection with this procedure the discriminant analysis might prove useful. Examples how to apply this tool to stability problems are given in (6)

* It will be seen later that only a finite number of certain sets of situations is practically relevant.

** It would not be too difficult to develop methods similar to those used for the weather criterion to deal with the two last mentioned cases.

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MARINE STABILITY CRITERIA

by

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1. INTRODUCTION

While there is evidence that marine stability criteria have been in existence, in a practical sense, since man first ventured on the oceans, some accidents which still occur routinely indicate that we have not yet paid enough attention to the stability aspects of ships. This paper attempts to examine in a functional manner the need, physical properties, and present methods of solving the problem. Finally, several areas for new development are suggested.

Intact stability characteristics of any floating system are largely determined by the shape of the displaced liquid and the size and distribution of the waterplane area. This geometrical relationship can be given the overall term FORM. In addition, the SERVICE the ship enters will help define cargo weights, external forces, and variations in loading. Finally, the high seas locations visited must be considered in order to determine the maximum EXPOSURE to sea or weather forces that the ship is expected to encounter. Only with a great deal of pure good fortune can any of these three considerations be ignored.

Damage stability evaluation begins by requiring the naval architect to be rather exact with his estimate of the intact stability in the ship. Damage stability analysis, to be complete, requires a two-fold examination. First, the loss

of buoyancy needs to be checked to determine whether reserve buoyancy or trim is lost. Second, transverse stability must be checked to determine whether the system will capsize or come to rest at a calculated heel angle. Until recently, governments have been concerned with damage stability only as a means of protecting passengers (persons who are considered innocent of a knowledge of the sea so that they must depend on seafarers for their safety while on the sea).

The damage stability problem has, of necessity, been rather rigidly handled on passenger ship design in order to be certain that passenger evacuation from the ship is always possible. This has required the ship to withstand the assumed damage without heeling more than seven degrees.

When considering damage or accidental flooding on cargo vessels, governments should be able to state the goal of the ship design in a manner commensurate with the differing reasons for damage protection. Within the past decade, governments have begun to require subdivision protection as a means of protecting the environment. Shipping firms have sometimes provided subdivision protection as a logical method of protecting the large investment in men and materials which goes into today's ships.

However, the process of selecting the relative severity of an assumed

accident is just beginning to be considered by the international community of nations, and it is still not fully defined in terms of the degree of safety expected.

I am sure that the Intergovernmental Maritime Consultative Organization (IMCO) will be mentioned many times during this Conference as the body which holds responsibility for developing marine criteria satisfactory to all nations. To date, in approximately fifteen years of activity, IMCO has developed several stability criteria. The 1960 SOLAS Convention included damage stability criteria for passenger ships and an intact criteria for bulk loads of grain. In 1966, the International Load Line Convention extended the stability assumptions of the earlier Load Line Convention (1930) and required that all administrations party to it satisfy themselves that each loadlined ship have adequate stability and be provided with full stability information.

In the late 1960's, a resolution was adopted recommending an intact stability criteria for ships less than 100 m. in length, based on the recommendations of J. Rahola. (1). This has been modified slightly to develop another recommended standard for fishing vessels. Finally, the Pollution Convention of 1973 has damage standards depending on the type of cargo carried.

Notice that the action to date has been in different areas with different solutions. Not all conventions have included stability criteria. Further, the criteria for fishing vessels is being challenged as to its applicability above 70 m. or below 24 m. in length.

There is a need, in this writer's opinion, to establish a fuller plan for development of stability criteria. All nations should share their experiences to assure that stability criteria are developed in systematic fashion, not just for the sake of orderly development, but particularly to help define the necessary goals and move toward them in a logical manner. Doing so will help avoid overlapping criteria which bewilder marine designers and sometimes seem to act at cross-purposes.

2. NEED

The need for continuing (or accelerating) the search for proper marine stability criteria can be quickly est-

ablished by reciting a few recent casualties known to the author which can serve as examples. Names and flags of casualties are deliberately omitted, since investigations may not be completed. Additionally, we can examine the physical differences in the newer floating systems which need fuller evaluation.

Several years ago, a small ferryboat capsized in good weather with only a small seaway. Investigators ascertained that there was little or no control over the number of people allowed on board for each voyage.

Within the past year, a ship carrying a cargo of bulk fertilizer raw material disappeared in heavy seas only half-day from safe harbor. This kind of bulk cargo has recommended stowage procedure, and there is a recommended standard GM, but no evaluation of the shift in cargo is necessary.

Within the past year, a small coastal cargo ship approximately 60 m. long was loaded with a cargo of bulk sugar which had been saturated with water during a previous voyage. The sugar was not completely dry, and it began to shift as soon as the ship got to sea. The ship was not lost, but only because its course was so close to land that it was able to put in to shore. It was deliberately grounded on a beach to keep it from turning over.

A ship carrying bulk sugar was lost several years ago in the Pacific after the sugar got wet and a liquid free surface was created by this hygroscopic cargo.

A small oceangoing freight vessel had been fitted with an anti-roll tank to ease the ship motions in a seaway. The vessel attempted to tow another vessel and was almost capsized when the free liquid accumulated in the low side tank due to the steady off-centre pull of the towline. Fortunately, the towline was slacked off and the liquid dumped before resuming the tow.

Shortly after a towing vessel was re-powered with an engine almost twice the horsepower of the former one, it turned itself over while maneuvering on a short towline. Apparently, the new HP developed a heeling action so quickly that it completely surprised the experienced captain, who did not reduce the thrust in time to save the ship from capsizing.

The well decks of ships can momentarily

contain many tons of sea water. This has three effects:

- 1) It increases displacement
- 2) It changes KG.
- 3) As a free surface supported by the floating system, it produces a virtual loss of metacentric height.

Depending on the size of the well deck relative to the ship, the above effect may be of little consequence, or it may directly cause the loss of the ship (usually for small ships).

Occasionally, we hear of stability problems which could be explained by a free surface, but the Master can prove that all tanks are quite full. Sometimes, an examination of the liquid tells us that it is not a true solution, but a mixture of two liquids which will separate, leaving the heavier liquid on the bottom with a surface interface the entire width of the tank. If a dense liquid is involved, a large free surface reduction in stability occurs.

The following situation applies to at least four recent accidents which come to mind regarding collisions in three different countries. Two ships in a bow-to-bow passing situation in a river or a harbor collide. In the last-minute attempt to avoid the collision, the alignment of the two ships changes and one strikes the other. The "attacking" ship is usually able to remain afloat and undergo repairs in drydock. The "struck" ship often sinks, capsizes, or is beached to prevent sinking. At this time, there are international subdivision/damage stability requirements to withstand such accidents only on passenger vessels. (NOTE: The 1973 Pollution Convention is not yet in force).

So much for known stability accidents. Let us also consider the stability problems expected from the newer ocean systems which will become the need of the future.

One newcomer to the list of stability problems on oceans is the problem of lifting heavy and relatively large objects at sea. This causes a loss of stability while the object is suspended (in addition to the change of location and adding or subtracting the weight itself from the displacement of the ship). The recent advent of continental shelf exploration systems has changed the naval architect's design problems from an occasional 5-

10-, or 15-ton lift at sea to the daily loading or offloading of several hundred tons. The naval architect has many problems with regard to safe movement of such a lift in a seaway. If the lift is attempted when the supporting ship is reacting to the seaway system by more than a few degrees of heel or trim, the suspended load may develop dynamic forces greater than the factor of safety used in intact stability calculations. Further, these dynamic forces can be expected to be out of alignment with the structure of the lifting equipment, so they may cause a structural failure.

Containerships have introduced not only a new way to transport the world's goods, but also a new way to spread the loading diagram of a ship. Prior to the development of the containership, deck loading was usually reserved for barges on coastal routes, except in time of emergency. Most cargoes, both dry and liquid, were contained inside the envelope of the ship's hull. Now it is possible to load containers to a height of several layers above the weather deck.

High weights can be balanced by weighted containers low in the ship so as to obtain the same average (KG) location as a homogeneously loaded ship. But what effect does this change in gyration have on the century old assumptions governing rolling motion? Might it not be a sinusoidal motion in still water, but susceptible to delay in the cycle due to seaway action?

Thus far, designers have recognized that severe accelerations sometimes occur when loading is spread vertically, but administrations have not found it necessary to offer guidance or limiting criteria. The average Centre of Gravity is still the only guide used.

The newer VLCC vessels have a much simplified cargo movement system, utilizing bulkhead valves. It seems evident that if all such valves were left open, an unsatisfactory condition of stability would exist. How many valves can be left open and still allow unlimited safe passage on the oceans? This author has no knowledge that this has even been investigated.

If a few valves cannot be closed prior to leaving port, the Master should know the extent of stability degradation he has suffered.

The new ships that carry liquefied petroleum at very low temperatures must be loaded and offloaded using a special pipeline of quite limited flexibility. Pumping liquid cargo and liquid ballast

at the same time causes some of these ships to approach a condition of negative stability which could cause a heel of several degrees. Thus, the designer must consider an in-port stability evaluation for these vessels.

Loading variation has also become a very critical part of the stability of any surface effect ship (i.e., foil-borne, air cushion, etc.). Unless the weights are distributed with great care, the foil loading may become too great on one side such that the ship cannot maneuver or cannot even support the load on one foil. In the case of the air cushion vehicle, the distribution of weight is directly linked to the location of the Centre of Pressure underneath the hull. (NOTE: There is no centre of buoyancy and no K, so there can be no KB or KG in the traditional sense). The Centre of Pressure can be changed by a variation in weight distribution or by a momentary change in the seaway. The intact stability criteria developed for this system must consider both the loading of cargo and the severity of the seaway the ship will transverse.

Thus, stability criteria should be developed for all surface effect ships systems, which include the allowable variations in speed in the maximum seaway. Considerable research will be required before the seaway limits in which these ships can operate safely can be accurately predicted in terms of speed, resonance, angle of impact, slope, and character of waves.

The external forces acting on a ship can be categorized as weather forces, as gear forces, or finally as velocity and maneuvering forces.

With regard to weather forces, perhaps the oldest criteria had to do with a ship's resistance to the wind. Sailing vessels have been judged by their ability to stand up to a particular strength of wind for many centuries. Early in the present century, the construction of large high-sided passenger vessels made it natural to utilize a wind criteria for transverse stability on these vessels. In the U.S. Coast Guard, this wind heel criteria eventually was used as the basic stability criteria for all ships. In the light of the many different kinds of ships built in the past two decades, the amazing part is the realization that the wind heel criteria actually served very well in the roll of all-inclusive intact stability criteria until very recently, when many new marine systems began to be used worldwide.

There are, of course, seaway forces that act upon the floating system as

well as wind forces. These forces have never been ignored, but until recently, it has been quite difficult to find an accurate definition of such forces, so that a meaningful criteria could be set up as a guide for ship designers. The mathematical and graphical expressions governing the rolling of a ship have been known for the last century, but it has been difficult to describe the many moods of the sea in mathematical or graphical expressions. Indeed, at this writing, the energy imparted into a ship's hull by a given wave has not been quantitatively determined.

External equipment forces acting on a ship at sea are not few in number, but almost all can be critical in their effect on the floating system. Therefore, they must be given consideration in the design of the system. Examples of such equipment forces are anchors, mooring lines, towing hawsers, fishnets, submersible transfer vehicles, ramps, buoy handling equipment, and structural support legs. This is not offered as a complete list, but it should serve to introduce the importance of these items on stability.

A new field of concern for marine stability is the effect of high-speed or high-velocity hydrodynamics. It was mentioned previously in connection with surface effect ships. The new 30-knot cargo ships also have to maneuver with care when travelling at full speed. Additionally, the very high THRUST now being obtained by small ships with oversized engines, propellers and rudders with extra area for maneuvering in close quarters creates a need for special analysis by the designer.

Solving the hydrodynamics involved in these new designs involves either full-scale or model testing to determine force coefficients, since they cannot be obtained from the standard naval architectural textbooks.

3. PHYSICAL PROPERTIES

Many textbooks have been written about the relationships between the mathematical concepts involved and the form of the ship. What is now necessary is an updating of the physical concepts in each type of ship (or marine system).

With regard to ship FORM, there are three categories which need recognition:

- 1) Displacement Hull (Single Hull) - Large Waterplane Area Ratio.
- 2) Displacement Hull - Small Waterplane Area Ratio.

3) Non-Displacement Hull (Planing Hulls, Surface Effect Ships, Foilborne Ships).

The first FORM area is the so-called "normal" ship which we naval architects call the cargo or passenger ship with FORM characteristics.

$C_b = 0.55$ to 0.80

$C_{wp} = 0.70$ to 0.90

$C_{midship} = 0.85$ to 0.99

The above hull shape fits a standard load line concept. The stability criteria needed for such vessels can be standardized as long as the ship does not have excessive freeboard or very large superstructure and it carries its cargo inside the hull, not as a deck cargo.

However, included in this first category are ships which are not "normal" in the stability sense above. Some are barges, which carry cargo mainly on deck. Some have excessive freeboard, because a very light cargo is carried. Some short vessels have extremely low actual freeboard, although they meet the minimum requirements of the International Load Line Convention. Others carry cargo stacked from the tank top inside the ship to well above the weather deck, which allows the chance that a poor distribution of weights will be used. See Figure (1). Others have been proposed with very high sides and no weather deck, which would be considered a novel form under the International Load Line Convention. Such ships must be specially designed so as to survive when an abnormally large wave puts excessive water in the well.

The second major difference of FORM that must be considered is a very small waterplane area displacement floating system which does not respond greatly to the seaway. This may be in the form of a floating buoy or a very tall floating tube. It may be a floating system with a suspended weight beneath, in which case it is difficult to speak in terms of KG. The most common example of this FORM is the mobile off-shore unit.

The shape of the righting arm curve is often a very high value in the first 15 to 20 degrees, followed by an equally sharp drop with a total range of 30 to 40 degrees. Such a righting arm curve does not lend itself to a standard righting arm evaluation such as is used on ships of high waterplane coefficients (C). It is possible for some of the forms

in this category to achieve a condition wherein G is lower than B.

The third major category of FORM is that which exists when the ship in operation is supported either partially or completely out of the water. Included in this category are the planing hull, the foil-borne ship, and several types of surface effect ships.

The chief physical characteristic of concern in these systems when in full operation is the absence of a centre of buoyancy. Since "G" and "B" are absolutely necessary for any marine system, we must invent some evaluation of support to substitute for "B". Of course, designers of these systems have been well aware of the different types of support. So far, no published design criteria have been accepted as the proper approach for safe passage, to this author's knowledge.

In order to establish criteria for these ships, we can identify at least three sub-groups. The planing hull and the foil-borne ship require speed of motion to create the lift and are supported by the sea by a summation of instantaneous pressure which equals their weight. Therefore, we might create a criteria utilizing Centre of Pressure.

The foil-borne ship is supported by hydrodynamic wings, the lift of which equals the weight of the system, and this weight is balanced by the geometric relationship of the foils.

The surface effect ship (air cushion type) is supported by a distribution of air at slightly above 1.0 atmosphere. The location of the center of air pressure is only partly dependent on the shape of the ship, and more dependent on the distribution of total weight above the air space. A stability criteria created for this type of ship would seem to require a limit on the allowable deviation of G from the ship's centerline at amidships.

Each of the systems above is meant for high speed. This complicates the stability criteria selection for these ships, as they may react violently to wave impact, depending on the relative angle of intercept with waves. At the present time, they are limited operationally by an inability to proceed in waves above a given significant height.

The noted differences make it necessary to consider more than GM or the Statical Righting Arm curve for such vessels. An adequate criteria for these ships should consider an assumed overturning moment caused by the high-speed impact of a

steep wave against one side of the ship.

Thus, it is evident that every marine system should be examined for stability reserve to withstand loading distributions (both vertical and horizontal), internal shifting of weights (deliberate or accidental), addition of seawater or change in buoyancy (temporarily or permanently), and exposure to severe weather and seaway conditions in all anticipated service conditions of loading.

4. BACKGROUND

From the foregoing examples, we see that the NEED for more stability criteria is both increasing drastically due to the many new oceangoing systems, and demanding a refinement of the judgement applied in settling on a particular criteria. Indeed, the variety of ocean systems presently in use shows a need for many different stability evaluations that have never existed until this past decade.

Let us briefly examine the criteria which are in use today. All such criteria can be grouped into two major groups: those that examine the adequacy of the Initial Metacentric Height and those that examine the characteristics of the Righting Arm Curve. The mathematics of these two evaluations has been known for more than a century and thoroughly discussed in many technical papers.

The GM evaluation depends on the geometrical relationships between G and B for a floating body. Some of the formulas used are valid only at small angles of heel, yet sometimes they are inadvertently used for large angle calculations. The Righting Arm evaluation can be valid at all angles of heel, but it is NOT automatically valid. If any change in displacement or trim occurs with heel, the curve should be evaluated for the magnitude of change. However, assuming for the moment that the curve does truly represent a correct summation of the attitude of the ship at each and every angle of heel, the Righting Arm Curve offers the better opportunity to ascertain the statical ability of the ship to resist overturning.

What is the proper variation of GM and of the statical Righting Arm Curve?

Historically, until the 1930's, stability criteria had been almost entirely left with the ship's designer rather than with the classification societies or the administrations. Today the responsibility for setting a minimum

criteria for the designer is clearly with the administrations.

Professor Peabody (2), writing between 1904 and 1917, stated that there were "two distinct types of curves of stability, those for sailing ships and those for steamers". He characterized the sailing ship curve as extending to or beyond 90° angle of heel, while the steamer was acceptable at 60°. No mention of downflooding was made in this range. Also, in mentioning these different curves, Professor Peabody felt obliged to defend the steamer range of 60° by pointing out that "the stability must not be excessive, as it leads to quick and violent rolling which may be very unpleasant or even dangerous".

The Coast Guard has a current stability research project with Hydronautics Corporation (3), who with Nickum and Spaulding, Inc. (4), have conducted a stability literature search in the English language. They have concluded that only three technical papers out of 212 published between the years 1746 and 1929 had actually been bold enough to propose a stability criterion. One of these was by Sir John Biles (1922), (5), who recommended that GM in a lightship condition be not less than 1.0 feet. In 1925 Mr. C. Frodsham Holt (6), proposed minimum and maximum GM values and related them to safe rolling periods at sea.

The 1930 Load Line Convention mentioned that the stability of the vessel was presumed to be adequate, thus establishing an undefined responsibility for providing adequate stability and leaving open the question of degree of responsibility between the designer, the ship's Master, and the administration. Thus, the designer has retained responsibility for providing a ship which can be operated safely in all conditions of loading, the Master retains responsibility for adequate stability at any particular point of a voyage, and the administration retains the responsibility for setting standards and reviewing the applicability of ships of various types to meet the standards. Almost all actual criteria in use today have been developed within the past thirty years.

The criteria used in the U.S.A. since early in the 1940's has been an initial GM criteria based on wind pressure on the total area above the waterline.

$$GM = \frac{PA \bar{h}}{\Delta \tan \theta}$$

where:

$$P = .005 + \left(\frac{L}{14,200}\right)^2 \text{ tons/ft.}^2 \text{ Ocean}$$

L = length between perpendiculars in feet

A = projected lateral area in square feet above waterline

h = vertical distance in feet from centre of A to centre of underwater lateral area or 1/2 draft

Δ = displacement in long tons

θ = angle of heel to 1/2 the free-board to deck edge or 14 degrees, whichever is less

This criteria has been used to satisfy all intact stability requirements for passenger ships in weather at sea, and for all cargo ships since 1952. Of course, Damage Stability has been reviewed, because of well-publicized accidents like the TITANIC and the LUSITANIA, on passenger ships since the 1930 Safety of Life at Sea Convention. Also, due to several capsizings caused by sudden movement of large numbers of passengers on multi-deck excursion steamers, an analysis of resistance to heel caused by the movement of passengers was required.

$$GM = \frac{Nb}{24 \Delta \tan \theta}$$

where:

N = number of passengers

b = distance in feet from centerline to geometrical center of passenger deck area on one side of centerline

Δ = displacement in long tons

θ = angle of heel to deck edge or 14 degrees, whichever is less

The three calculations are plotted on a graph of DRAFT vs. REQUIRED GM (minimum). No maximum GM is set, this being left to the ship's designer. If a passenger ship is designed with excessive GM, the customer would soon regulate the condition.

Although it was not specifically stated at the time, the latter two evaluations (damage and passenger heel) were "surprise" criteria as opposed to the weather criteria in which it is presumed that the ship's crew can prepare the ship by closing all weathertight openings in the several hours prior to the storm. It was not necessary to label these as "surprise" criteria in the 1930, 1948, and 1960 SOLAS Conventions regarding passenger vessel subdivision because the Margin Line concept was used as the

flooding and heeling limit.

Theoretically, this concept would not allow the deck to get wet.

However, when it became necessary in 1969 to define the limit of flooding for the subdivision concepts adopted in the 1966 Load Line Convention, which allowed deck edge immersion, the IMCO delegations agreed that weathertight doors, etc. should be considered open when evaluating a damaged condition. Thus, a new element of judgment (i.e., degree of crew preparedness allowable) has been recognised in the quest for stability criteria.

The use of the required GM criteria (wind, passenger, damage) in the U.S.A. as the all-inclusive stability formula was reasonably successful until 1960. After this, it became increasingly apparent that the newer variations of floating systems were presenting new problems for this criteria.

Only thirty years ago, we could classify almost all oceangoing ships in three broad services (e.g., passenger, cargo, tanker). Now, many passenger ships have become specialized cruise ships or combination passenger-vehicle ferries; tankers vary from 1000 tons to 500,000 tons displacement, and their cargo density may vary from a specific gravity of 0.5 to 1.3. Additionally the bulk chemical ship has joined the ranks of bulk liquid carriers with cargoes that may vary from 0.5 to 2.0 in specific gravity.

Cargo vessels are no longer only built as multi-purpose vessels. Many are built for specialized trades, such as the container ship, the roll-on/roll-off (no passengers) cargo ship, the grain ship, the bulk/oil carrier, the steel products ship, etc. Such a variety of ships makes it increasingly difficult to call any shape or service the "normal" ship. Indeed, to this writer, it seems no longer possible to reach an understanding with an audience by using the term "normal or "average ship". Some have high lateral area which makes the wind criteria important; others are so beamy as to make the wind criteria no real test of the ship at all.

In an initial attempt to compare the initial GM criteria with the Rahola values for Righting Arm criteria recently accepted at IMCO, Mr. W. Magee, former Head, Hull Scientific Branch, Coast Guard Headquarters, examined the righting arm curves of a cargo vessel (LIBERTY) in the load line condition with the minimum GM as required by the Wind Heel formula. The result was a righting arm curve which almost exactly fulfilled the recommendations of Dr. Rahola. While this was gratifying

in view of the good stability record of the LIBERTY, it was recognised that we could NOT adopt the comfortable attitude that the U.S. Wind Heel formula is always equivalent to the most widely accepted righting arm criteria. However, recognizing the great variety of ships in service today with block coefficients varying from 0.50 to over 0.90, some having low freeboard while others have huge excesses of freeboard, and some having two or three times the beam formerly associated with a specific length, it would be quite extraordinary to expect that the Wind Heel criteria would produce 0.08 metre-radians under the righting arm curve for every ship in every condition of loading.

Indeed, our recent experience has shown us that the Wind Heel criteria definitely has practical limits. We are reasonably sure, for instance, that the Wind Heel criteria for a VLCC has very little use, since the ship has a much greater requirement to meet the damage stability criteria which are increasingly evident in international thinking.

To date, unless a unique SERVICE condition such as grain loading, towing service, etc. required a special analysis, most U.S.A. flag vessels have been examined only to be certain that they had the minimum GM for Wind Heel, which varies with draft. A sample Wind Heel graph such as is provided in the stability information to the Master is shown in Figure (2).

The stability criteria which have been mentioned in this section have been utilized over the past two decades by various administrations and by designers. In the past two or three years, however, it has become evident to some administrations that the existing stability criteria are not always appropriate for the very small vessels and may not be appropriate for the very large vessels. The U.N. agency IMCO has found it necessary to modify slightly a recommendation which was published as Assembly Resolution No. 167, which was entitled "Recommended Stability Criteria for Cargo and Passenger Ships Less than 100 m. in Length". This criteria needed a modification in order to be considered proper for fishing vessels, and a subsequent criterion especially for fishing vessels was recommended.

Next, it was realised that this criteria would be inappropriate for the ships in the oil industry which carry pipe and supplies to the drill rigs out at sea. Because of their form, the statical stability curves

for this shape vessel cannot meet the criteria.

The fishing vessel criteria just mentioned have been challenged twice: First, with regard to the large fishing vessels, 75 to 100 m. in length, the need for this much stability has been questioned. Secondly, as a criterion for fishing vessels less than 24 m. (79 feet) in length, it appears to be less than enough to provide adequate stability in all operating conditions. Specifically, the trouble is suspected to lie in the use of righting arms rather than a righting moment curve. Figure (3) compares identical righting arm curves which look adequate, but the small ship may be inadequate when the actual righting moment and available righting energy are pictured against the expected energy in a seaway. Since the indication is that the smaller ship reacts to a greater degree to sea spectrum which are present in the ocean most of the time, it is only possible to state the problem and not the full solution, as the representatives have not fully discussed the technical problems involved.

At the other end of the spectrum of marine vehicles, the VLCC and ULCC are also testing the ability of designers and regulatory authorities to provide a satisfactory intact stability criteria, because of their extreme size, the moment righting curve may show a high resistance to heeling under force of wind and seaway that the present criteria may offer very little protection and may be unnecessary. Let us hasten to qualify the previous sentence. Before dropping all intact stability criteria from large vessels, the authorities must be satisfied that none of the service conditions have an inherent stability reduction which must be accounted for. One such stability reducing condition which requires continued examination is found in large combination oil-bulk carriers which are designed with full free surface in the holds, or in oil carriers with cargo tanks which may be interconnected. For these reasons, slosh bulkheads and tunnel piping systems must be considered very carefully before finishing the design.

5. RATIONALE FOR MARINE STABILITY CRITERIA

In order to produce a design which is well qualified with regard to stability, the designer should

examine the FORM he has chosen for the SERVICE intended in the most severe weather predicted for the area of operation (i.e., amount of EXPOSURE).

In addition, it is important to the validity of his assumptions that each event examined be clearly distinguished as either a surprise event for which the crew may not be prepared (in which case the ship must save itself) or an event which the crew can be expected to prepare for. Examples of the "surprise" and "prepared for" stability events are:

"Surprise"

- 1) Passenger Heel
- 2) Bulk Cargo Heel (Slack)
- 3) Towline
- 4) Damage (Collision)
- 5) Free Surface (Deck Well)

"Prepared For"

- 1) Storm (High Seas)
- 2) Bulk Cargo (Full Holds)
- 3) Lifting Cargo
- 4) Floodings (Doors or Hatches, etc.)
- 5) Free Surface (Tanks)

The reason that it is important to make the distinction above is that we must consider an angle of downflooding as a termination of the righting arm curve. The openings (hatches, doors, air pipes, vents, etc.) on the deck and in the superstructure must be considered open in dealing with a "surprise" stability criteria, whereas we can logically assume most openings secured weather-tight when considering an event for which competent seamen can prepare the ship, given several hours' notice.

With regard to the form of analysis used for the stability examinations, the usual choice is whether to examine a GM value or a righting arm curve.

In the opinion of this writer, ships should be examined by the designer for adequate stability on the basis of the righting arm curve, including an analysis of the full range of stability.

The GM value or KG value can be just as useful when it is used to present the stability information to the master of the ship. Often this can be shown in easily read tabular format

or in graphs. The master has less (or no) time to calculate stability while he is attending to navigation, radio communications, personnel problems, etc. Still, he is responsible for providing adequate stability (within the capabilities of the ship itself) at any point in the voyage.

Returning to the examination of stability in the design and construction stages, all of the heeling effects which may logically be considered of sufficient magnitude should be plotted against the righting arm curve for each loading condition.

This should always include Wind and Seaway energy heeling moments and the effect of shifting weights, such as bulk cargoes, and free surface corrections for slack tanks. Additional heeling moments to consider are:

1. the effect of water in a deck well
2. rudder force
3. loading or offloading cargo at sea
4. special equipment over the side of the ship at sea
 - (a) fishing
 - (b) dredging
 - (c) towing

The remark above about heeling moments of "sufficient magnitude" will naturally raise the question "What is sufficient magnitude in each case?"

In the writer's opinion, the following figures could serve as an interim guide to signal the need for an examination of stability.

Regarding deck well water momentarily trapped on deck, we can consider each of the three effects noted earlier:

1. Increase in Displacement - greater than 5%
2. Raise in KG - greater than 10% of original KG
3. Free Surface - greater than 10% of waterplane inertia

If any of these is high, the total effect on stability should be realistically examined over the full range of stability.

Regarding Rudder force, the condition should be checked if the rudder area is great or 5% of the underwater lateral area or if the speed of the vessel exceeds a Fronde No.=1.0. There should also be a ratio of SHP/Displacement which could give the designer warning to examine steering stability, but the writer has no idea at the moment what the threshold figure should be. The range of this ratio in ships now sailing varies from approximately 0.1 SHP/Ton Displacement for VLCC to almost 50 times this figure or approximately 5 SHP/Ton Displacement for some modern tugboats. Hopefully, research will provide this answer.

With regard to loading cargo at sea, this is at present almost entirely within the fishing and dredging industry, unless one includes the transfer of materials from ship to offshore mobile unit. In each case, the ship needs to be examined in a beam seaway.

The offshore mobile units are often the subject of extensive model tests for seaway characteristics prior to being built. The ships which supply and receive items from these units are less fortunate. Also, they must be ready to moor almost under the mobile unit in order to be within the swing radius of the lifting booms provided on the mobile unit.

The mooring system cannot be shifted each day in order to remain directly into the wind and sea; thus, the effect of partial load removal and load addition on the trim and heel of the ship in a moored condition in other than head seas must be reviewed. Again, research is needed to tell us the design limits in a seaway.

With regard to dredging or mining at sea, the ship will be subject to extreme variations of free surface. These should be examined against both light and loaded righting arm curves.

A special note should be added here regarding the calculation of free surface. The normal method of free surface adjustment has been to use the mathematical equivalent of a vertical change in the position of G. This creates a new righting arm curve. However, the new righting arm curve is only accurate for the approximate range of 0-10 degrees heel, for which the trigonometric relationship of B, G, and

M remains valid. For greater angles of heel, an error in the righting arm curve develops, which is often a substantial error. The cure for this problem is to establish the basic righting arm curve using the "virtual" change in G and then to adjust the curve at larger angles of heel by calculation of the actual off-center liquid weights by the moment of transference method.

The cargo which is being dredged or mined out of the sea may have more than one free surface in each tank, since it is a mixture of water and solid material. Thus, even if the cargo tank is filled to the top with water, the material which is in the hold may shift to one side and cause capsizes.

With regard to ocean towing and fishing gear over the side, the righting arm curve should be evaluated in a beam seaway against a heeling moment generated by the towline or fishing gear. With regard to fishing vessels alone, it is recommended that the designer check the stability in a beam seaway when the loaded gear is suspended from the lifting boom. The ship will respond to the sea in the heaving mode while the load will remain in position.

Another item in the stability evaluation which appears worthy of consideration on some floating systems is the large gyradius due to either extreme vertical dimensions such as a mobile offshore unit or due to the ability to load cargo equivalent to a very high deck load. The present system of stability analysis begins with the determination of average centre of gravity KG. For the purposes of constructing the righting arm curve, all weight is initially assumed at G. The judgment and experience used in looking at a righting arm curve or in selecting an arbitrary figure of area under the curve as the safe limit has been under the assumption that all cargo was inside the FORM of the ship. In the containership trade, this is no longer the case. Also, in the instance of a jack-up mobile offshore unit while in transition on the ocean, there is a great difference in gyradius compared to treating this unit as a barge.

When we examine only the difference in rolling period in still water, the change in gyradius does not appear to affect the situation greatly. However, these two new developments on the oceans (jack-up units in transit and containerships)

should be considered in a seaway from several angles of heading, including head, beam, and following seas at the speeds indicated in each design.

It is hoped that after these designs have been examined in the full environment, designers should gain an insight as to which are the important features of a righting arm curve for these ships. Purely as a guess, we may discover that it would be desirable for a high speed, full displacement ship to have a sharp rise in the righting arm curve between approximately 15 and 25 degrees of heel. Perhaps other features will arise which will help the evaluation of large gyrodium systems and high speed/hull displacement ships.

Another subject which needs renewed discussion among naval architects is the degree of flooding protection required, including the stability of partially flooded ships.

To date, the assumptions used for flooded ships have been a mixture of two different philosophies. First, the empirical approach is utilized, assuming that the ship was flooded on a compartment basis arbitrarily. Second, it is assumed that the ship is flooded because of damage. (Most often a side collision has been envisaged.)

The compartment concept was originally understood to mean the entire space from side to side of the ship between two adjacent transverse bulkheads. The effect of longitudinal bulkheads was not spoken to. Thus, the compartment concept has become somewhat vague in application. Also, the use of the compartment concept on floating systems such as mobile offshore units which may extend 100 feet or more above sea level makes it difficult to conceive the accident which will result in the assumed flooding.

The damage concept can be less empirical, provided agreement can be reached on the most probable size and location of damage.

There are other possible flooding situations which could be used in a logical damage stability criteria.

The lowest order flooding that might be envisioned is that from the loss of watertight integrity of topside fittings such as doors and hatches. This type of flooding would be of value in a damage criteria which envisaged storm damage to the weather deck area. This explanation of flooding has not yet been used in a

criteria, to the author's knowledge.

The next step is a minor damage criteria wherein the ship is holed anywhere on the side or bottom shell, but the damage is not extensive. Such damage would be a logical selection for a criteria which called for low energy collisions or dock damage. This concept has been recently introduced in the concept of damage protection for liquefied gas carriers in the latest IMCO code.

Finally, when high-energy collisions are envisaged for damage criteria, an extent of damage is specified by depth of penetration, longitudinal extent, and vertical extent of damage. Usually they are $B/5$, $L/3$ and base line upwards without limit, respectively. These limits are defensible when speaking of full displacement single hull ships, but not when addressing some newer systems. If the designer desires or is required to protect his design against a high-energy collision situation on the high seas, logically, the limits of the damage assumption should be applied randomly anywhere along the length of the ship. There is statistical evidence that the collision is more likely in the forward half of the ship, but this does not mean that there is no danger to the stern.

It is not logical, and it is quite unnecessary, in this writer's opinion, to presume "damage" when it has been decided to adopt a one-compartment flooding criteria. To do so presumes that the ship will be struck at only a few precise locations along its length, due to the assumed longitudinal extent of damage overlapping the transverse bulkheads.

The foregoing thoughts on damage stability rationale can be completed by recommendations for progressive development. In order to develop the more logical approach of "size of damage", it is necessary to consider all FORMS (single hulls, catamarans, semi-submersibles, etc.) versus probable collision partners and to conduct a combined stability and structural evaluation.

Next we might turn to the rationale for stability criteria on high-speed ships. Included are planing boats, foil-borne ships, and surface effect ships. Since these ships operate at high speed and their hulls are partially or completely out of the water, the process of adopting a righting arm criteria becomes difficult because the displacement changes.

The approach taken by the author's administration thus far has been a combination of the full displacement righting arm evaluation versus a wind heeling arm, but requiring a greater reserve area under the curve in an attempt to provide some degree of protection for the high-speed/low displacement condition.

Additionally, such a ship is limited operationally by the human criteria of an inspector riding the vessel and setting a limit on the height of seaway in which the ship may operate. Obviously, this approach does not allow the ship to be used anywhere in the world at any season of the year.

The new marine stability criteria should be developed for each of the general FORMS of high-speed ship. In each criteria, account should be taken of the overturning effect of a high-speed impact with a short-crested wave top on one side of the hull at $B/2$ from the centerline.

Account should also be taken of the effect of high-speed traverse of a beam seaway, including the possibility of synchronous roll leading to capsizing.

It is evident that the clear height above still water is significant to the latter two types of craft considered above. If the ship operates in a seaway of significant wave height (H_s) equal to one-half the clearance height, the ship will statistically be impacted by no more than one wave in 3000. (NOTE: H_s is taken equal to the average of the one-third highest waves.)

Until research shows that closer limits are acceptable, administrations faced with the decision as to level of safety for passenger craft might be tempted to limit such high-speed ships in a probabilistic manner such as this. Thus, if a designer contemplates a full ocean design, he would have to consider one with an 80-ft. clearance instead of developing these designs gradually in size. This would cause the designer to jump from the existing designs, which are generally limited to coastwise or semi-protected service, to hugh ships without the benefit of experience gained in the design, construction, and operation of intermediate sizes. It would be an extremely onerous design condition, so let us open several ideas for alternatives.

First, the designer might provide a craft that is both light enough to

be lifted and yet strong enough to withstand repeated very high velocity impact in waves, and which will not be tripped into a capsize when impacting on waves.

Another possible answer could be in developing a real-time ocean mapping system which had at all times a composite view of weather and the seaway systems, including the effect of a confluence of seaway storm systems, and which could predict the development of seaway systems for twelve to twenty-four hours in advance along the projected route of any such high-speed craft.

Third, the naval architectural, oceanographic, and materials research worldwide communities could work on the problem from all sides and, over the next decade, develop a combination of the above suggestions which could assure the ocean passenger that such an ocean crossing would be comfortable at all times and assure the administrations that it would be safe.

The writer feels that such ocean travel systems are the next step in high-density passenger travel as well as high-value cargo transport, but only after the necessary development work has given us the answers to proper design.

6. CONCLUSIONS AND RECOMMENDATIONS

Adequate stability is dependent upon a proper analysis of the FORM of the floating shape in the maximum EXPOSURE (seaway or weather) for which the shape is intended, taking into account whatever is unique about the SERVICE of the shape.

None of these categories may be completely overlooked. Parts of the latter two categories may sometimes be eliminated, only if it is obvious that another criteria governs the shape of the desired righting arm curve.

Looking back through recent history, often it has been the practice to presume that one criteria or another was enough to provide an adequate level of safety in all areas, even though they may not have been examined by the designer. It is no longer valid to assume that one criteria will be sufficient to cover all stability needs of the ship if its form, speed, or area of operation have enjoyed the ship design innovations of the last twenty years. It is, therefore, incumbent upon the marine designer

to examine his ship or system in each of the three broad areas mentioned above and to provide a reserve of stability so that all uses of the floating system are covered.

When administrations are concerned with establishing a stability criteria of a sufficient level of safety to assure the citizens and industry that lives and property are safe at sea, the above conclusions should be stated in another way, as follows:

It is not sufficient to publish one criteria for passenger ships, another for cargo ships, another for tankers, another for oceanographic ships, etc. The error occurs in placing all ships of a single SERVICE in the criteria. The most important of the three categories is FORM. Thus, a stability criteria for passenger ships must be different if the passenger ship is 100 ft. long versus 1000 ft. long, or if the ship is a catamaran instead of a full displacement mono-hull, or if the ship is a high-speed, surface effect ship.

The major conclusion of the discussion on Rationale is, in the writer's opinion, the realisation that so much research is necessary if naval architecture is to provide the measuring stick of safety commensurate with the many new oceangoing systems in use today.

Almost all of the ideas in the section on Rationale indicate a need for continuing research to test newer FORMS in both model and full-scale seaway situations and to correlate calculated results with actual motions of full-scale floating systems.

The results of such tests and calculations must then be plotted systematically on the Righting Arm or Righting Moment Curves to determine the proper design parameters. Righting Moment curves will be necessary to adequately define parameters of small ships and high-speed ships, since the dynamic aspects of such ships in a seaway are much more critical than a static evaluation of stability.

In summary, it seems abundantly clear to this writer that the existing, largely empirical marine stability actually in use have already been severely challenged by the wide variety of new marine systems. It is time for a much bolder approach if we are to provide meaningful criteria for the future.

In conclusion, the writer would like to state that while taking full

responsibility for the opinions expressed here, he cannot claim credit for originality of all of the ideas shown here. Many of the ideas expounded here are gleaned from hours of discussions with naval architects of several countries, as well as other naval architects from his own country.

What is expressed here is the author's summation of ideas on marine stability criteria which he has seen, heard, or thought about, and which, in his opinion, offer the most logical approach to future development of marine stability criteria.

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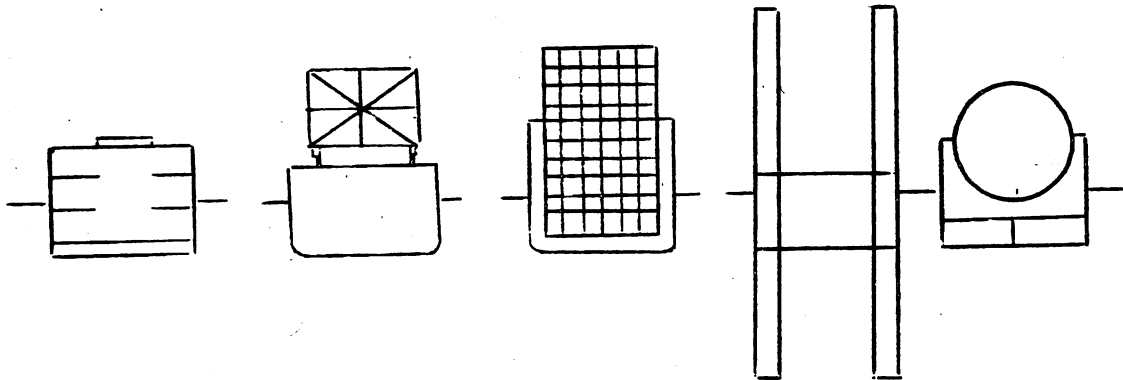
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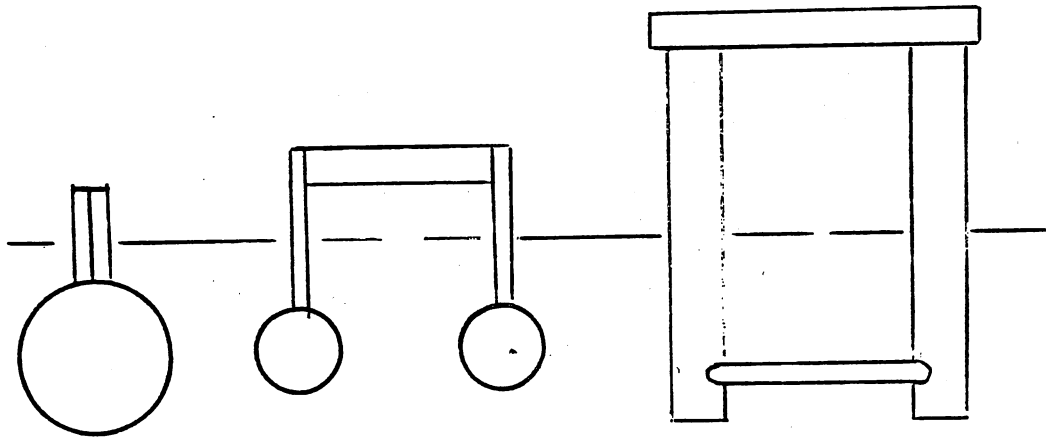
S.N.A.M.E.

TABLE I

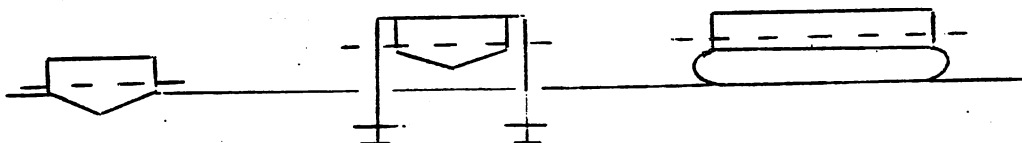
<u>LIST OF FORCES</u>	
<u>INTERNAL</u>	<u>EXTERNAL</u>
<u>INTACT STABILITY</u>	
PASSENGER SHIFT BULK CARGO SHIFT LIQUID SURFACE EFFECT WEIGHT MOVEMENT AT SEA CARGO LOSS CARGO GAIN ICEING LOADING VARIATIONS	WIND HEEL SEAWAY FORCES GEAR FORCES MOORING TOWING DREDGING FISHING MANEUVERING FORCES RUDDER THRUST VARIATION
<u>DAMAGE STABILITY</u>	
EFFECTS OF FLOODING CHANGE OF C. G. or (if damage is envisaged) CHANGE OF C. B. plus (when cargo is lost) CHANGE OF C. G.	WIND HEEL SEAWAY FORCES GEAR FORCES MOORING TOWING DREDGING FISHING MANEUVERING FORCES



DISPLACEMENT HULL — Large Cwp Ratio



DISPLACEMENT HULL — Small Cwp Ratio



LOW or NON DISPLACEMENT HULL

FORM CATEGORIES

Figure (1)

Cleary

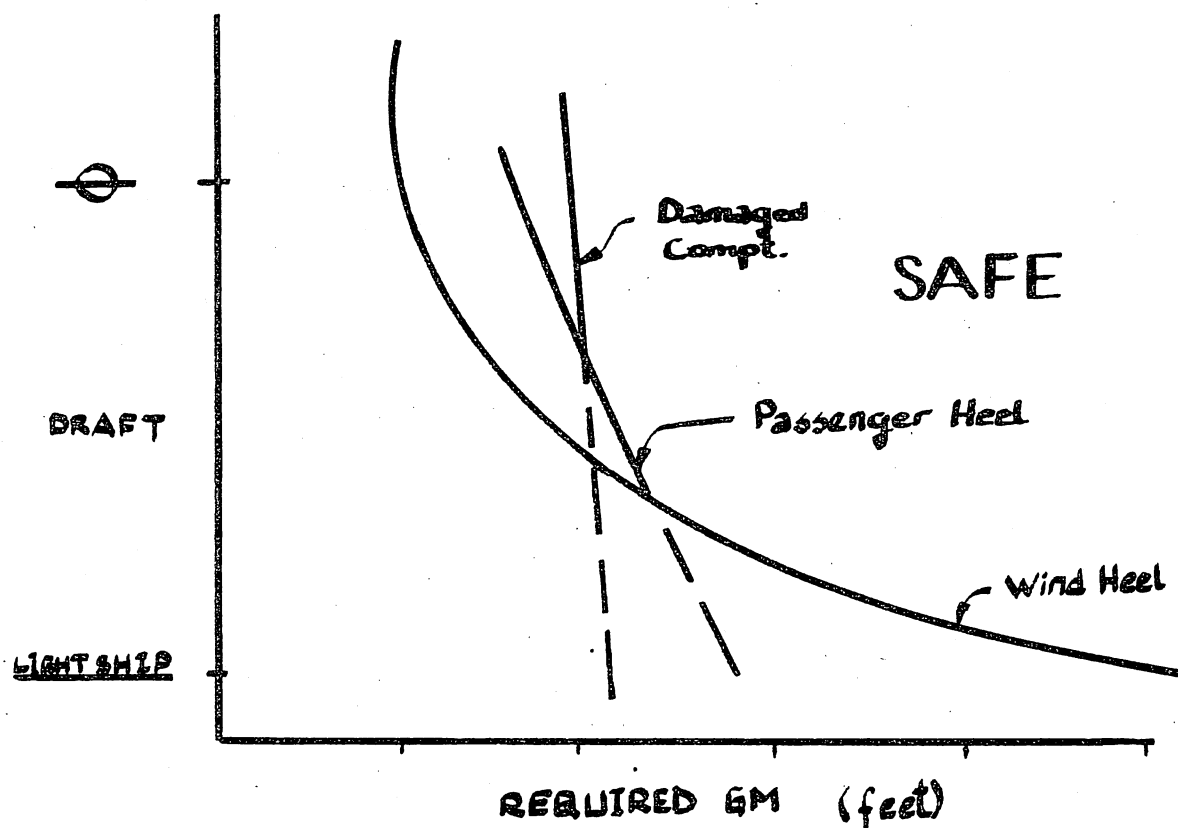
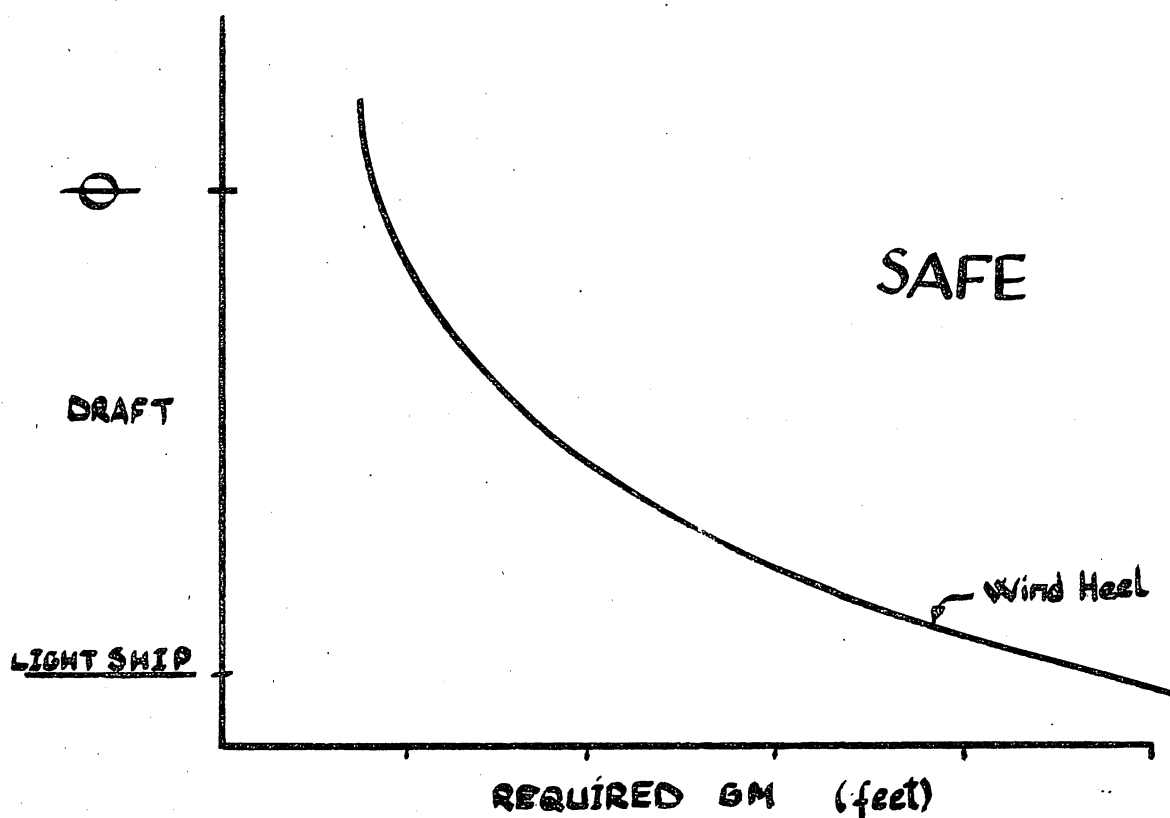
**PASSENGER SHIPS****OTHER SHIPS**

Figure (2)

Cleary

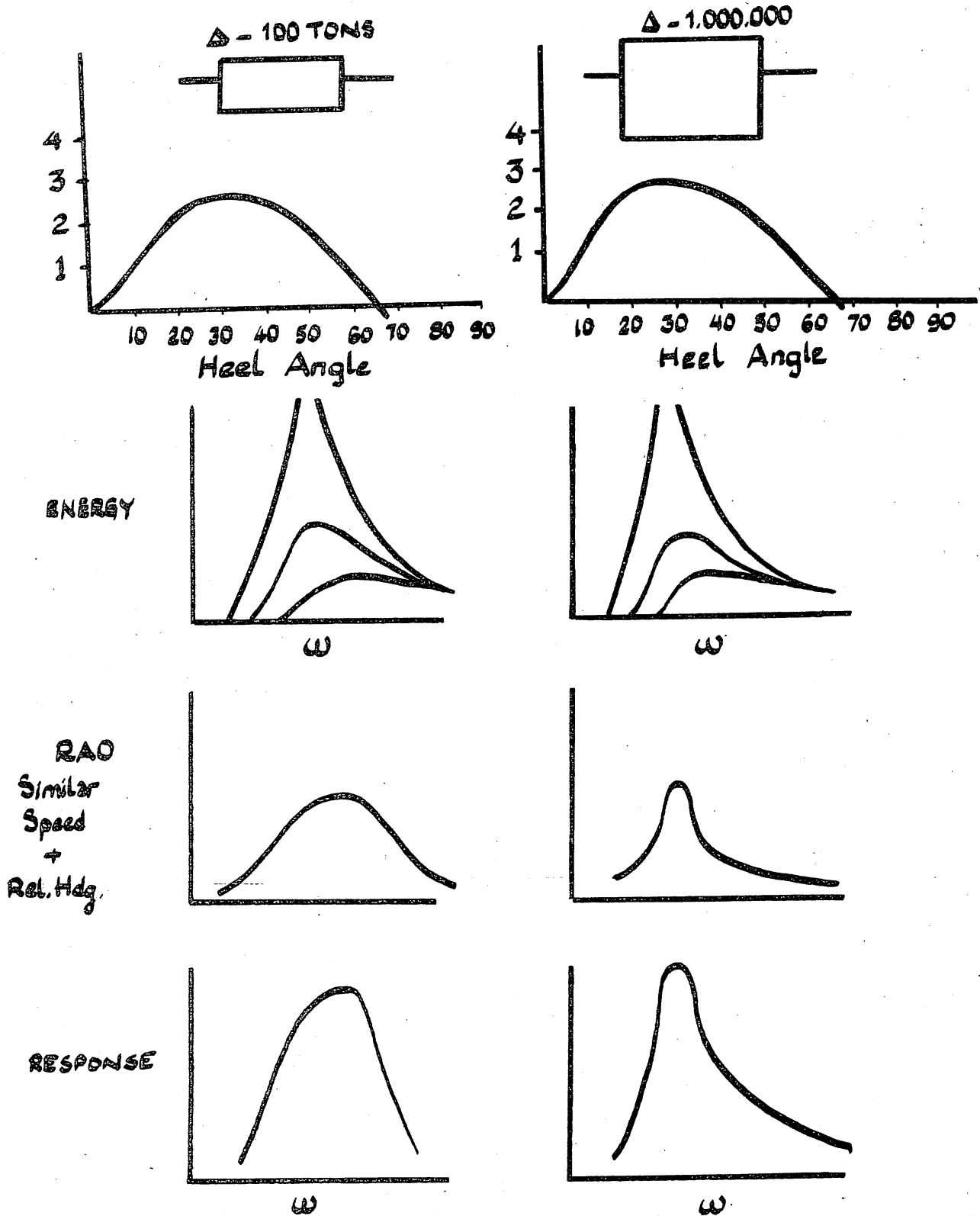


Figure (3)

RATIONAL STABILITY CRITERIA AND PROBABILITY
OF CAPSIZING

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1. INTRODUCTION

In 1962 IMCO started its work towards the development of stability criteria for fishing vessels and for small passenger and cargo vessels of less than 100 metres in length. The work was completed in 1968 and the criteria were introduced by IMCO as recommendations [1].

At the time IMCO had started its work towards elaborating international stability criteria several countries introduced some stability criteria going beyond the requirements of 1960 SOLAS Convention [2]. All national requirements and regulations were carefully analysed, but the main source of inspiration for the evaluation of IMCO-Criteria was an analysis of casualty records and a comparison of the various stability parameters for vessels which capsized with those which were found safe in service [3]. From all the stability parameters which could be used as stability criteria, the ones chosen for further analysis were those which lead to the lowest position of KG. This was decided on the basis of statistics. The details of the approach adopted at IMCO are well known from many references, such as [1], [2], [3], and will not be discussed further. It should be underlined, however, that the approach was a pure statistical one. Its main drawback was that the available data constituted only a small population of vessels. In consequence, the statistical analysis was not satisfactory.

During the discussions at IMCO, the view was expressed several times that in future more rational stability criteria are needed. Rational stab-

ility criteria are understood to be those that can take into account the physical phenomena occurring during the ship's service and all external forces exerted on them. The development of such rational criteria is a long-term task and for this reason simpler statistical approaches are first adopted at IMCO.

2. PROBABILITY OF CAPSIZING

Considering the rational criteria, it is possible to come to the conclusion that the most universal stability criterion should be the probability of non-capsizing of the vessel during its whole life.

However, in the calculation of the probability of non-capsizing of a vessel many serious difficulties are encountered. In such a calculation it is necessary to take into account:

- a) the variation of stability characteristics with time,
- b) the variation of external loads with time and in the various areas
- c) the period of service,
- d) areas in which the vessel is intended to operate.

Therefore, if in the definite period of time t_1 , which belongs to the period of service T , there exists a set of conditions W_1 , which consists of any combination of the sea state, of wind force and direction and of any other external loads, of heading of the vessel, its speed and stability, then the probability of

non-capsizing of the vessel, X , can be calculated as a conditional probability:

$$P_{ti}(X) = P(X/W_i)$$

If, in the period of service T , there exists the possibility of n sets of conditions W_i , and each of them can be repeated m_i times, then the probability of non-capsizing of the vessel can be calculated by the formula: see [4]

$$P_T(X) = \prod_{i=1}^{i=n} [P(X/W_i)]^{m_i}$$

Discussing the probability concept of stability criteria, it should be noticed that the probability can never reach the value of 1.0, so the probability of non-capsizing will never be 1.0. From the point of view of safety a very slight deviation from 1.0 may be allowed. If we take for example the probability of non-capsizing equal to 0.99, it means that one vessel out of a hundred could capsize. This can not be accepted. Therefore, a much smaller deviation should be adopted: equal to a small fraction of one per cent.

A question now arises on whether it is possible to calculate the probability with such great accuracy; moreover, in the vicinity of 1.0, the dependence of the probability upon stability characteristics is strongly linear. On the other hand, occurrence of extremely severe conditions of wind and waves is a very rare phenomenon, and at present the probability of its existence cannot be calculated on the basis of statistics.

Therefore, the error in the calculation of the probability of non-capsizing should be of a lower order than the probability deviation which could be accepted as a criterion.

So the usefulness of the probability concept in establishment of the stability criteria may be questioned. This was pointed out by Sevast'yanov [4].

3. POSSIBLE METHOD OF DEVELOPING OF RATIONAL CRITERIA

The impracticability of using the probability of non-capsizing of the vessel as a universal stability criterion leads to the consideration of

other possible methods of estimation of rational stability criteria.

Any "rational" stability criteria should take into account all possible external loads and the behaviour of the vessel under the influence of such loads. The view has been expressed many times that a definite "safety margin" should be estimated for each situation.

From all possible external loads part of them can be calculated with sufficient accuracy. These loads may be static or dynamic and they appear as random events but not as random functions. The probability of their occurrence is rather low, but nevertheless the vessel should be able to meet their full value and the possibility of their occurrence has to be taken into consideration. In general, the probability of the occurrence of the combination of two or more of these loads, or the combination of these loads with loads which are random functions, is supposed to be extremely low as compared to the probability corresponding to the remaining allowable risk.

There are, however, some cases when the probability of occurrence of two or more different loads is supposed to be significant, and they should both be superimposed. It is recommended that such loads should be taken into account in way of balance of levers, and a certain safety margin should be left in each case.

There are, however, other loads, such as wind and sea, which are random functions.

The necessary margin of safety for such loads can be estimated only in probabilistic way. This statement can be further explained. If we take the simple balance of heeling and righting levers with respect to loads which appear as random functions, then the extremely severe conditions should be included in the balance.

However, this will lead to a large safety margin which will never be used throughout the entire life of the vessel. Moreover, a safety margin which is too large has many negative effects: poor seaworthiness, unsatisfactory economic factors, etc.

Therefore, when developing stability criteria it is more useful to analyse actual possible situations of the vessel, taking into account the

characteristics of external loads. These characteristics, e.g. wind pressure and wave heights, always appear together with a certain probability. Therefore, the description of various analysed situations can be made only on the basis of the concept of probability. In this concept, the stability criterion should be the probability of non-capsizing of the vessel in several selected dangerous situations. The most important part of the method is obviously proper selection of these situations. Once the situations are selected, it is necessary to formulate a mathematical model of capsizing for each of them, to solve this model and to evaluate the safety of the vessel - in terms of probability of non-capsizing or, in simple cases, in way of direct balancing of heeling and restoring moments.

This procedure was partly adopted in the development of national stability regulations of some countries, such as Reference [5]; it was also suggested at IMCO Conferences and by some authors, see References [4], [6].

The procedure, although giving a chance of completion in a reasonable period of time, still needs considerable research work.

Firstly, the selection of typical dangerous situations which characterises a ship's safety is a very difficult task. For this purpose it is necessary to enumerate all possible situations with respect to external forces exerted on vessels of various types, to evaluate the degree of risk in each situation and to evaluate the probability that the vessel will be found in each situation. The performance of such an analysis requires thorough statistical analysis based on a large number of data. Secondly, mathematical models for various situations of the vessel are far from perfect. Only some simple cases are solved satisfactorily. More complex situations are being investigated but, in most cases, the results achieved are not sufficiently exact.

4. CONSIDERATION OF DANGEROUS SITUATIONS OF THE VESSEL

The dangerous situations of the vessels can be selected on the basis of:

- a) casualty records and statistics,
- b) model tests,
- c) nautical experience and practice.

The data collected so far with respect to all above three items are far from complete, however, materials available at present enable us to draw some preliminary conclusions as to the selection of dangerous situations.

The most useful in this respect are casualty records and statistics. There are many descriptions of stability casualties available in world literature; some of them are rather detailed, see "Pamir" casualty [7]. Also available are collated casualty statistics such as the work of Manley [8]. The most valuable material, however, is contained in two reports prepared for IMCO, [2], [3], because an attempt was made to analyse the causes of casualties from the point of view of the situation of the vessel. These two reports, supplemented by other records, can be used for selection of dangerous situations.

For the purpose of this paper an extract from both reports was prepared which is shown in form of Tables 1 and 2. Table 1 was prepared on the basis of the Table 1 in [2]. All cases in which shifting of deck cargo or cargo in hold occurred and the vessel was not lost were omitted. Other cases were divided into two groups - without shifting of cargo and with shifting of cargo. The same method was used in the preparation of Table 2 for fishing vessels, on the basis of Table 1 in [3]. Although the total number of vessels analysed is not large, and data supplied are in many cases obviously not very exact, some conclusions can be drawn from the analysis of both tables.

In the majority of cases, the sea and wind were the main effects at the time of casualty. In only a few cases did the vessel capsize in calm sea, and then mostly the action of helm was observed. So the action of helm should be treated as important factor, but not necessarily in connection with wind and sea effects.

Water on deck and rushing in of water in connection with free surface effect are in some cases reported as the main causes of casualty. Both are connected with rough sea conditions. The last effect which should be included in the whole picture is icing which in one case was reported as the cause of casualty in connection with the action of helm, in some other cases in connection with wind and sea effects. From other reports,

see [7], it is known that in some cases additional weight (caused for example by moistening of deck cargo) or heeling moment caused by crowding of passengers were the main causes of accident.

With regard to the way the casualties took place, in the majority of cases gradual capsizing was reported. The tables do not show that this is necessarily connected with water on deck. However, bearing in mind that the majority of vessels analysed are small low-freeboard full scantling cargo vessels or low-freeboard fishing vessels, it could be assumed that there was water on deck in each case. In many cases the casualty occurred in beam sea and as it is seen from model tests, in such cases, water on deck is the main cause of gradual capsizing.

Model tests of capsizing phenomena form a very good basis for studying various modes of capsizing and consequently for the selection of dangerous situations. Results of model experiments of this kind were reported several times during the last few years, see [9], [10], [11]. The results of experiments will not be discussed here as most of them were published. The conclusions which could be drawn from them are of great value in proper selection of dangerous situations. An analysis of the behaviour of the vessel sailing amongst the waves shows that the heading with respect to direction of wave propagation is of utmost importance. It is possible to distinguish three main positions of the vessel with respect to waves:

- a) beam seas,
- b) following seas,
- c) quartering seas.

The probability of capsizing in head seas is very small and this is in agreement with the IMCO statistics. In model tests, the models never capsized in head seas. Therefore this condition may be ignored. If we take beam seas under consideration first, three possible ways of capsizing can be observed:

- a) sudden capsizing in wind gust,
- b) gradual capsizing with water trapping on deck,
- c) capsizing in breaking waves.

Sudden capsizing in wind gust may

occur even in comparatively calm water, if the lateral area exposed to wind is sufficiently large and stability comparatively low. If such a vessel is steaming in waves, the conditions are even less favourable and capsizing may occur.

The sudden capsizing in beam waves occurs mostly in the case of high freeboard vessels and sailing vessels. Nevertheless similar capsizing phenomena were observed for low-freeboard vessels with high position of the centre of gravity, see Figure 1.

Gradual capsizing with water trapping on deck occurs in cases of low-freeboard vessels. In this category we find mostly small vessels, coasters, fishing vessels, tugs, etc. If such a vessel is steaming in beam seas, subsequent waves can trap on deck causing additional heeling moment and progressively increasing heeling angle. There are two possibilities - the vessel capsizes, or after some time, the condition of equilibrium establishes in which the vessel with the deck partially flooded oscillates around the heeled position. Both cases are shown in Figure 2. The mean angle of heel in a quasi-static equilibrium condition is called the pseudostatic angle of heel. Both large pseudostatic angle of heel and capsizing are dangerous from the safety point of view.

The history of capsizing, as observed in Figure 2, which was obtained from model tests in irregular waves in the open water of a lake, always shows that the origin of progressive increase of heeling angle occurs when a group of large waves is encountered.

From this point of view, the probability of occurrence of such groups in irregular seas is of great importance.

The third mode of capsizing is observed in breaking waves on shallow water. If the vessel is considered to be broadside to waves it is rolling on steep shallow-water waves. Moreover, it is observed that under the additional impact of oncoming waves the vessel can capsize. This phenomenon can be observed in model scale but it is often met in reality also.

When the ship is moving in following seas, two modes of capsizing can be observed, see [11]. A low cycle resonance phenomenon in which the vessel is overtaken by a group of especially steep waves. With the crest of the wave amidships, stability is greatly reduced and may become negative resulting

in large heel. Then, when this wave moves on, stability is sharply increased. The vessel then "snaps" back upright and beyond, and the second wave again reduces stability and increases the heel. The capsize may occur if the ship is forced to roll far enough past the upright position when the wave crest moves into the amidship position.

This phenomenon may be assigned to the phenomenon of "parametric resonance" which occurs in non-linear systems when a parameter in differential equation is a function of time. Another possible mode of capsizing in following seas, as observed in model tests, is the "pure" loss of righting arms. In this situation the model was observed to be travelling in following seas with relatively little rolling motion and was overtaken by two or three exceptionally steep waves, usually of about the model's length. If the deck was immersed in the crest, the model was observed to simply lose static stability and to capsize without preliminary rolling motion. Rolling records obtained from model tests are shown in Figure 3.

The last mode of capsizing may occur in quartering seas - this phenomenon is called "broaching". It occurs when the vessel is struck by two or three steep breaking waves. The vessel loses some of its directional control and large yaw is usually observed.

The vessel capsizes under the combined influence of dynamic forces of the breaking waves coupled with dynamic forces associated with the turning motion of the vessel. Capsizing takes place usually with the crest of the wave amidships, hence in position of reduced transverse stability.

In proper selection of dangerous situations a very important role should be assigned to nautical experience and practice. This is mostly incorporated in national rules and regulations, or recommendations of various countries as well as in many proposals of stability criteria which can be found in world literature.

This material was partly analysed at IMCO [12] and will not be discussed further. The results of this analysis are of great importance for proper assessment of various additional heeling moments which should be taken into account for vessels of different types.

5. SELECTION OF TYPICAL DANGEROUS SITUATIONS

Selection of typical dangerous situations of the vessel for the purpose of developing stability criteria can be made as a result of the considerations given in the previous paragraph. This is very responsible work which has not, so far, been carried out. At this stage, some preliminary suggestions can be made which are far from complete and are limited to conventional cargo vessels only.

It can be concluded from the previous considerations that the major effects are those of wind and sea. If so, then all situations should be categorized according to the heading with respect to the wind and wave direction. In some cases action of helm may be of importance.

All other effects, such as shifted weights (partially filled tanks) and additional weights (additional cargoes and ballast, icing, deck cargoes) should be considered as changing stability characteristics (position of the centre of gravity) rather than as external heeling moments. This leads to the selection of dangerous situations as shown in Table 3. Obviously, icing should be considered only for vessels sailing in high latitudes, and situation 5 should be checked exclusively for very small vessels sailing in shallow coastal waters. Table 3 should be considered only as an example as further studies will probably reveal other dangerous situations.

For vessels other than conventional cargo vessels, additional situations can be selected which should take into account specific features of construction or type of service of these vessels. It is well known, for instance, that for passenger vessels the situation must be considered in which all passengers are crowding on one side of the deck. As it is seen from the IMCO statistics, in a great number of casualties the shifting of deck cargo or cargo in holds, and flooding of the hull have a very important effect. It is assumed, however, that such effects should not be included in typical situations on which stability criteria are to be based. It is much better to include in stability regulations necessary precautions against shifting of cargo or flooding through non-watertight openings.

6. PHYSICAL MODELS OF DIFFERENT SITUATIONS

According to the proposed procedure, proper physical models should be chosen for each situation of the vessel.

This paper does not have the scope to discuss all possible physical models in detail. It must be said, however, that from all situations given in Table 3, the situations 1 and 2 are those that are most thoroughly investigated and included in the national regulations of some countries.

With respect to other situations, much work has been done during the last few years towards investigating them fully. However, further experimental and theoretical studies are needed before specific stability criteria could be established and based upon them.

In particular, many studies have been done recently with respect to the situation 1a, see [13], [14], which is rather important for many types of small vessels; however, no satisfactory physical model has been found so far. It is obviously a rather difficult problem to solve because of the complexity of phenomena involved. Other non-linear problems connected with the situations 3 and 3a are not solved sufficiently well although better results have been achieved in recent years, see [11], [15], [16].

Broaching has been extensively studied by Boese [17], and shallow water effect is being investigated by both the University of Leningrad and the University of Gdańsk [18]. The prediction of the behaviour of the vessel in various situations under the action of external heeling moments is not only a matter of developing a proper physical model but also of collecting sufficient statistics and data on external heeling moments including wind and sea effects. While large amounts of work have been done in this direction, statistical data on many heeling moments are still lacking. In the opinion of the author, however, the concentration of effort towards establishing physical models and investigation of the behaviour of vessel in different situations, together with collecting necessary statistical data, may lead to the development of stability criteria in a comparatively short time. Obviously, these will not be strictly "rational" criteria, in the absolute sense, but criteria which should improve the safety of the vessel considerably.

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TABLE 1

Number	Sea	Wind	Additional effects		Way of casualty	Heading against wind and sea.		Notes
	Smoath Moderate or rough Still weather	Moderate stormy	Water on deck Water intrush Free surf effect	Jcing Action of helm	Swadren capsizing gradual capsizing Flooding	Bearn Head	Following Quartering Broaching to	
1	X				O		-	
2	X							
3	X							
4	X				O			
5	X							
14	-							
17	X							
22	X							
27	-							
28	X							
32	X							
33	X							
34	X							
36	X				O			
38	X							
42	-							
43	X							
46	X							
51	X							
57	-							
62	X							
68	X							

13								
21	O							
23	-							
24	-							
25	X							
28	X							
29	X							
30	X							
31	X							
37	X							
44	-							
45	X							
48	X							
49	X							
63	X							
65	X							
67	X							

No	Shifting	aft	cargo	hold
1				
2				
3				
4				
5				
14				
17				
22				
27				
28				
32				
33				
34				
36				
38				
42				
43				
46				
51				
57				
62				
68				

POSSIBLE DANGEROUS SITUATIONS FOR CONVENTIONAL
CARGO VESSELS.

Table 3

N u m b e r	Description of the situation	Changes of stability characteristics				External heeling moments			
		Partially filled tanks	Additional cargoes and ballast	Icing	Deck cargoes	Sea	Wind	Deck well flooding	Action of helm
1	High-freeboard vessel in beam sea No speed	x	x	x	x	x	x	-	-
1a	Low-freeboard vessel in beam sea No speed	x	x	x	x	x	x	x	-
2	Vessel in calm sea. Full speed	x	x	x	x	-	x	-	x
3	Vessel in following sea. Full speed	x	x	x	x	x	-	-	-
3a	Low-freeboard vessel in follo- wing sea. Full speed	x	x	x	x	x	-	x	-
4	Quartering sea at full speed	x	x	x	x	x	x	-	-
4a	Broaching at full speed	x	x	x	x	x	x	-	x
5	Vessel in beam sea, shallow water	x	x	x	x	x	x	-	-

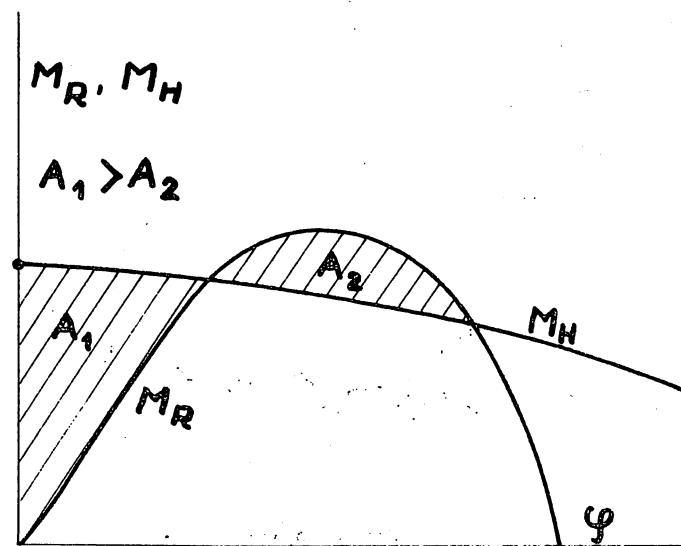
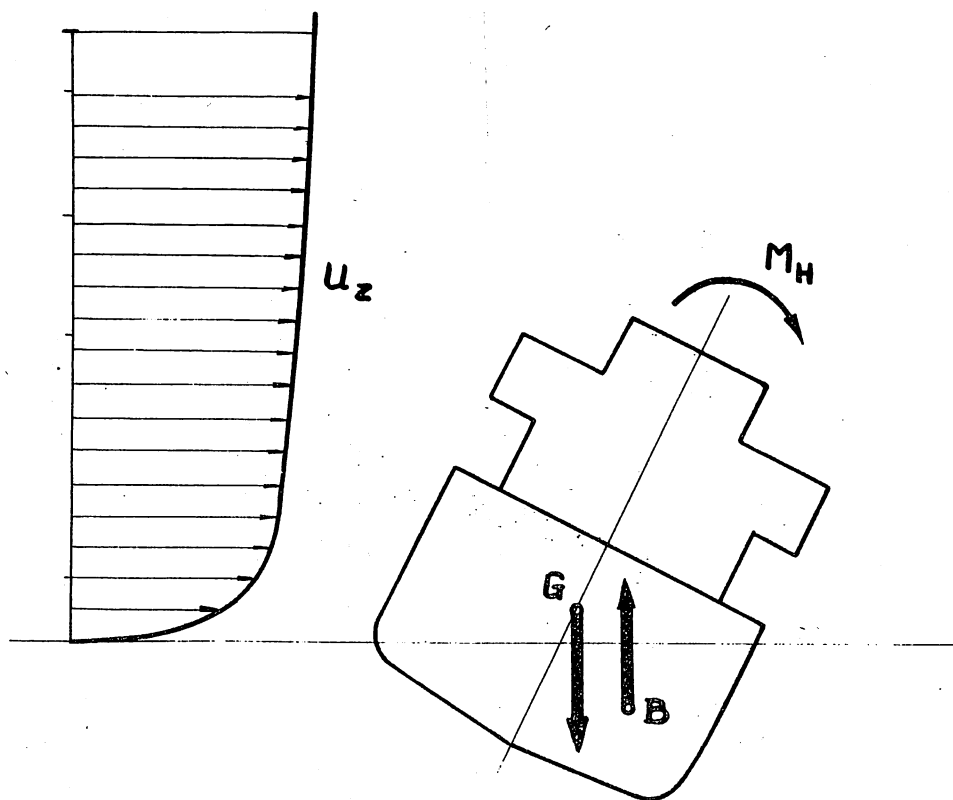
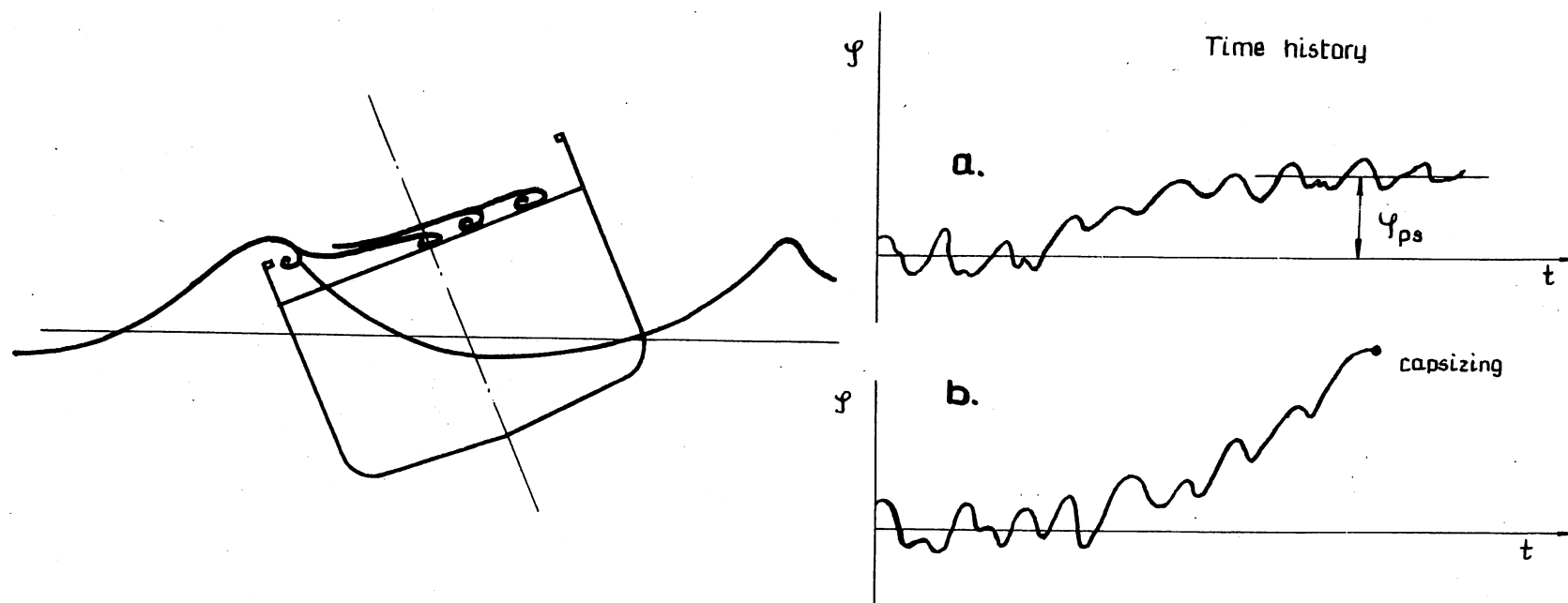


FIG. 1

Fig. 2

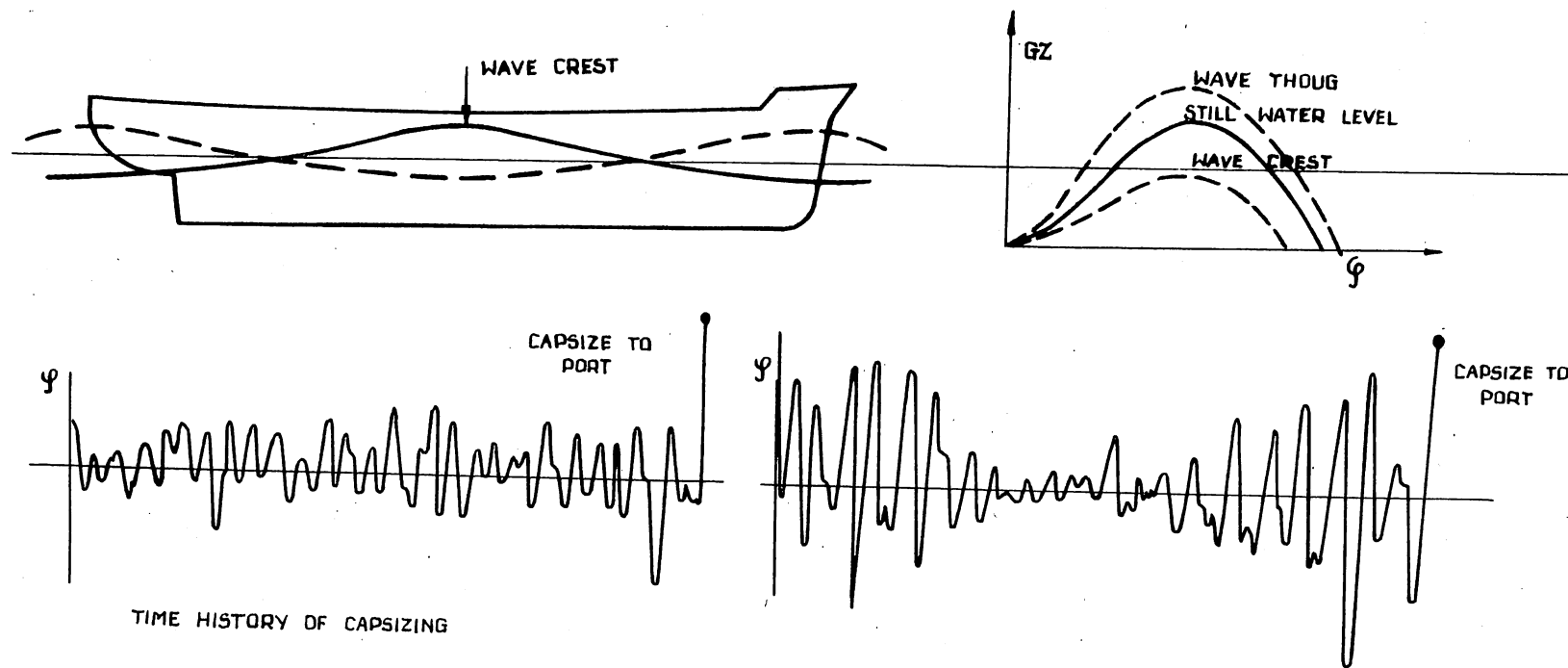


Fig. 3

SESSION 2

ENVIRONMENTAL CONDITIONS & EXPERIMENTAL STUDIES

<u>Paper No.</u>	<u>Author</u>	<u>Title</u>
2.1	Ewing, J.A. (U.K.)	"Environmental Condition Relevant to the Stability of Ships in Waves"
2.2	Fujii, H., Takahashi, T. (Japan)	"Experimental Study on Lateral Motions of Ship in Waves"
2.3	Takaishi, Y. (Japan)	"Experimental Technique for Studying of Ships Achieved in the Ship Research Institute"
2.4	Miller, E.R. (U.S.A.)	"A Scale Model Investigation of the Intact Stability of Towing and Fishing Vessels"
2.5	Dudziak, J. (Poland)	"Safety of a Vessel in Beam Sea"

ENVIRONMENTAL CONDITIONS RELEVANT TO THE
STABILITY OF SHIPS IN WAVES

by

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1. INTRODUCTION

Waves are usually the most important environmental factor which has to be considered by engineers in the design of ships and marine structures. Other factors such as the direct influence of winds and currents are of lesser importance in the context of ship stability and they will not be considered in detail in this short paper except in so far as winds are the primary mechanism for the generation of waves and that ocean currents can cause wave refraction.

Wave data have been collected for a number of years so that there is a basic knowledge of wave conditions over the world's oceans and seas which can and has been used by naval architects in the structural design of ships. Three classes of wave data are available according to the origin of the data. These consist of wave data from visual observations, data from direct measurements, and data hindcasted from known, past, meteorological situations. Of these types it seems likely that hindcasted wave information obtained from numerical wave prediction models may be the most important for ship stability considerations, for the analysis of ship casualty data indicates the existence of local sea areas where the influence of currents and bottom topography can cause steep crossing wave systems which are especially hazardous to small ships and structures. It is very unlikely that a wave recorder will have been deployed for any length of time at one of these locations so that the only way of describing the conditions in such areas is by using numerical wave prediction methods based on meteorological information as input.

Ship casualties often arise in situations where several high waves occur in a group and cause excessive rolling. The statistics of groups of high waves have been studied in a number of recent investigations and should be of importance in describing certain modes of capsizing.

2. SUMMARY OF WAVE DATA SOURCES

Comprehensive surveys of available wave data have been compiled and referenced by the Environmental Conditions of the International Ship Structures Congress (ISSC) in reports to the 3rd, 4th and 5th Congresses held in 1967, 1970 and 1973 respectively. This paper will therefore not duplicate this work, but will be concerned with the use of wave data in the particular context of ship stability.

2.1 Data from visual observations

The most extensive amount of wave data has been derived from visual observations made by observers on merchant ships during their passage over the oceans. One of the most complete presentations, giving coverage of fifty different sea areas with the exception of the North Pacific Ocean, is the compilation carried out by Hogben and Lumb (1967). Many other sources of visual wave data have been analysed and presented in papers which are documented in ISSC reports.

Visual observations are necessarily subjective estimates of wave height, period and direction so that individually they are liable to large errors. Comparisons between

visually observed wave heights and measured wave heights show that visual estimates of wave height are fairly reliable and quite well-correlated with measurements. On the other hand visual estimates of wave period have low correlation with average and modal periods obtained from the spectral analysis of wave recordings. For these reasons there does not appear to be a reliable way of estimating wave spectra by fitting visual observations to certain spectral characteristics.

When a large number of observations are available visual observations can be used to estimate the long-term distribution of wave heights or the 'wave climate'. This seems to be the main use for this type of wave data.

2.2 Data from wave measurements

Data from wave measurements have been analysed and presented in two ways. The first and main source of measured wave data is available in the form of statistics of wave height and period obtained by simple analysis techniques (for example, Tucker (1963)) based on the theoretical studies of Cartwright and Longuet-Higgins (1956). Wave data analysed in this way is now available from weather ship measurements in the North Atlantic and North Pacific oceans and an increasing number of other areas including the North Sea. (References are given in the various ISSC reports noted earlier).

The second and smaller class of measured wave data is that of spectral measurements. The motion of a ship in a seaway can only be calculated accurately if the waves are described in terms of an energy or power spectrum. This is particularly important for the rolling motion of a ship where the response is confined to a narrow band of wave frequencies near the natural roll period. Wave spectra measurements may therefore be the best input to use in ship stability calculations but as yet they are not available in sufficient numbers except for certain weather ship stations in the North Atlantic Ocean (Moskowitz, Pierson and Mehr (1962, 1963, 1965), Miles (1972)).

Measurements of the one-dimensional (frequency) spectrum of waves at O.W.S. 'India' and 'Juliett' have been used by Darbyshire (1963) to

derive a spectral formula in terms of wind speed and fetch. More recently Pierson and his associates have analysed a subset of about fifty spectra from North Atlantic weather ship stations to correspond to fully-developed seas with wind speeds up to 40 knots. The resulting spectral form, called the Pierson-Moskowitz (P-M) spectrum (1964), is now widely used to represent fully-developed ocean waves in ship design calculations. It should be noted however that ship losses and damage do not necessarily occur in extreme conditions. Indeed dangerous motions may occur in quite moderate sea conditions when the ship is in quartering or following waves.

The JONSWAP study (Hasselmann et al (1973)) has measured the development of waves under fetch-limited conditions off the island of Sylt in the southern North Sea. Wave spectra obtained under these conditions did not have the same shape as the P-M spectrum. Fetch-limited spectra were found to have much sharper peaks than the P-M spectrum; the high frequency tails of such spectra were also found to be fetch-dependent and not constant as first suggested by Phillips (1958). If empirical spectra are used in ship stability calculations then a distinction must be made between using spectra characteristic of fully-developed conditions (the P-M spectra) and spectra which describe fetch-limited waves (the JONSWAP spectrum) in limited areas like the North Sea.

A complete description of ocean waves requires the measurement of the two-dimensional spectrum, that is the energy spectrum of waves with respect to frequency and direction. Due to the difficulty in recording and analysing waves in the open ocean only a few such measurements have been made. Furthermore the measurements have usually been taken as part of fundamental research on ocean waves or special ship trials and so cannot be considered as an adequate statistical sample for wave data purposes. At present there does not seem to be a suitable way of collecting directional wave spectra information on a routine basis for wave climate studies.

In the absence of wave spectral data for a particular area the best course may be to retrospectively generate the wave climate using numerical wave models. This is discussed in the following section.

2.3 Hindcasted wave data derived from numerical wave models

Modern methods of wave prediction use as their basis a fundamental equation which expresses the propagation of waves together with their generation, interaction and dissipation (reviews of presently available numerical methods are given in ISSC reports). Two main uncertainties arise in this approach for hindcasting ocean wave spectra and statistics. The first limitation lies in the difficulty of specifying the wind field to the accuracy needed in spectral wave calculations. However, with the advent of computers large enough to handle advanced weather models and the increase in observing stations over the oceans it should be possible to give a more accurate specification of the wind field. The second uncertainty is in the representation of the terms in the Source Function or those terms governing the dynamics of waves. Here our knowledge of wave growth and interaction processes is being rapidly advanced with studies such as JONSWAP (Hasselmann et al (1973)).

The present status of deep water numerical wave models shows that it is possible to produce reliable hind-cast wave heights (the r.m.s. error of predicted significant wave heights is about 1 m compared to measurements for wave heights less than about 15 m) and reasonable estimates of the one-dimensional spectrum. The few comparisons that exist of hindcasted estimates of the directional wave spectrum show that, as expected, this is not easy to predict accurately. Nevertheless it seems likely that, in due course, hindcasted wave data (in the form of spectra as well as the simpler statistics) may well supersede wave spectra measurements. In many sea areas where no measurements are available, hindcast wave methods offer the only way of estimating the long-term distribution of extreme wave conditions by an analysis of past meteorological events.

In shallow water the prediction of waves is more difficult than in deep water due mainly to the complications of wave refraction and the (at present) uncertain processes of dissipation by bottom effects. For these reasons the development of numerical wave models for shallow seas, such as the North Sea, is only now being started.

Hindcasted wave data may be especially important in the case of ship casualty analysis. If, as will be discussed in subsequent sections, local effects

are important in a particular area where a casualty has been reported then it is most unlikely that any measurements will have been made at that particular location. In this case the only way of describing the wave conditions at the time of the ship loss or damage is by using a numerical wave model based on the past meteorological data for that time.

3. LOCAL FEATURES OF WAVES CAUSED BY BOTTOM TOPOGRAPHY AND CURRENTS

In a study of the loss of two British trawlers in the North Sea, Pierson (1972) believes that the circumstances causing the loss may well have been due to high crossing-wave systems caused by refraction around shoal areas. One of the ships was lost south of the Dogger Bank and the other ship between North Ronaldsay and Fair Isle; in both locations wave refraction is important. Indirect confirmation of the existence of high waves during strong northerly winds in the area south of the Dogger Bank is provided by Ewing and Hogben (1971). In their analysis of over 500 wave measurements taken by British research trawlers in the seas around the British Isles, it was found that the record with the highest waves (the significant wave height was 8.7 m) was taken at a location near to that where the trawler was lost south of the Dogger Bank. In a situation where northerly winds can cause waves to be refracted around shoal areas like the Dogger Bank, wave energy can come simultaneously from two different directions making it very difficult for a small ship to take avoiding action for the ship would experience severe rolling and pitching at any heading. Experiments in a tank by Chao and Pierson (1972) show that waves when travelling from shallow to deep water are turned round on themselves to form an intersecting pattern where the wave heights are about twice as high as they were at the point where they were generated. The effect is analogous to the formation of caustics in optics.

Intensification of wave energy by refraction must occur to some extent in certain areas of all shallow seas and is therefore a potential source of danger to small ships. The existence of such areas can be foreseen through a study of the bathymetry, but, as noted previously, a thorough analysis of the waves requires the use of a shallow-water

numerical wave model.

Unusually steep and high waves have been reported in locations where strong currents interact with wind waves. Mallory (1974) has documented the conditions off the east coast of South Africa where some ships have been damaged by unexpectedly high waves. In this area locally generated waves and swell waves from the Southern Ocean can encounter the opposing Agulhas current (which may reach speeds as high as 5 knots) with the result that the waves become higher and steeper. Longuet-Higgins and Stewart (1961) have shown that the nonlinear interaction between an opposing current and waves results in a shortening of the wavelength and an increase in the wave height. According to their theory the wave height becomes very large at the critical point where the current velocity is equal and opposite to the local group velocity or one-quarter of the initial phase velocity of the waves. (In practice the waves will break before this point is reached) The wave periods and current speeds characteristic of the area off the east coast of South Africa often occur to satisfy this condition.

Any local variations in the strength and direction of the current - such as occurs in the Agulhas current - can, in a similar way to bottom topography, cause the waves to be refracted with the formation of steep waves and caustics. Wave and current conditions similar to those which occur off South Africa must exist in other parts of the world and can therefore cause damage or be dangerous to small ships.

4. STATISTICS OF GROUPS OF HIGH WAVES

Groups of high waves are known to cause severe rolling motions and are therefore important in the study of the capsize condition. The statistics of high wave groups have recently been studied by Goda (1970) and Ewing (1973). A run of wave heights can be defined as the sequence of waves the height of which exceed a particular level. Goda derived the mean length of runs of wave heights described by a random process where successive wave heights are uncorrelated (this assumption is equivalent to representing the waves by a spectrum with a large bandwidth, that is, a typical wind-generated sea), and compared his results with measured run lengths

taken from simulated wave records. For spectra with a large bandwidth the agreement was found to be good.

The longest run lengths occur for narrow-band spectra characteristic of swell waves and, in this case, Ewing showed that the mean length of runs of high wave heights was inversely proportional both to the width of the spectrum and to the crossing-level being considered. For swell waves values of the mean run length (for a level corresponding to the significant wave height) are from about 4 to 10 depending on the width of the spectrum. Long run lengths are of direct interest in the case of the rolling motion of ships since it is often in this situation, where there are a succession of high waves, that capsize occurs. The phenomenon of groups of high waves seems to be associated in some way with the mode of capsize called 'low cycle resonance' by Paulling et al. (1972)

5. BREAKING WAVES

The phenomenon of breaking waves is of great importance in the study of the stability of ships for it is well known that the forces due to breaking waves are considerable and often exceed those in non-breaking conditions.

Experimental studies of breaking waves in shoaling water have led to a practical description of the breaking process which has been used successfully by coastal engineers but in deep water wave breaking is more difficult to observe because of the transient and sporadic nature of the phenomenon. When waves break in deep water the crest of the wave spills forward forming a strongly turbulent region on the forward face and a less turbulent wake behind the crest. Whitecaps are observed when air is entrained by the turbulence on the forward face of the wave. An initial description of the dynamics of a spilling breaker in deep water, made by Longuet-Higgins (1973a) and Turner (1974), is able to describe some laboratory observations but a full description of the breaking process is theoretically difficult because of the highly nonlinear nature of the motion. From another point of view, Hasselmann (1974) has considered whitecapping to be represented by a local, strongly nonlinear interaction in a random wave field. In this

case the Source Function for the dissipation due to whitecapping can be derived and is found to be consistent with the structure of the energy balance derived from JONSWAP.

Other related work on the limiting form of progressive waves in deep water and the dynamics of solitary waves in shallow water are the subject of recent investigations by Longuet-Higgins (1973b, 1974) and Longuet-Higgins and Fenton (1974).

In open ocean conditions the processes of wave breaking in a random, three-dimensional seaway remain largely unknown. An understanding of the intermittent nature of wave breaking requires the combination of a realistic fluid mechanical model of wave breaking together with a stochastic model of sea waves.

6. CONCLUSIONS

Wave spectra measurements offer the best possibility for understanding the motions of a ship in a seaway but they are not yet available in sufficient numbers to define conditions over the world's oceans. In the future it should however be possible to use numerical wave

models to hindcast wave spectra and statistics from known past meteorological conditions. This is of particular importance in the case of ship casualty analysis where it is very unlikely that any measurements will have been made at the particular location of the casualty where wave refraction and other factors may be locally important. Strong ocean currents are also of importance in their influence on waves and can produce steep breaking waves which are hazardous to small ships.

The statistics of groups of high waves have been studied recently and may be useful in understanding the mode of capsizing known as low cycle resonance.

The basic difficulty in understanding the motion and stability of ships in waves is due to nonlinearities both in the waves themselves and in the resulting ship motions. Certain approximations can be made, such as assuming the linear spectral theory of waves, but these approximations may not be valid for high breaking waves. It will require a combined effort by both oceanographers and naval architects to solve the problems of ship stability in extreme wave conditions.

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EXPERIMENTAL STUDY ON LATERAL MOTIONS OF A SHIP IN WAVES

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SUMMARY

In order to apply the theoretical calculation of ship motions and wave loads to the practical ship design, it is necessary to confirm the agreement between prediction and experiment.

Tests were carried out using ship models and the results were compared with the theoretical predictions based on the strip method. The kinds of tests were forced oscillation model tests in calm water to check the coefficients of the equations of ship motions, restrained model tests in oblique waves to check the terms of wave excitation and free model tests in oblique waves to check the amplitudes of ship motions.

It is pointed out that the estimation of roll damping is the most important and its proper approximation method should be developed for making a better prediction. Moreover, it is pointed out that investigations should be carried out for ship motions among the waves of larger wave height so as to be able to predict and maintain the stability and safety of a ship in rough sea.

1. INTRODUCTION

In view of safety of ship operations in a seaway, it is important and necessary to predict the ship motions and the wave loads at the initial ship design stage. At present, theoretical prediction method based on the strip method is available and computer programs are now prevailing. Although there still remains a certain amount

of refinement to be done, and a need for more precise evaluation, the strip method has been known as one of the most reliable methods so far for practical use.

In order to apply the theoretical calculation to the practical ship design, however, it is necessary to confirm experimentally or modify theoretically the important terms in the equations of ship motions. In fact, fairly good accurate ship motion calculations will be necessary, because the prediction of wave loads is carried out on the basis of the calculated results of ship motions.

Experimental confirmations should be conducted in connection with each term in the equations of ship motions; that is, tests are made using models of various ship forms and comparisons are made between the test results and the computed ones. Thus, the key point will be found for the improvement of the calculation method. The kinds of tests are:

- 1) Forced oscillation model test in calm water to check the coefficients of the equations of ship motions, particularly for the lateral motions, (1),
- 2) Restrained model test for wide variety of the wave incident angle to check the terms of wave excitation, (2), (3), and
- 3) Free-running model test in oblique waves to check the amplitudes of ship motion corresponding to the solution of the

equations of motions.

In the present study, the above mentioned approach was followed and attention was concentrated on the rolling motion. Supplementary model tests were also conducted in the waves of large wave height to evaluate the non-linearity of rolling motion.

2. MODEL EXPERIMENTS

Three kinds of test were conducted for models of several ship forms to confirm the prediction method of ship motions and hydrodynamic pressure acting on hull surface.

2.1 Forced oscillation model test in calm water

It was considered that there are many points to be improved in the calculation method of lateral motions: swaying, yawing and rolling. A forced oscillation test was conducted for these lateral motions only.

In order to carry out forced oscillation tests, it is essential to have an oscillating mechanism in swaying, yawing and rolling or their coupled motions with prescribed period and amplitude and a measuring system for corresponding hydrodynamic reactions. This apparatus is called "Forced rolling dynamometer" from the viewpoint that rolling is the most important of the lateral motions in waves. The outline of dynamometer used is shown in Figure 1.

Hydrodynamic reactions were measured for each mode of motion, advance speed and circular frequency of motion by use of this dynamometer. Simultaneous measurement was made for hydrodynamic pressure, so-called radiation pressure, acting at several points on the hull surface.

2.2 Restrained model test in oblique waves

A restrained model test was conducted for a wide variety of the wave incident angle by towing the model with a constant speed using an apparatus called "Wave force dynamometer". The outline of dynamometer used is shown in Figure 2. Hydrodynamic forces and moments due to wave were detected in the form of the strain of semi-conductor type resistance gauge on the spring plate or block gauge. The mechanism, which allows the ship motions with suitable combination of ship motion components, is provided in the dynamometer so that only the wave exciting forces and moments

in lateral direction can be measured.

Wave exciting forces and moments: heaving force, pitching moment, swaying force, yawing and rolling moment, were measured for differing circular frequency of regular wave, direction of wave incidence and advance speed. Simultaneous measurements were made for hydrodynamic pressure, so-called the diffraction pressure and the pressure based on Froude-Kriloff's hypothesis at several points.

2.3 Free-running model test in oblique waves

A free-running model test was conducted in regular oblique waves using a self-propelled model. Six components of ship motion: heaving, pitching, surging, swaying, yawing and rolling, were measured and the total pressures were measured simultaneously at several points.

3. TEST RESULTS AND CONSIDERATION

Coefficients of main terms in left hand side of equations of motions, wave exciting force and moments and amplitudes obtained by each experiment are compared with the computed results based on the strip method.

3.1 Representations of test results

Linearized equations of coupled lateral motions: swaying, yawing and rolling, are expressed as in equation (1).

$$\begin{aligned}
 & a_{11}''\ddot{y} + a_{12}'\dot{y} + a_{13}\ddot{\psi} + a_{14}\dot{\psi} + a_{15}\ddot{\phi} \\
 & + a_{16}\dot{\phi} = Yw \\
 & a_{21}''\ddot{y} + a_{22}'\dot{y} + a_{23}\ddot{\psi} + a_{24}\dot{\psi} + a_{25}\ddot{\phi} \\
 & + a_{26}\dot{\phi} = Nw \\
 & a_{31}''\ddot{y} + a_{32}'\dot{y} + a_{33}\ddot{\psi} + a_{34}\dot{\psi} + a_{35}\ddot{\phi} \\
 & + a_{36}\dot{\phi} + a_{37}\ddot{\phi} = Lw
 \end{aligned} \tag{1}$$

1) Coefficients in left hand side of equations of motions

Important coefficients are shown for main terms as follows:

- a_{11} virtual mass of swaying
 $\hat{a}_{11} = a_{11} / \rho \nabla$
- a_{12} damping force coefficient of swaying
 $\hat{a}_{12} = (a_{12} / \rho \nabla) \sqrt{B/2g}$
- a_{24} virtual mass moment of inertia of yawing
 $\hat{a}_{24} = a_{24} / \rho \nabla L^2$
- a_{25} damping moment coefficient of yawing
 $\hat{a}_{25} = (a_{25} / \rho \nabla L^2) \sqrt{B/2g}$
- a_{37} virtual mass moment of inertia rolling
 $\hat{a}_{37} = a_{37} / \rho \nabla B^2$
- a_{38} damping moment coefficient of rolling
 $\hat{a}_{38} = (a_{38} / \rho \nabla B^2) \sqrt{B/2g}$
- 2) Amplitudes of wave exciting force and moments
- Y_{WA} amplitude of swaying force, Y_w
 $Y_{WA} = Y_{WA} / \rho g L d h$
- N_{WA} amplitude of yawing moment, N_w
 $N_{WA} = N_{WA} / \rho g L^2 d h$
- L_{WA} amplitude of rolling moment, L_w
 $L_{WA} = L_{WA} / \rho g L d^2 h$
- 3) Amplitudes of ship motions
- y_A amplitude of swaying displacement, y
 $\hat{y}_A = y_A / h_A$
- ψ_A amplitude of yawing angle, ψ
 $\hat{\psi}_A = \psi_A / k h_A$
- ϕ_A amplitude of rolling angle, ϕ
 $\hat{\phi}_A = \phi_A / k h_A$

where

- L ship length
 B breadth
 d draft

- ∇ displacement volume
 ρ density of fluid
 g acceleration of gravity
 h_A wave amplitude (= wave height/2)
 k wave number

The results were expressed in non-dimensional form on the basis of the non-dimensional circular frequency of oscillation $\hat{\omega} = \omega \sqrt{B/2g}$ or wave length to ship length ratio λ/L .

3.2 Coefficients of equations of lateral motions

The calculation method of lateral motions based on the ordinary strip method was presented by Tasai.⁴⁾ In the present study, modifications were made of the coefficients of main terms in equation (1) based on the Tasai's formulations by adding extra terms as indicated in the following formulae by underlining and they were calculated using the values of virtual mass and damping coefficients from the table given by Tamura.⁵⁾

$$\begin{aligned}
 a_{11} &= m_0 + \int m y d x \\
 a_{12} &= N y d x + \frac{U}{L} \int m y d x \\
 a_{24} &= I_{zz} + \int m y (x - x_G)^2 d x \\
 a_{25} &= N y (x - x_G)^2 d x + \frac{U L}{4} \int m y d x \quad (2) \\
 a_{37} &= I_{xx} + \int (m y l y l \phi - 2 O G m y l y + \frac{37}{2} O G m y) d x \\
 a_{39} &= \rho g \nabla \overline{GM}
 \end{aligned}$$

where

- m_0 mass of ship
 I_{xx} mass moment of inertia about x-axis
 I_{zz} mass moment of inertia about z-axis
 $m y$ sectional added mass in the y-axis direction
 $N y$ sectional damping force coefficient in the y-axis direction

- x x -coordinate of the centre of gravity of ship
- ly lever of sectional added mass inertia force due to rolling motion with respect to O' which is the project of O on the transverse section
- $l\phi = i/myly$
- i sectional added mass moment of inertia with respect to O'
- lw lever of sectional force due to rolling motion with respect to O'

The underlined terms in a_{12} , a_{25} represent the effect of advance speed supplemented by referring to the non-steady wing theory.

For the roll damping coefficient a_{38} , the calculating formula based on the ordinary strip method has remaining amounts still to be solved and it may be allowable to say that there is still no prediction method established. Therefore, from the viewpoint of practical use, the coefficient a_{38} was assumed to be simply composed of three terms, that is,

$$a_{38} = a_{38w} + a_{38v} + a_{38u} \quad (3)$$

where

- a_{38w} coefficient corresponding to the damping due to wave-making calculated by usual potential theory
- a_{38v} coefficient corresponding to the viscous damping, which is a part of N coefficient obtained experimentally
- a_{38u} coefficient representing the effect of advance speed

The percentage of the damping due to wave-making and the viscous damping is not yet clarified; accordingly it was supposed that each component should be multiplied by modification factor. N coefficient, proposed by Watanabe and Inoue⁶⁾ and usually regarded as roll damping coefficient, is in itself effective near the point of synchronism of roll and, therefore, the effect of frequency of motion should be introduced in it. Further, as the increment of damping moment by the bilge keels can be almost attributed to the viscosity, the increment of

coefficient N due to the bilge keels should be added. When a ship has advance speed, the roll damping moment increases in general, but the amount of increment can not yet be clarified correctly. In the present study, the effect of advance speed on roll damping moment was supposed to be the same form as the coefficients a_{12} and a_{25} .

Adopting N_{10}^0 , the value of naked hull at $\phi = 10$ degrees as the representative value of N coefficient, and N_{8k} as the value of increment of N coefficient due to the bilge keels, each term of a_{38} is expressed as follows:

$$a_{38w} = kw \int Ny(OG-lw)^2 dx,$$

$$kw = 0.5$$

$$a_{38v} = (kvN_{10}^0 + N_{8k})a_{37} \frac{\phi_A}{\pi} \omega_e,$$

$$kv = 0.5$$

$$a_{38u} = ku \frac{U}{L} (OG-d/2)^2 \int mydx$$

$$ku = 1.0$$

(4)

Each value of modification factors was tentatively determined by considering the results of comparison of the calculated values with the experimental ones. Therefore, their values have no definite physical meaning because they are based on many experimental data only. The ratio of these components is shown in Figure 3. Adopting the roll damping coefficient defined by formulae (3) and (4), equation of motions can be solved by the successive calculation as a_{38} becomes the function of rolling amplitude ϕ_A only.

The calculation of wave exciting terms was also carried out by using the table given by Tamura, considering that only the lateral components are effective as the approximate treatment in oblique waves.

3.3 An example of test results on a tanker model

Three kinds of test were conducted for several types of ship models. The test results on a tanker model are shown for example. Principal particulars and test conditions of the tested model are shown in Table 1.

The quantities related to swaying motion are shown in Figure 4 as compared with the computed values, and those to yawing motion in Figure 5 and rolling in Figure 6. Each figure includes the representative results of forced oscillation model test, restrained model test and free-running model test.

For the coefficients of main terms of swaying, yawing and rolling motion, the experimental values show good agreement with the computed one and a fairly good improvement is observed in the value of \hat{a}_{38} computed by the proposed formulae (3) and (4).

For the terms of sway exciting force and yaw and roll exciting moments, the experimental values roughly coincide with the computed ones, but in yaw exciting moment some differences are observed, especially in oblique waves at $F_n = 0.1$.

For the amplitudes obtained by free-running model tests, the experimental values and the computed ones show comparatively good agreement except in the case of yawing amplitude. The computed values of yawing amplitude in quartering waves differ from experimental ones. The reason why the yawing amplitude does not agree so well as in the case of swaying and rolling may be attributed to the fact that for the computed values some problems are included in the calculation of yaw damping coefficient and for the experimental values the check helm was given by auto-pilot system to keep the prescribed course in oblique waves.

3.4 Measurement of lateral wave exciting force and moments

In order to measure the wave exciting force and moments in lateral direction (anti-symmetric motion), the restrained model test was conducted in the condition of heaving and pitching motions free. The test results obtained for a container ship model are shown in Figure 7 as compared with the one obtained in the fully restrained condition.

Both measured values show good agreement and the assumption of the linearized theory was certified that the hydrodynamic cross coupling force between symmetric and anti-symmetric motions are negligibly small. However, in the longer wave length range the amplitudes of roll exciting moment with heaving and pitching motions free are smaller than those where they

are fully restrained, so further investigations will be necessary.

3.5 Data of roll damping coefficient

Forced oscillation test has been known as an effective mean to obtain the basic hydrodynamic data useful for examining the prediction method; therefore, it is being carried out in many experimental tanks. With this test method, in fact, it is possible to investigate systematically and analytically the various effects on roll damping such as that of advance speed, rolling amplitude, motion frequency, appendage, position of rolling axis and so on.

Nagasaki Experimental Tank has also been carried out the forced rolling model test for many types of ship form from the viewpoint of practical applications. Some data on roll damping coefficient \hat{a}_{38} are shown in Figure 8, comparing the cases with and without advance speed. These are the results for ship models of a single screw container ship A, a twin screw container ship B and a triple screw ferry boat C.

Comparing \hat{a}_{38} with each other at the point of the natural rolling period, $\omega = 0.3 \sim 0.4$ for these ship models, it is found that the increment of \hat{a}_{38} due to advance speed at ship models B and C are larger than that of ship model A. This fact was also confirmed by N coefficient obtained from free rolling tests. That is, the ratio N_{ω}^{θ} with advance speed to N_{ω}^{θ} without one is $2.5 \sim 3.0$ for ship model B and $1.5 \sim 1.8$ for ship model A. The reason why \hat{a}_{38} of ship model B with advance speed increases remarkably with increasing frequency may be that she has comparatively large propeller bossing and fillet at both sides of the stern and they induce non-steady lift due to periodical change of attack angle of water flow and, thus, it contributes to \hat{a}_{38} .

Coefficient \hat{a}_{38} of a tanker model is shown in Figure 9, in which the effect of motion frequency, advance speed and rolling amplitude are represented. The effect of advance speed on \hat{a}_{38} is not so large because of the comparatively low advance speed, but the effect of rolling amplitude is large. Coefficient \hat{a}_{38} increases in proportion to rolling amplitude at $\omega = 0.73$, corresponding to the natural rolling period, and the non-linearity in rolling amplitude is obviously found.

Observing these data, it is pointed out that further investigation should be carried out:

- 1) to separate the damping due to wave-making and the viscous damping in the roll damping term,
- 2) to investigate the origin of increase of roll damping due to advance speed with increasing frequency,
- 3) to introduce the appendage effect in the prediction method.

In order to develop more rational and practical prediction method based on the strip method, it will be essential to conduct basic investigations as those mentioned above.

4. EFFECT OF WAVE HEIGHT ON ROLLING MOTION

The strip method, which has been widely used as an approximation expedient to predict ship motions and wave loads, is based on the calculation of linearized hydrodynamic force, therefore, the applicability has mainly been confirmed for the case where wave height is comparatively small. The long term predictions of the ship motions and the wave loads have been carried out using the data obtained by the strip method according to the linear superposition principle.

In order to keep sufficiently safe even in the extreme sea conditions a ship will encounter in her life, it is necessary to improve the reliability of long term prediction. In fact, the ship motions and the wave loads in rough sea are calculated using their response function by linear extrapolation of which validity has not yet been well clarified. The first step of the improvement of reliability may be theoretical or experimental investigations into the response function taking account of the non-linear effect due to wave height and the speed drop in the extreme sea conditions. In the case of severe sea such as where the wave height exceeds 15m, shipment of water will be observed even on a very large tanker and in such a case it is supposed that the ship motions are large and substantial amount of speed drop may be observed due to the resistance increase in waves; consequently there will be the non-linearity of wave height in the ship motions.

Model experiments, therefore, were

conducted for differing the wave height, including very high ones, on the tanker model mentioned in Section 3. Figure 10 shows the rolling amplitude for wave length to model length ratio and wave height together with the amplitude of relative motion at weather side of S.S. 8 $\frac{1}{2}$ for wave height. Non-linearity in rolling amplitude is observed with increasing wave height and shipment of water occurs at the condition where the amplitude of relative motion $3r_a$ exceeds 5.46 cm. In this condition considerable non-linearity appears in the relative motion and also the hydrodynamic pressure.

The response amplitude of ship motions in severe waves can thus be obtained by experiment. It is difficult, however, to evaluate theoretically the correct value of long term prediction by use of the non-linear response function. In fact, the long term prediction for the range of large wave heights would be overestimated when the response function for the range of small wave heights was used in the calculation, so far as the method is based on the linear superposition principle. From the viewpoint of practical use, therefore, there must be some convenient method which can correctly present the way to the practical long term prediction for the range of large wave heights. One of the possible approaches may be obtained from the consideration of the maximum amplitude of roll and the non-linearity of the response function.

5. CONCLUDING REMARKS

As a step to the improvement of the calculation method of lateral ship motions in waves, experimental confirmation was made by three kinds of tests together with supplementary experiments to clarify the rolling motion in rough seas. From these investigations, the following conclusions are obtained.

- 1) The experimental values show fairly good agreement with the calculated ones for waves with various incident angles, and the present prediction method is found generally applicable to the prediction of the ship motion.
- 2) To obtain more reasonable prediction of the roll damping term, it is pointed out that

some proper method should be developed for the approximation of the effect of viscous damping, advance speed, bilge keel and motion frequency.

- 3) For the improvement of the prediction method, it is considered essential to continue experimental studies on the components of the equations of ship motions and the present approach is one of the most effective means for pointing out the important factors to be improved.

6. ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Dr. J. Fukuda, Professor of Kyushu University, and Dr. K. Watanabe, Vice Manager of Nagasaki Technical Institute, Mitsubishi Heavy Industries Ltd., for their continuing guidance and encouragement. The authors also wish to express their appreciation to Mr. K. Ikegami and all members of Seakeeping Research Laboratory who cooperated in carrying out this investigation.

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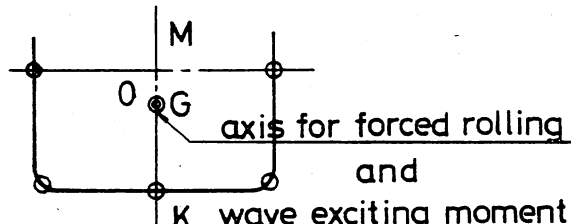
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Table 1 Principal particulars of the tested model

Ship	210 ^K DWT tanker
Scale	1/103.3
L _{pp}	3.000 m
B	0.4719 m
d	0.1828 m
Δa	220.6 kg
C _b	0.8521
L.C.B.	46.53 %
TKM	0.1917 m
GM	0.0597 m
T _{ϕ}	1.40 sec
K _{yy} /L	0.232
K _{xx} /B	0.311
Appendages	Bilge keels , Rudder
<p style="text-align: center;">Tanker</p> <p>o Position of pressure transducers</p>  <p style="text-align: center;">axis for forced rolling and K wave exciting moment</p>	

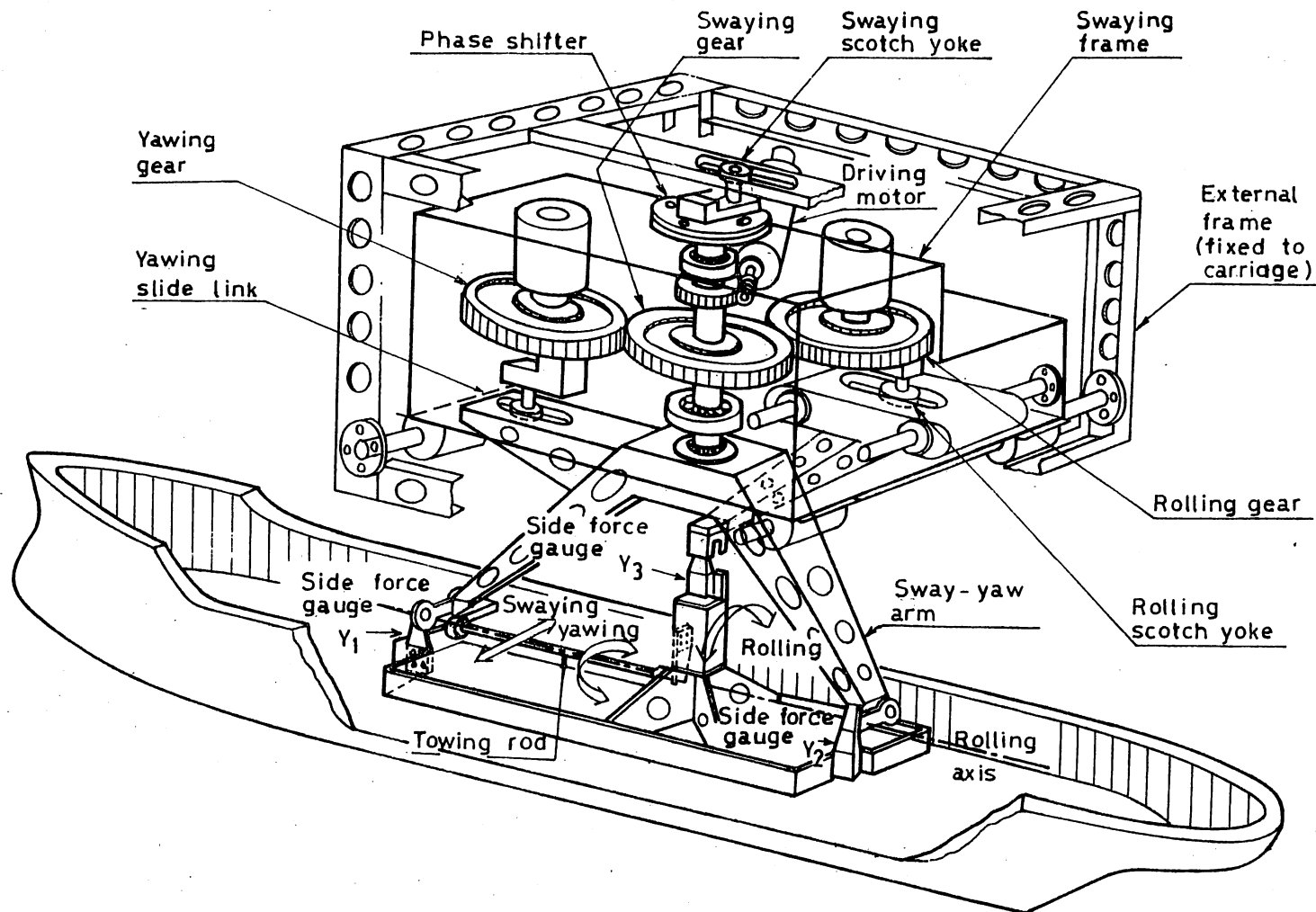


Fig. 1 Scheme of mechanism of forced rolling dynamoment

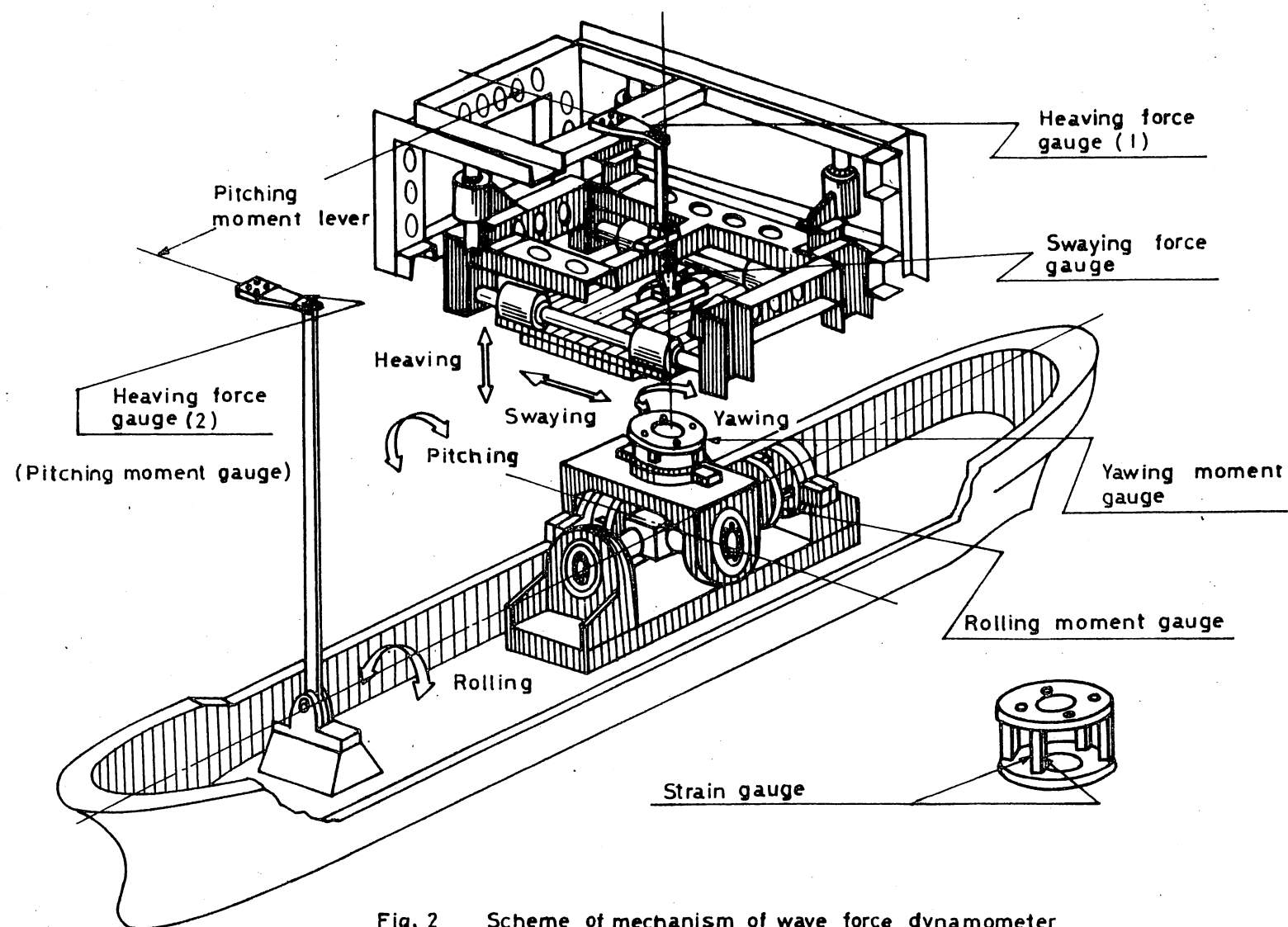


Fig. 2 Scheme of mechanism of wave force dynamometer

\hat{a}_{38} : Damping moment coefficient of roll

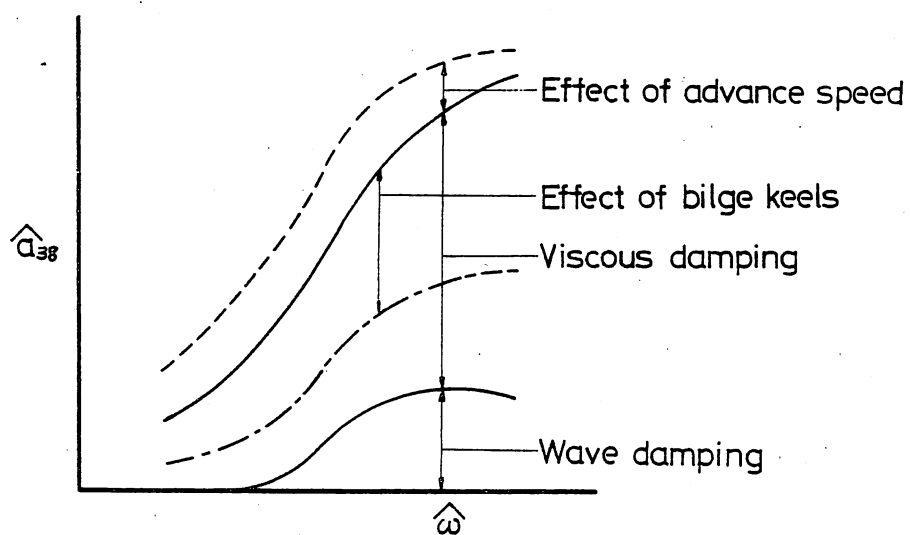
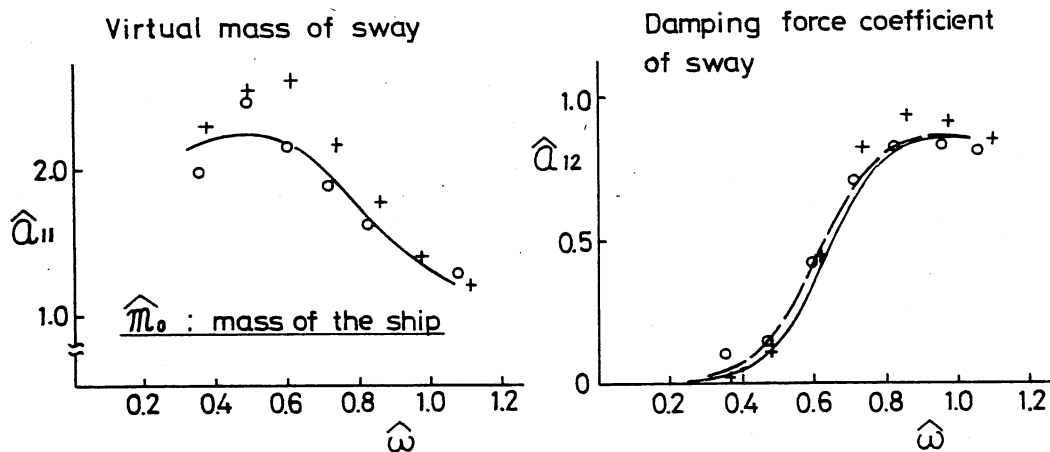


Fig.3 Components of damping moment coefficient of roll

(a) Coefficient of main term of sway

Swaying amplitude	F_n	Measured	Computed
$y_A/B = 0.0424$	0	+	—
	0.15	o	---

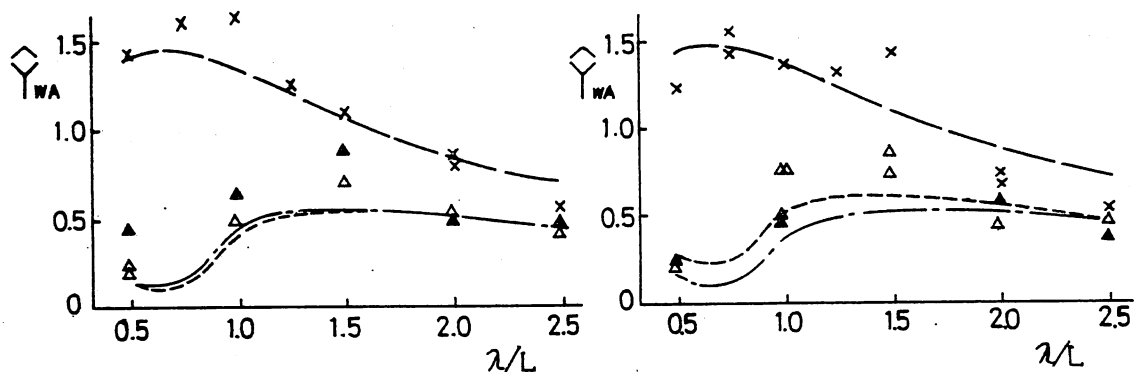


μ	Measured	Computed
135°	▲	---
90°	×	---
60°	△	---

$F_n = 0$

$F_n = 0.1$

(b) Amplitude of sway exciting force



(c) Amplitude of swaying motion

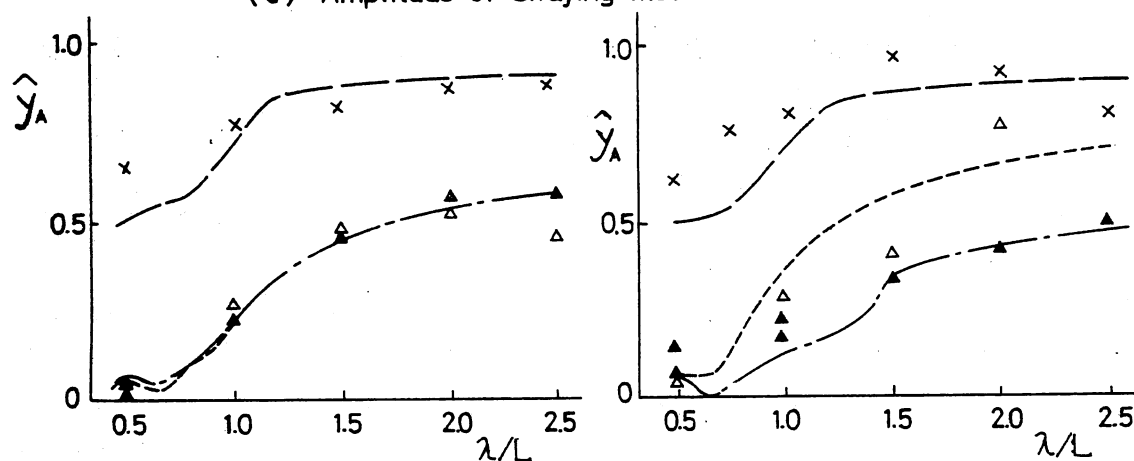
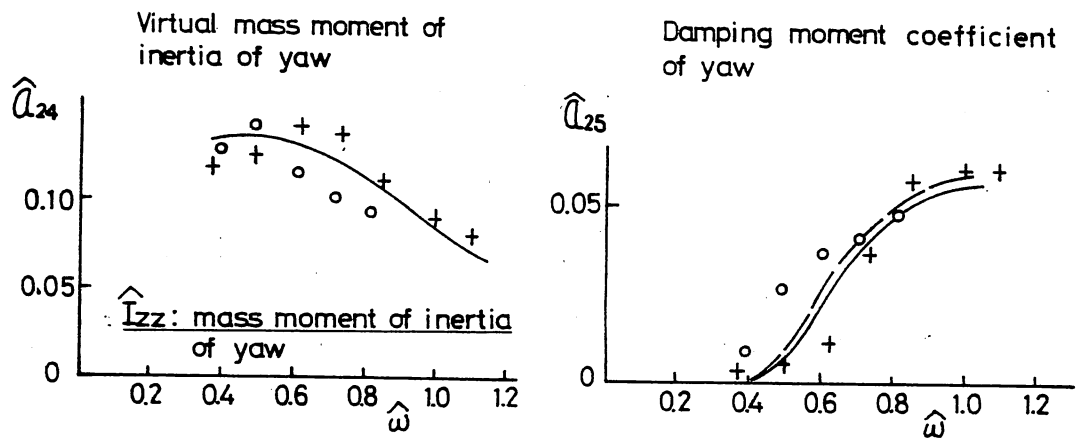


Fig. 4 Quantities related to the swaying motion : tanker

(a) Coefficient of main term of yaw

Yawing amplitude	F_n	Measured	Computed
$\psi_A = 2^\circ$	0	+	—
	0.15	o	---

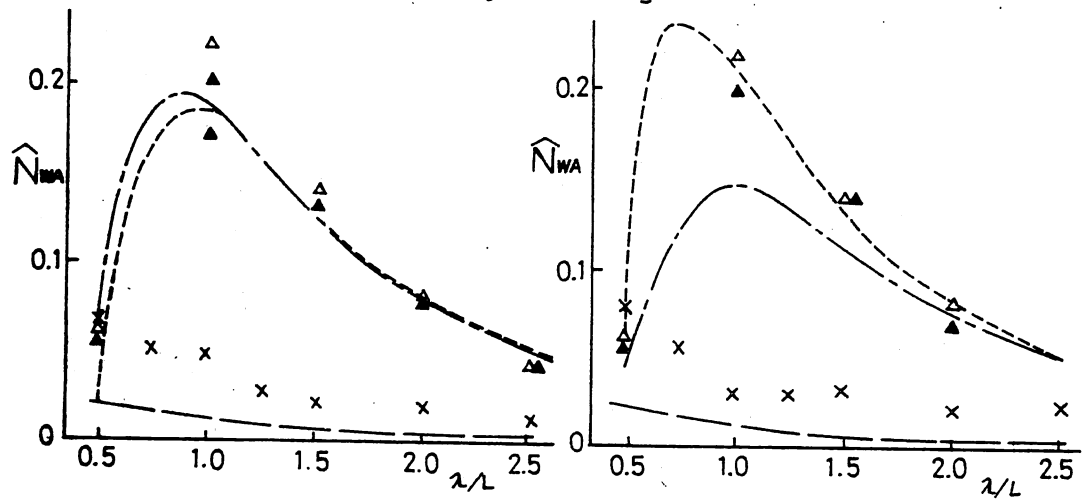


μ	Measured	Computed
135°	▲	—
90°	x	---
45°	△	----

$F_n = 0$

$F_n = 0.1$

(b) Amplitude of yaw exciting moment



(c) Amplitude of yawing motion

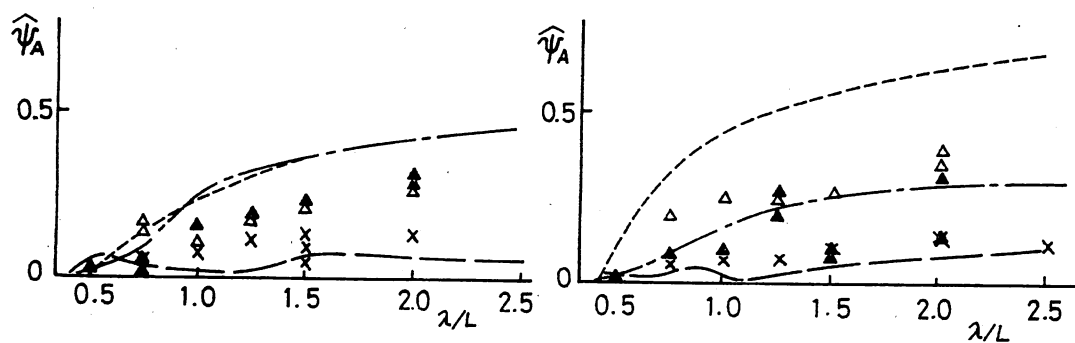
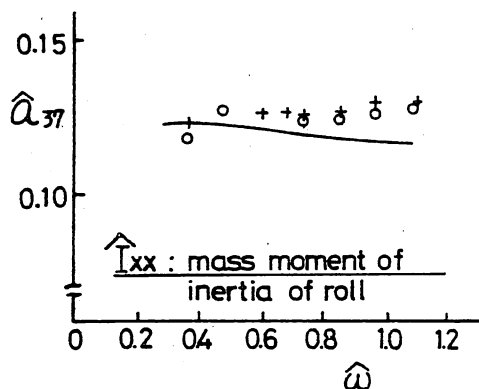


Fig.5 Quantities related to the yawing motion : tanker

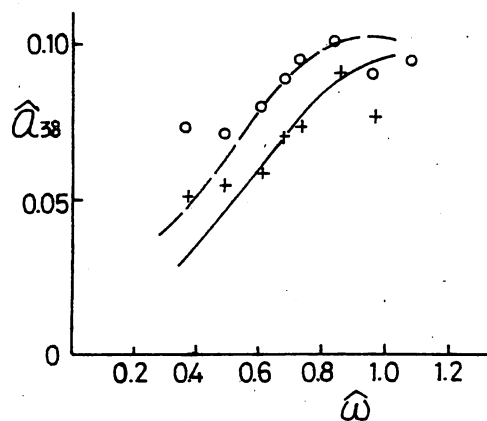
(a) Coefficient of main term of roll

Rolling amplitude $\phi_A = 10^\circ$	F_n	Measured	Computed
	0	+	—
	0.15	o	---

Virtual mass moment of inertia of roll



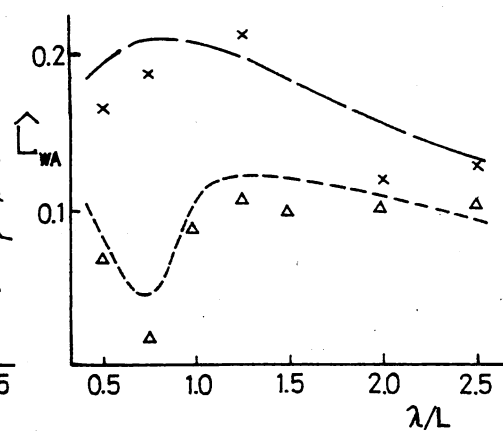
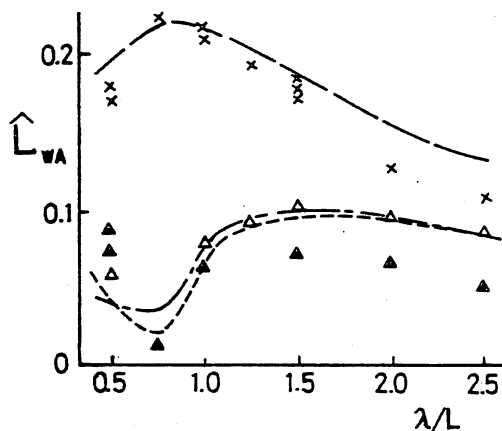
Damping moment coefficient of roll



F_n	Measured	Computed
135°	▲	---
90°	×	---
45°	△	---

 $F_n = 0$ $F_n = 0.1$

(b) Amplitude of roll exciting moment



(c) Amplitude of rolling motion

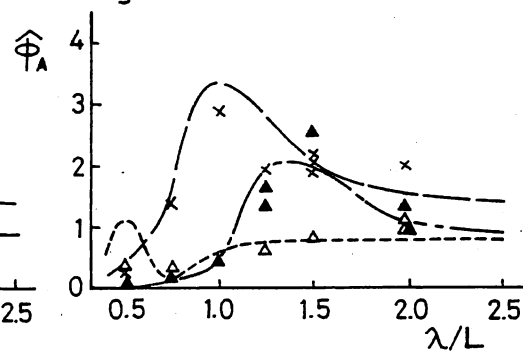
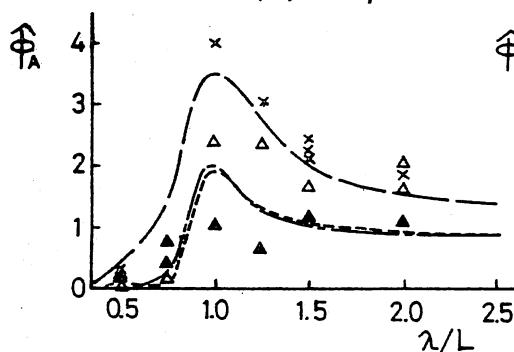


Fig. 6 Quantities related to the rolling motion : tanker

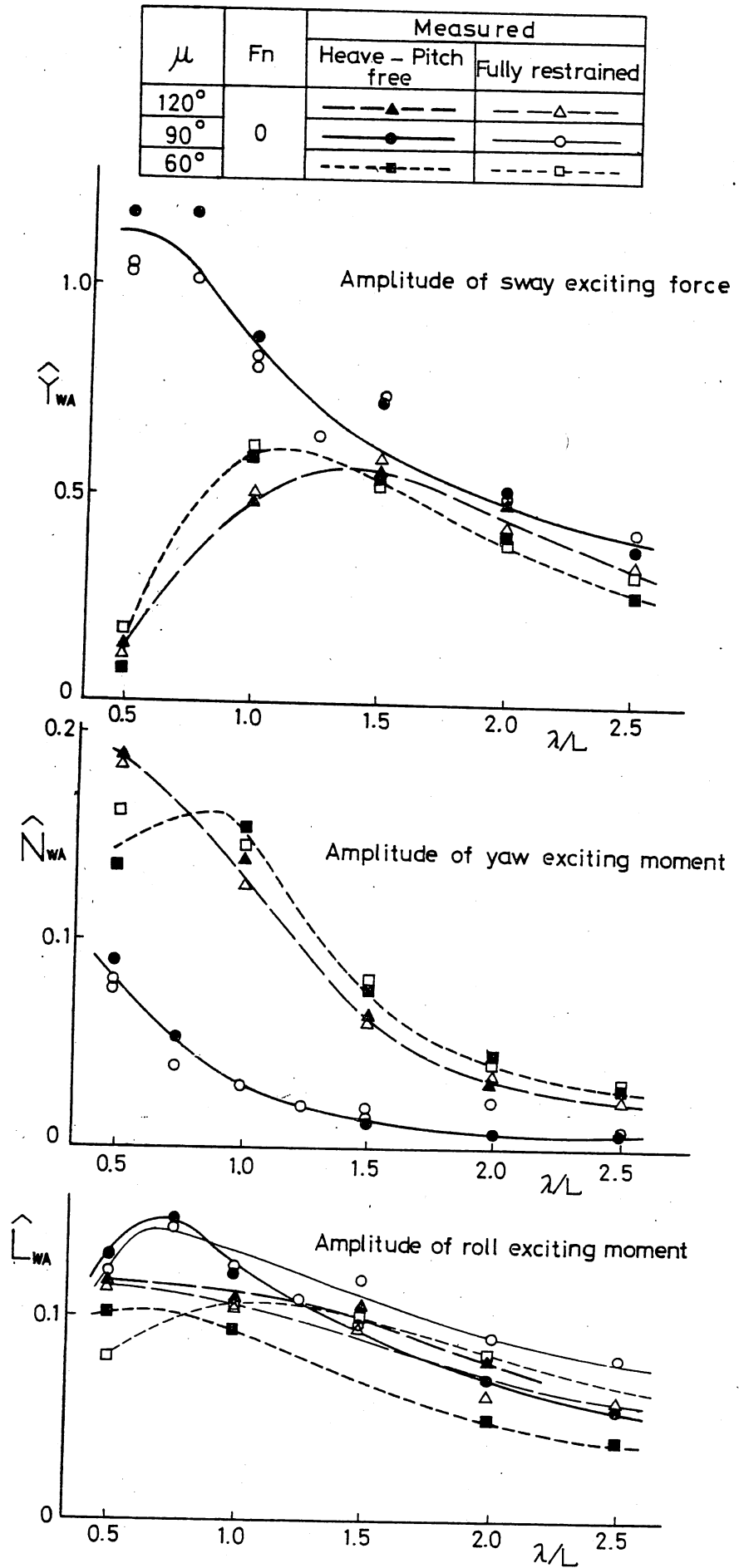
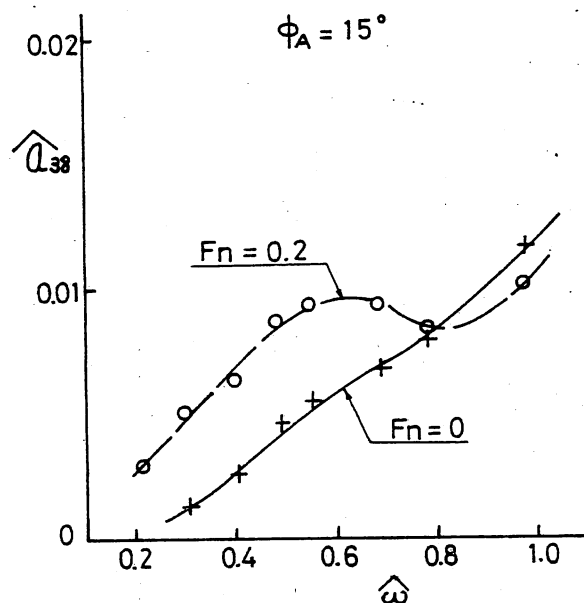


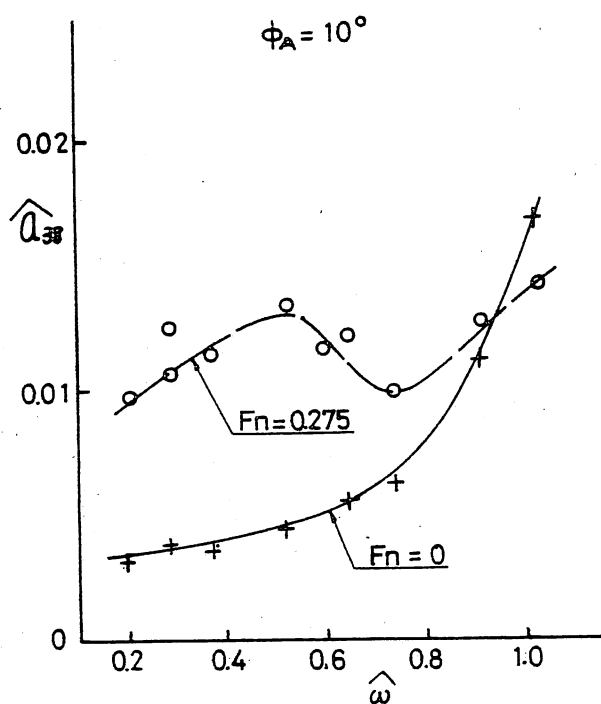
Fig. 7 Wave exciting force and moments : container ship

Ship	(a)	(b)	(c)
	Single Screw Container ship	Twin Screw Container ship	Triple Screw Ferry boat
Scale	1/5833	1/70	1/50
Lpp	3.000 m	3.500 m	3.600 m
B	0.446 m	0.461 m	0.520 m
d	0.163 m	0.158 m	0.140 m
C _b	0.557	0.573	0.509
Appendage	Bilge Keel Rudder	Bilge Keel Rudder Bossing Fillet	Shaft Shaft-bracket Rudder

(a) Single screw container ship A



(b) Twin screw container ship B



(c) Triple screw ferry boat C

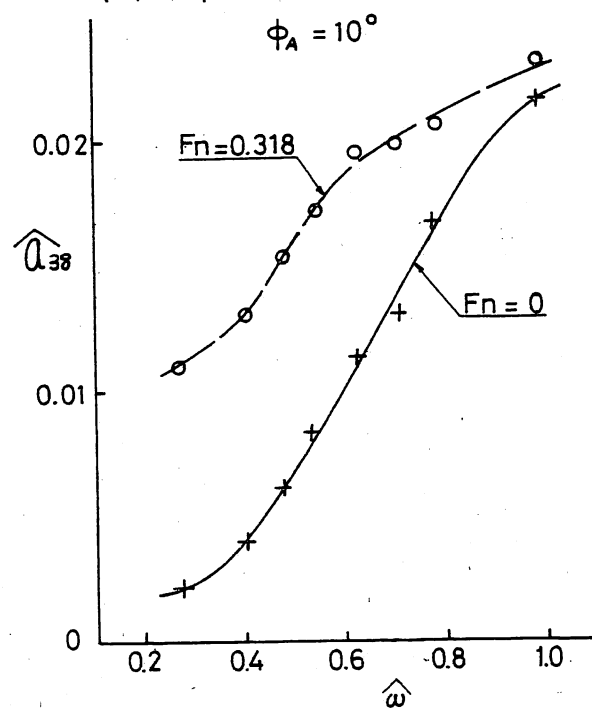
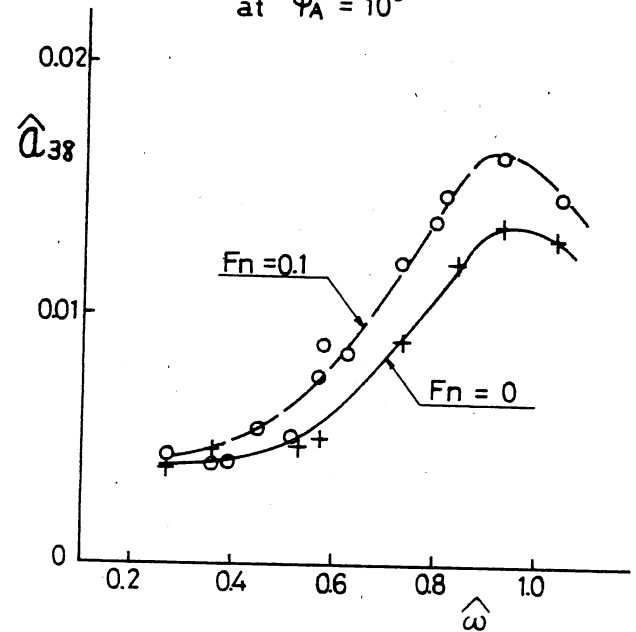


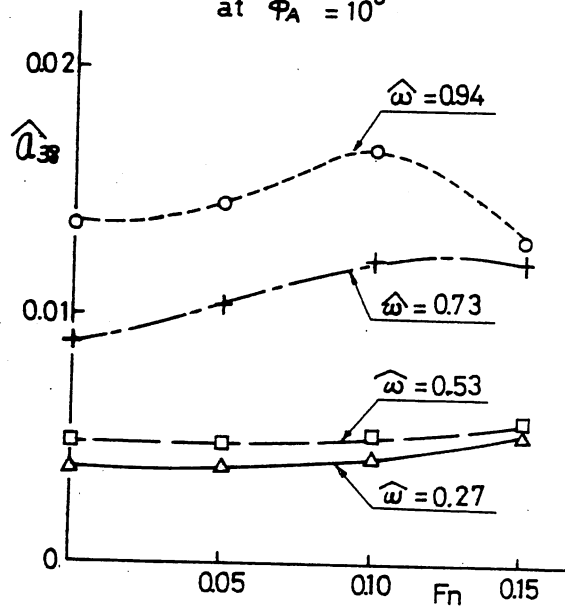
Fig.8 Roll damping moment coefficient of container ships and a ferry boat

Ship	Tanker
Scale	1/125
L_{pp}	3000 m
B	0.544 m
d	0.216 m
C_b	0.846
Appendage	Bilge Keel Rudder

(a) Effect of frequency
at $\phi_A = 10^\circ$



(b) Effect of advance speed
at $\phi_A = 10^\circ$



(c) Effect of rolling amplitude
at $F_n = 0.1$

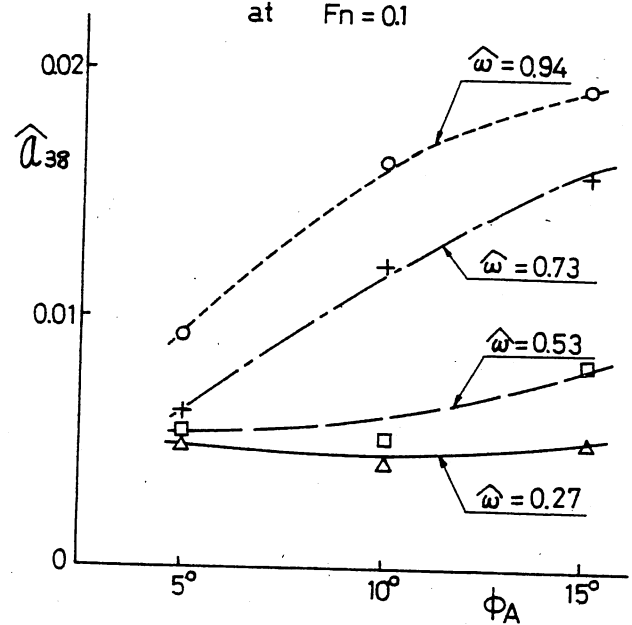


Fig.9 Roll damping moment coefficient of a tanker

Fn	μ	h_w	h_w/L	Marks
0.15	90°	6 cm	1/50	—○—
		10 cm	1/30	---△---
		15 cm	1/20	- - □ - -

Fn	μ	λ/L	Marks
0.15	90°	0.5	—○—
		0.75	---△---
		0.96	- - □ - -
		1.25	---◇---

(a) Non-dimensional rolling amplitude

(b) Rolling amplitude

(c) Relative motion

at s.s. 8 1/2 weather side

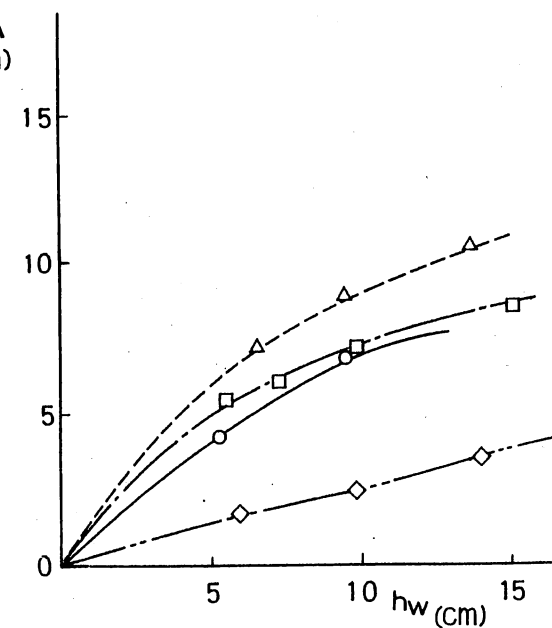
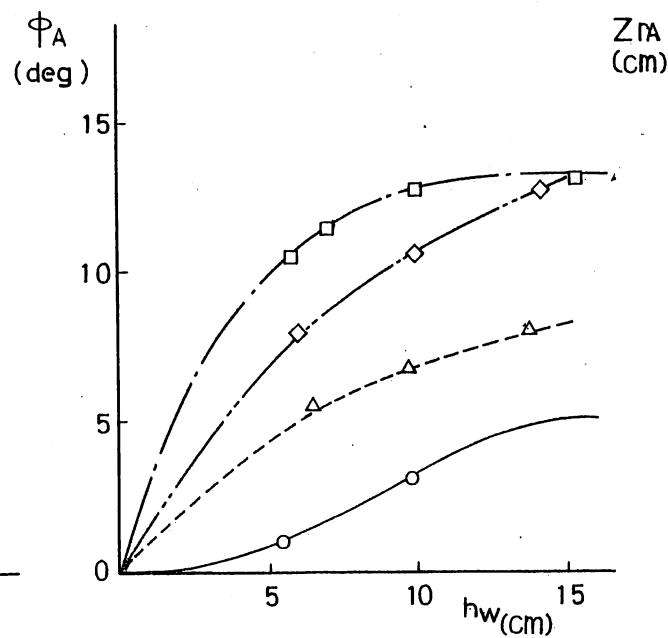
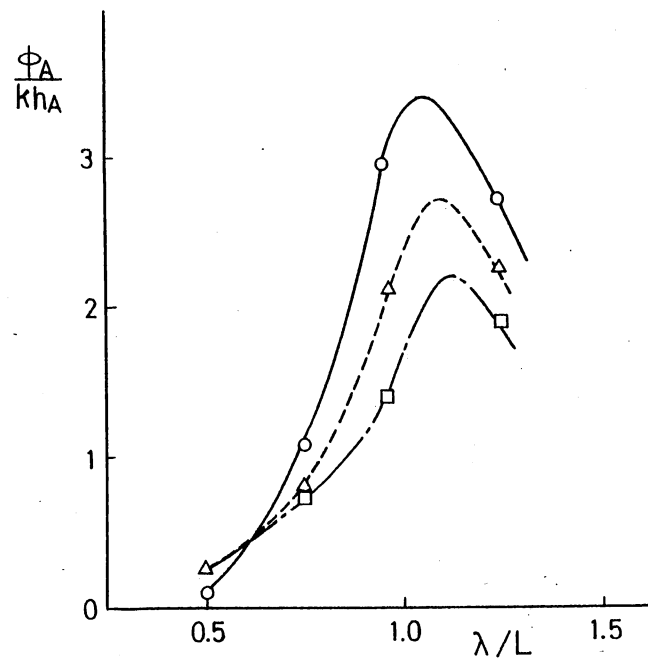


Fig.10 Effect of wave height on rolling and relative motions

EXPERIMENTAL TECHNIQUE FOR STUDYING STABILITY
OF SHIPS ACHIEVED IN SHIP RESEARCH INSTITUTE

by

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Japan

SUMMARY

To investigate the stability of ships operating under strong effects of wind and waves, such as in the case of a small fishing boat resisting a storm. The aerodynamic forces acting on the ship structure above water line, as well as the hydrodynamic forces acting on the ship hull, have been estimated by experiments at the towing tank which is equipped with a wind blower.

This paper describes the experimental techniques of this facility and some examples of applications of the test results, i.e. the directional stability of a ship in strong wind considering the effects of a sea anchor and the transverse stability of a ship drifting and oscillating under combined effects of waves and wind accompanied with gust.

1. INTRODUCTION

To maintain safety in extremely strong wind and waves, ships must have the ability to preserve stability against the effects of external forces induced by wind and waves. The responses of a ship resisting wind and waves can be represented schematically as in Figure 1. The responses affecting the transverse stability are largely governed by the ship's heading against wind and waves. The ship will steer to hold her bow as close as possible to wind direction in order to keep the transverse stability. The ability to keep the bow close to the wind is called the directional stability or the directional controllability and may be considered as the first

gateway to safety. This ability depends mainly upon rudder and propeller or sea anchor if used. The directional stability can be estimated by considering the equilibrium between external forces and the controllable forces of the ship. The transverse stability can be estimated by considering the dynamical responses of the ship induced by the dynamical forces of wind and waves which are superposed on the statical forces.

To solve the equations of equilibrium of motions, the aero-dynamic forces acting on the ship body above water line should be estimated for various conditions of wind on the ship together with the hydrodynamic forces acting on the ship hull under water line which are induced by drifting of the ship.

The first part of this paper will describe the experimental equipment and techniques for measuring these aero- and hydrodynamic forces acting on the model ship in the towing tank. The second part will show an example concerned with the transverse stability under the combined effects of waves and wind with gust.

2. MEASUREMENT OF THE AERO- AND HYDRODYNAMIC FORCES

Experimental Equipment Since the aero- and hydrodynamic forces acting on the ship body are largely influenced by the viscosity of fluid, the model experiments have been carried out for practical use in the various fields of engineering, (1). The hydrodynamic forces acting on the

ship hull can be measured by oblique towing of the model ship in the conventional towing tank. The aerodynamic forces are usually measured by wind tunnel tests which not only require skilled techniques but also laborious and expensive procedures for testing ship body which has a free surface corresponding to the loading condition. To overcome this difficulty a wind blower has been installed on an experimental model basin at Ship Research Institute.

The basin (30m long, 8m wide and 2.5m deep) is equipped with a wind blower in addition to a towing carriage and a wave maker as shown in Figure 2. The wind blower is cross-flow type with the nozzle 3m wide and 0.45m high and generates comparatively uniform air flow above the water surface. The blower is driven by an AC induction motor with an electric coupling for velocity regulation. The velocity of the air flow is 6-17.5 m/sec. The blower is settled on its own carriage which is connected to the towing carriage so as to follow the drifting model if necessary. The blower and the velocity distributions are shown in Figure 3 and Figure 4, respectively.

The model ship to be tested was 2m long and it and the superstructure was made as similar to an actual ship as possible. The model is attached to a dynamometer on the towing carriage through a connecting rod which receives forces to be measured. The dynamometer has four sensing elements for three components of forces in the horizontal plane and the rolling moment. Pitching and heaving motion are not restricted and are measured by the potentiometers. The dynamometer and the model can rotate in the horizontal plane around the vertical axis of the model so that the angle between wind direction and the longitudinal axis of the model is easily changed from 0 to 360 degrees. The connecting rod between model and dynamometer faces the air flow which induces the additional force to be subtracted from the total forces measured. The mutual aerodynamic interactions between rod and model as well as the interference by waves and flow of water surface according to air flow are ignored.

The merits of this equipment are as follows:-

- a) The aero- and hydrodynamic forces can be measured by using the same model ship and the same dynamometer.

- b) The draft, trim and heel of the model can be easily changed.
- c) The relative direction of wind or drifting can be easily changed.
- d) Dynamic forces by waves or gust can be measured under various conditions of the restriction of the motions.
- e) Dynamic responses of the model under simultaneous actions of waves and wind accompanied with gust can be simulated, if all restrictions for force measurements are loosened.

It is observed that the cross-flow type blower is suitable for generating uniform air flow throughout the entire range of the nozzle. The velocity gradient of the air flow on the water surface depends upon the height of the lower end of the nozzle. It suggests that the velocity gradient of natural wind at sea can be reproduced in the tank.

Test Results The wind forces as well as the hydrodynamic forces were measured for various vessels in this tank, (2). The results are shown here only for three Japanese fishing boats which will be dealt with in the next chapter. The forces and moments are defined in Figure 5 and represented by the non-dimensional coefficients as in Equation (1).

$$\begin{aligned}
 C_{R_x} &= R_x / \left(\frac{1}{2} \rho U^2 B \right) & C_{R_y} &= R_y / \left(\frac{1}{2} \rho U^2 B \right) \\
 C_{M_a} &= M_a / \left(\frac{1}{2} \rho U^2 B L \right) & C_{M_{R_a}} &= \frac{M_{R_a}}{\frac{1}{2} \rho U^2 B \left(\frac{B_0}{2} \right)} \\
 C_{L_x} &= L_x / \left(\frac{1}{2} \rho_w v^2 S \right) & C_{L_y} &= L_y / \left(\frac{1}{2} \rho_w v^2 S \right) \\
 C_{M_w} &= M_w / \left(\frac{1}{2} \rho_w v^2 S L \right) & C_{M_{R_w}} &= \frac{M_{R_w}}{\frac{1}{2} \rho_w v^2 S \left(\frac{B_0}{2} \right)} \\
 C_D &= D / \left(\frac{1}{2} \rho_w v^2 A_D \right) & K_T &= T / (\rho_w N^2 D P^4) \\
 C_R &= T_R / \left(\frac{1}{2} \rho_w N^2 P^2 A_R \delta \right) & & (1)
 \end{aligned}$$

The dimensions of the tested ships are shown in Table 1, and the results are shown in Figures 6 and 7. The wind forces of ship B are also

measured by the wind tunnel test using a mirror imaged model for comparison. Both results agree rather well with each other, with some slight fluctuations in the centre of force.

3. CALCULATION OF THE DIRECTIONAL STABILITY OF SHIPS WITH SEA ANCHOR

The equilibrium equations of the forces and moments in three directions in the horizontal plane can be represented as in Equation (2).

$$-L_x - R_x + T + D \cos \beta = 0$$

$$-L_y + R_y + F_R - D \sin \beta = 0 \quad (2)$$

$$-M_w - M_a + F_R \cdot l_R + D \cdot l_f \sin \beta = 0$$

or as the non-dimensional formulas (3)

$$-C_{Lx} - (C_1 C_{R_x} - C_t \xi) \xi - \eta \cos \alpha = 0$$

$$-C_{Ly} + (C_1 C_{R_y} + C_r \xi) \xi - \eta \sin \alpha = 0$$

$$-C_{M_w} - (C_1 C_{M_a} - \lambda_R C_r \xi) \xi + \eta \cdot \lambda_f \sin \alpha = 0 \quad (3)$$

where

$$C_1 = \frac{\rho_a \delta}{\rho_w S} \quad C_t = \frac{D_p^2 K_T}{S p^2} \quad C_r = \frac{A_R}{S} C_{R\delta}$$

$$\xi = \left(\frac{U}{V} \right)^2 \quad \eta = \frac{A_D}{S} C_D \quad \zeta = \left(\frac{NP}{U} \right)^2$$

$$\lambda_f = l_f / L$$

$$\lambda_R = l_R / L$$

Assuming that α , ξ and η are variables and δ and ζ or N are given as the controllable quantities, Equation (3) can be solved numerically by applying the experimental values for the coefficients which are shown in the previous section. The calculated results are presented in Figure 8 for the relative

direction of the wind. Consequently, a simplified formula has been derived for estimating the necessary size of a sea anchor which will keep the ship's bow within a certain heading angle to the wind, that is:-

$$A_D = (k_B / C_D) \{ 0.0055(90 - \beta) + \overline{c_b c_a} / L \} \quad (5)$$

where

$k = 1.9$ for ships A and B, 1.5 for ship C

C_D is the drag coefficient of the sea anchor, and

$\overline{c_b c_a}$ is the horizontal distance between the centres of the area B and A

4. EXPERIMENTS ON THE TRANSVERSE STABILITY

A concept of the dynamical stability of ships in waves and wind accompanied with gust has been adopted for the criteria of the transverse stability of ships in Japan. Watanabe and others have described this concept as follows (see (4)):-

"Let us assume that, as shown in Figure 9, a ship is listed θ_s under the statical transverse heeling moment (due mainly to steady wind), and rolls θ_0 and θ'_0 from side to side about θ_s , and then the ship is subjected to a gust when the ship is at the maximum angle of roll θ_0 windward. In order that the ship does not capsize,

Area ABC > Area BFGD or

Area AK'C > Area K'FG'

Let area AK'C = b, and area K'FG' = a, and $C = b/a$ (6)

It follows that this C must be greater than unit, if the ship is not to capsize."

In the application of this concept for general use, the wind forces and ship responses should be practically

determined for all sorts of ships. Some simplified formulas have been given in Reference (4) for passenger ships.

The experimental equipment described in this paper can be applied to estimating the forces as well as to simulating the responses to such a stability concept. Figure 10 shows a record of simulation of the transverse stability of a ship in wind and waves. The ship is oscillated by waves having a period synchronous with that of the rolling and is listed by a steady wind. The gust is supplied by suddenly opening the screen mesh covering the nozzle of the blower at the moment that the ship rolls maximum to windward.

The ship inclined leeward and the maximum angle of heel reached approximately the estimated value by the stability criteria but did not capsize.

Figure 11 shows an experiment of the statical stability of the model in strong wind with and without the effects of shipping water on deck as well as drifting by wind.

5. CONCLUDING REMARKS

In this paper, the experiments on the stability of ships carried out at Ship Research Institute are presented. These experiments were originated by the research which was carried out on the occasion of the large scale sea disaster involving modern Japanese bonito pole fishing boats and which was caused by a typhoon in October, 1965, in the Mariana Islands. Seven ships among ten sank and 209 human lives were lost. It has been deduced that the main cause of the capsize or foundering of the ships was that they had lost their controllability due to the extremely strong wind which exceeded 70 m/sec and were inclined so heavily by the wind and the additional effects of shipping water on deck that the end of the bridge of the ships reached wave surface, and the ships then foundered by flooding.

After this urgent research, the research projects concerned with the stability of fishing boats in rough weather and the guidance of applications of sea anchor for improving the directional stability were performed by the committees organised at the Japan Association for Preventing Marine Accidents.

In the author's opinion, future research activities should include stability experiments in irregular waves, in heavy following or quartering seas, as well as in shortcrested seas or in multi-directional waves.

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LIST OF SYMBOLS

A_D	Area of sea-anchor
D	Drag of sea-anchor
F_R	Transverse rudder force
L_x	Longitudinal water force
L_y	Transverse water force
M_a	Wind moment
M_w	Water moment
N	Number of revolutions of propeller
P	Pitch of propeller
R_x	Longitudinal wind force
R_y	Transverse wind force
T	Thrust of propeller
U	Velocity of wind
V	Velocity of drifting ship
c_a	Position of the centre of area of ship's profile above the water-line
c_b	Position of the centre of area of ship's profile under the water-line
k	Constant
l_R	Distance of the rudder from the midship
l_a	Distance of the centre of wind force from the midship
l_b	Distance of the centre of drifting force from the midship
l_f	Distance of the towing point of sea-anchor from the midship
α	Drift angle of ship
β	Towing angle of sea-anchor
δ	Rudder angle
λ	Distance between the centres of wind and water force
λ_0	λ at α and $\varphi = 90$ degrees
φ	Relative wind-direction
ρ_a	Density of air
ρ_w	Density of water

Table 1. Dimensions of the tested ships

ITEMS		SHIP-A	SHIP-B	SHIP-C
LENGTH Bet P.P.	L (m)	27.800	29.500	29.000
BREADTH MLD.	B_0 (m)	6.100	6.300	6.360
DRAFT FORE	d_f (m)	1.370	2.188	1.430
DRAFT AFT	d_a (m)	3.240	3.366	3.380
DRAFT MEAN	d_m (m)	2.305	2.777	2.405
DISP. VOLUME	Γ (m ³)	272	365	308
PROJECTED AREA (ABOVE WATER LINE)				
FRONT VIEW	A (m ²)	31.7	34.3	31.6
PROFILE	B (m ²)	90.2	100.0	100.9
(UNDER WATER LINE)				
PROFILE	S (m ²)	64.3	81.8	69.9
DIA. OF PROPELLER	D_p (m)	1.64	1.74	1.71
PITCH RATIO OF PROP.	p	0.56	0.56	0.56
RUDDER AREA	A_R (m ²)	2.64	2.08	2.28

Ship A: Drift netter

Ship B: Bonito pole fishing boat

Ship C: Two-boats stern trawler

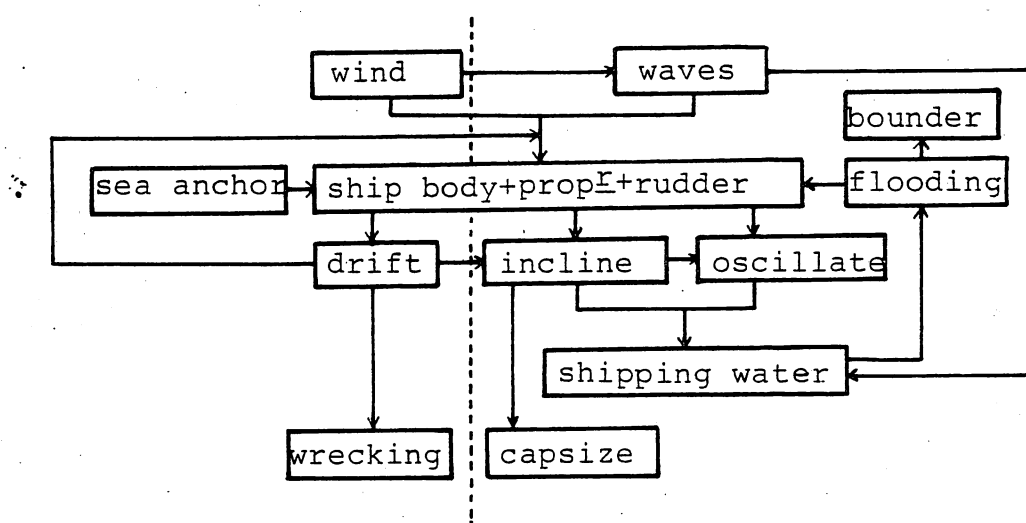


Fig. 1. Responses concerning stability of ship in rough weather

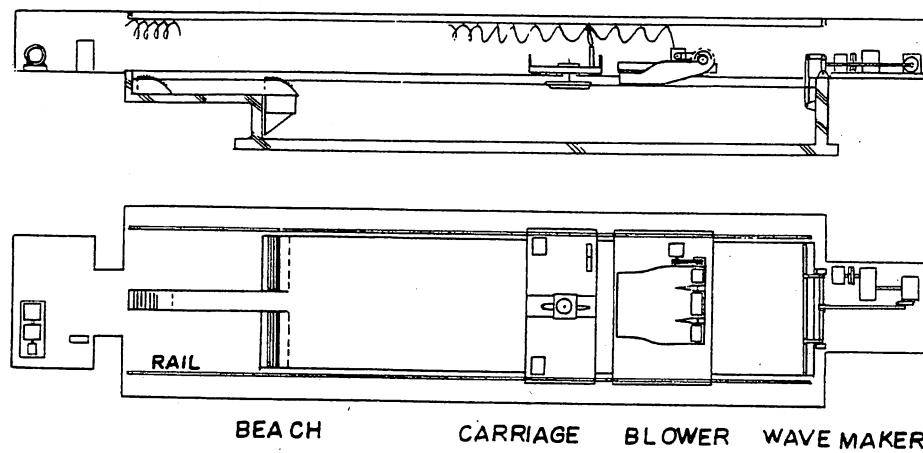


Fig. 2. Model Basin with a wind blower

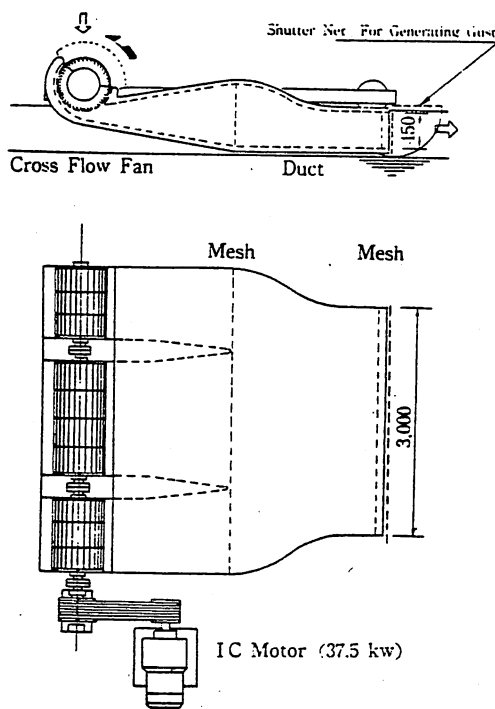


Fig. 3. Wind blower

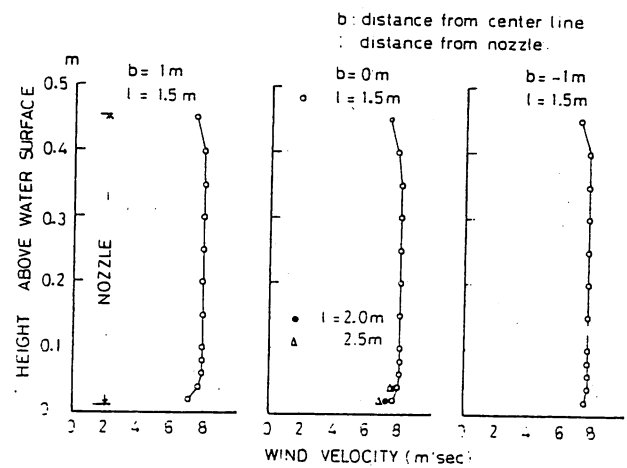


Fig. 4. Velocity distributions

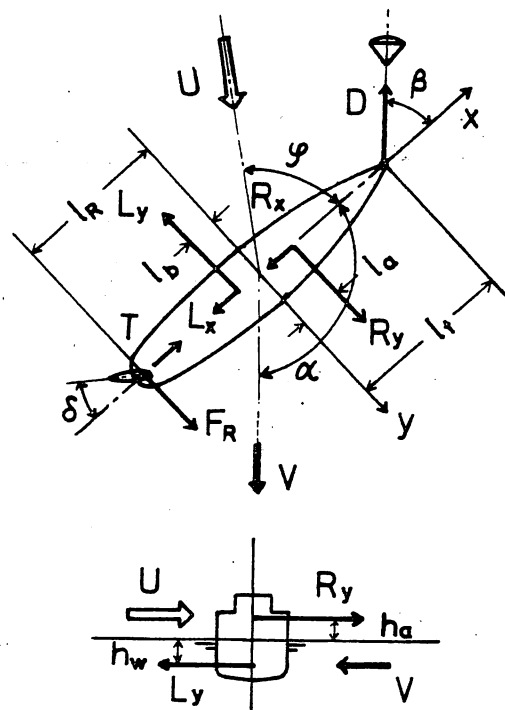


Fig. 5. Definition of forces and moments

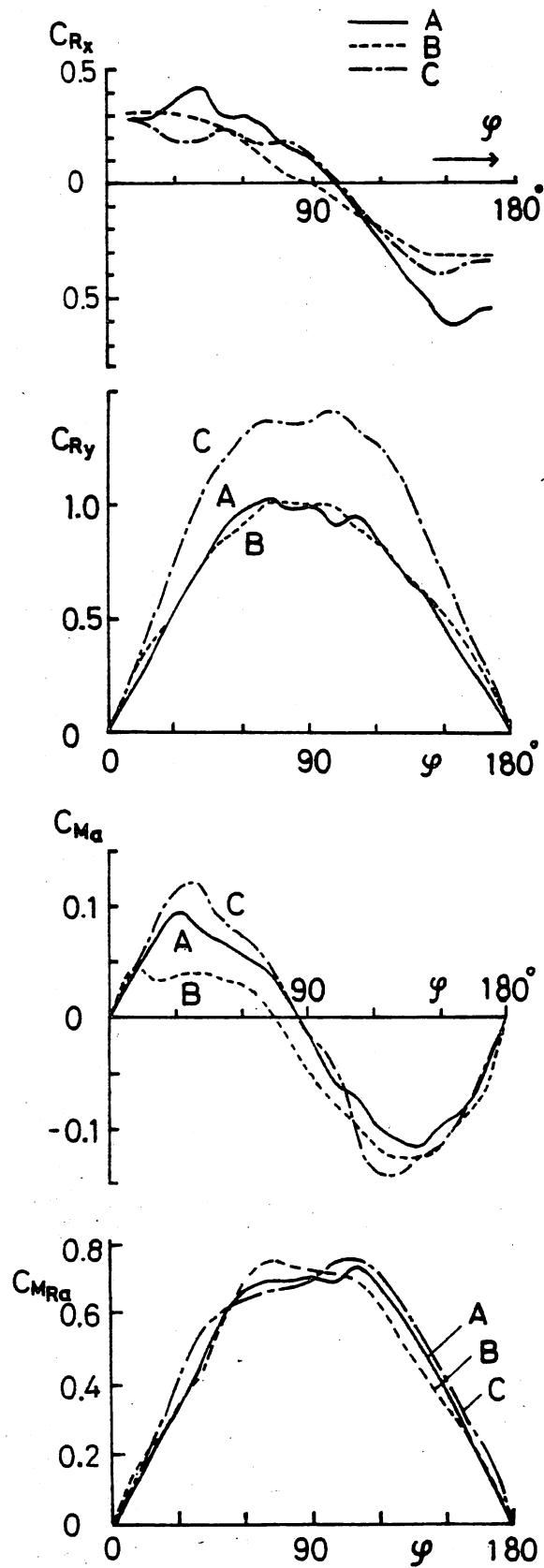


Fig. 6. Aerodynamic forces and moments

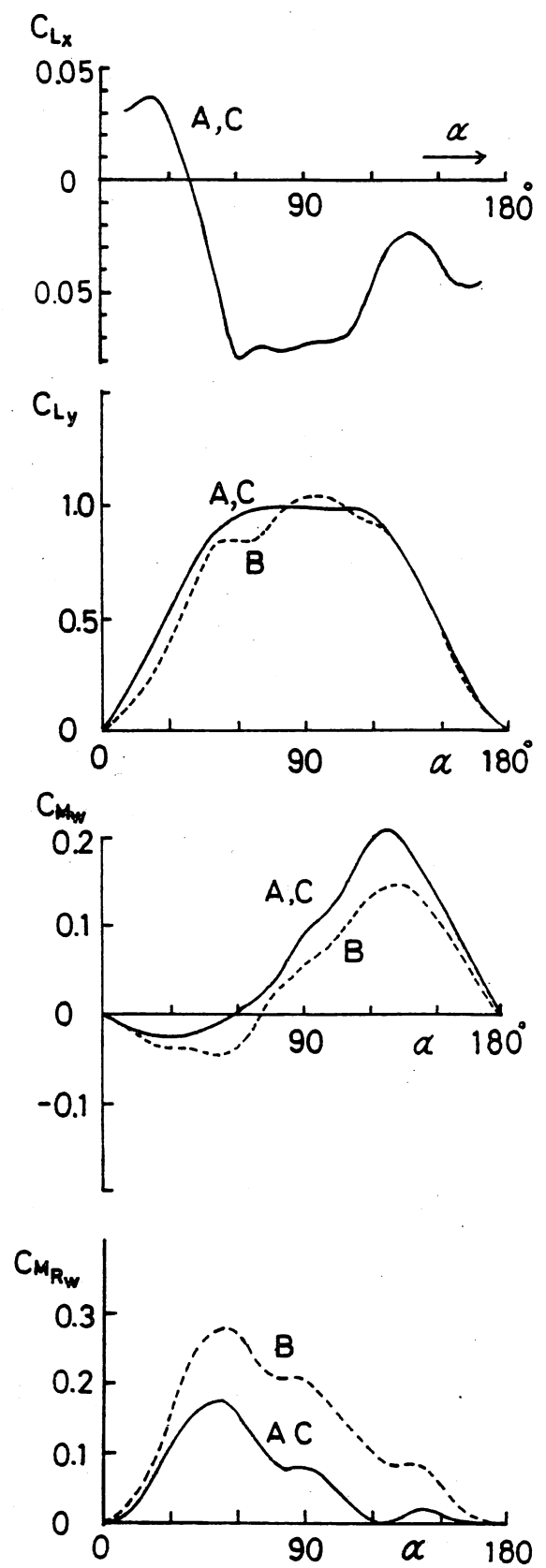


Fig. 7. Hydrodynamic forces and moments

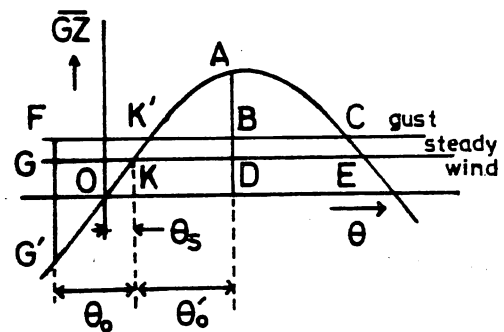
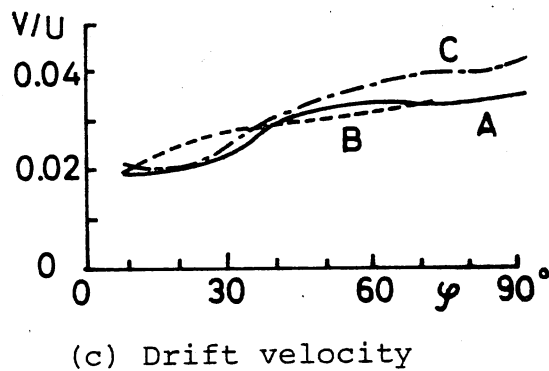
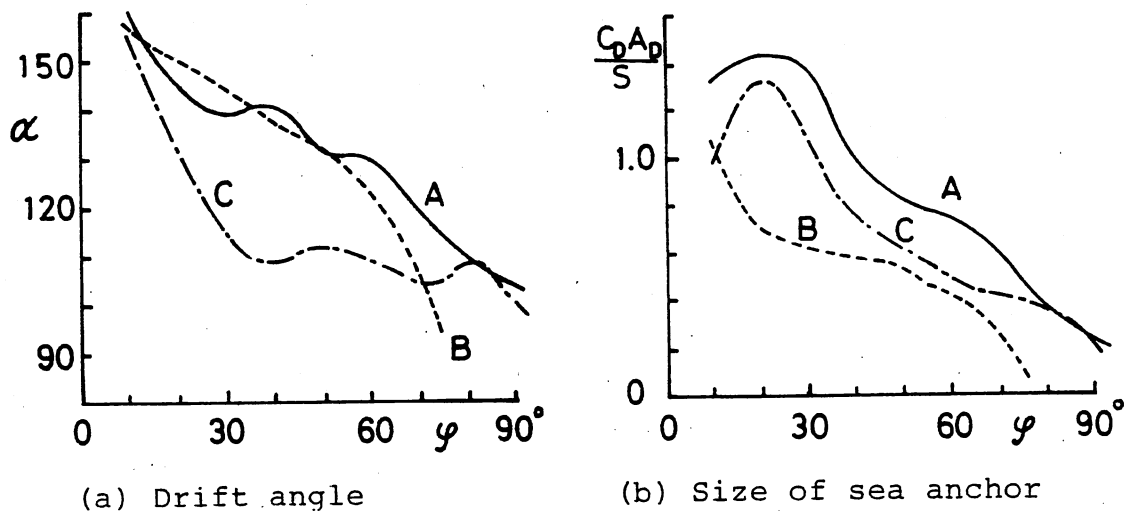


Fig. 8. Necessary size of sea anchor

Fig. 9. A concept of stability criterion

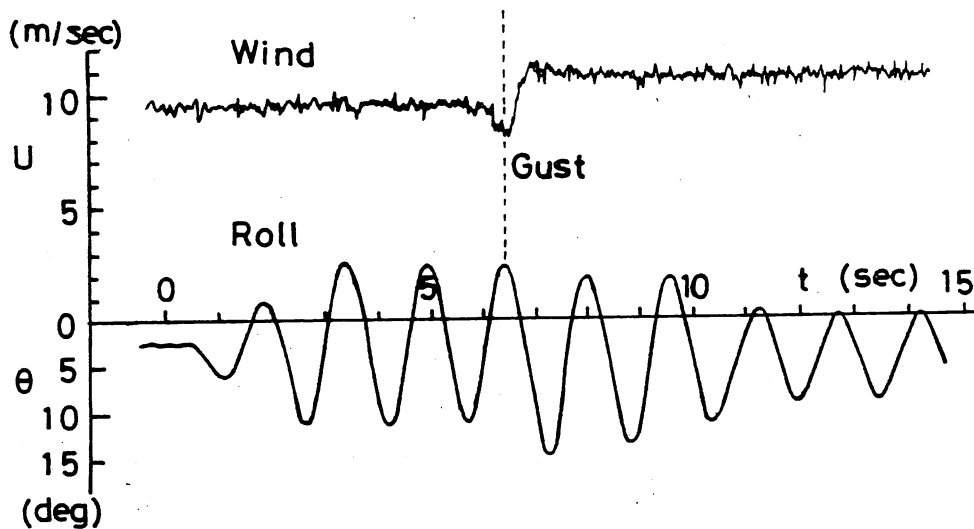
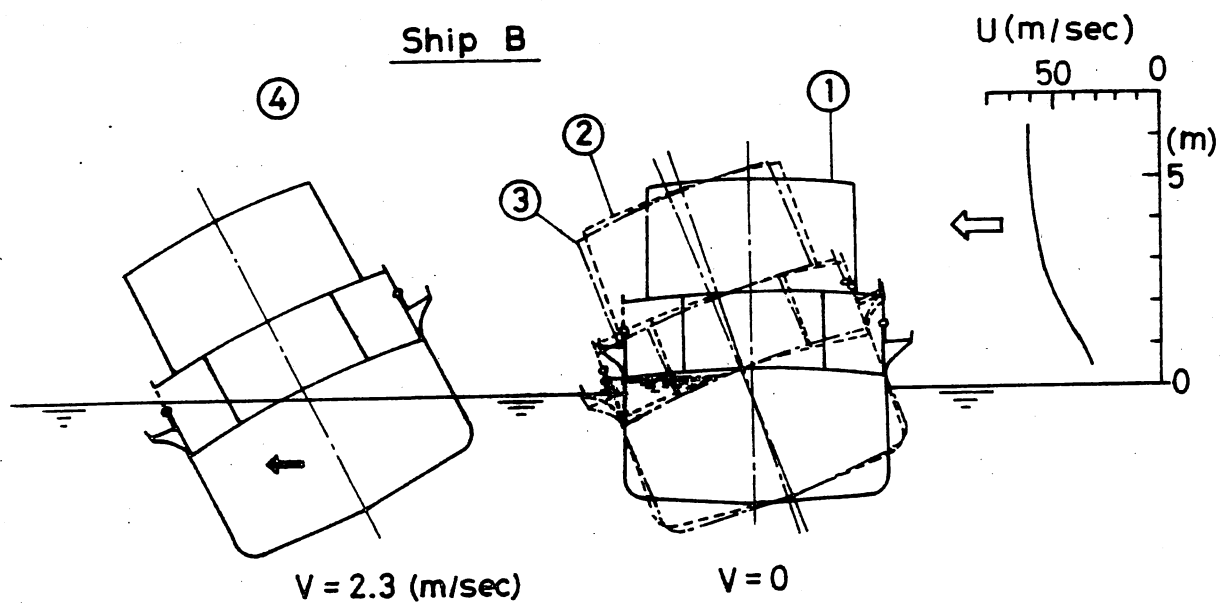


Fig.10. A record of ship motion in wind with gust and waves



- ① Ship is up-right without wind. $\theta = 0^\circ$
- ② Ship is inclined by wind but no drifting. $\theta = 17^\circ$
- ③ Ship is inclined by wind and shipping water on deck but no drifting. $\theta = 21^\circ$
- ④ Ship is inclined by wind and drifting after ③. $\theta = 30^\circ$

Fig.11. Stability experiment in strong wind

A SCALE MODEL INVESTIGATION OF THE INTACT STABILITY
OF TOWING AND FISHING VESSELS

by

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SUMMARY

A scale model investigation of the intact stability of towing and fishing vessels is described. The program was based on four models of typical U.S. towing and fishing vessels. The tests included tripping of towing vessels in calm water and towing and free route operation in head and following seas and in beam seas at zero speed. The objective, rationale and background of the test program are described. The test program are described. The test equipment, test procedures, and model dimensions and stability characteristics are given in detail. Selected results from the model tests are presented.

1. INTRODUCTION

Each year there are a number of casualties in the U.S. towing and fishing vessel fleets which can be attributed to a lack of sufficient intact stability. In general, these casualties are of two types. The first is the towing vessel tripping casualty in which the towing vessel is capsized by the towline forces generated by its own actions or the actions of its tow. The second is the seakeeping casualty in which the vessel is capsized by wave induced forces.

The United States Coast Guard is currently sponsoring a major research programme aimed at developing better intact stability criteria with the objective of reducing the number of stability related casualties. This programme includes studies on the capsizing of cargo liners in following seas, the analytical calculation

of capsizing in following seas, the capsizing of deck cargo barges in beam seas and the development of stability criteria for towing and fishing vessels. The program to develop improved stability criteria for towing and fishing vessels is being carried out for the Coast Guard by Hydronautics Incorporated with major subcontract support by Nickum and Spaulding Associates. This programme is divided into three major tasks. The first is programme formulation which includes a literature survey, a census of the U.S. towing and fishing vessel fleets, detailed stability calculations on some 51 representative towing and fishing vessels and the formulation of a model test programme. The results of this task are reported in (1). The second task, which has just been completed, is an extensive model test program to investigate capsizing due to tripping and wave action. The third task, which is now in progress, is an evaluation study leading to the formulation of improved criteria based on the results of the first two tasks.

This paper presents a limited report on the model test programme which was conducted as the second task of the total project. The objectives and scope of the test programme are briefly discussed and followed by a description of the test equipment, the test procedures and the models. The results of the seakeeping and tripping tests are then presented in general terms and followed by conclusions and recommendations.

2. OBJECTIVES AND SCOPE OF THE TEST PROGRAMME

The objectives of the model test programme were to define the nature of the capsizing phenomena and to obtain quantitative data for a range of vessel characteristics, stability levels and environmental conditions for use in the formulation of improved criteria. The test programme included four models, an ocean tug (Model T-24), an offshore supply boat (Model S-04), a U.S. West Coast fishing vessel (Model F-34) and a harbour tug (Model T-14). A number of different types of tests were conducted with one or more of the models. Tripping tests included:

- a) Tripping due to actions of the towing vessel, i.e., self-tripping.
- b) Tripping due to actions of the tow, i.e., tow tripping.

Seakeeping tests included the following:

- a) Free running in regular following waves.
- b) Towing in regular following waves.
- c) Free running and towing in regular head waves.
- d) Zero speed in regular beam waves.

No tests were conducted in irregular waves.

3. TEST EQUIPMENT AND PROCEDURES

Facilities, Models and Instrumentation

The model tests were carried out in the Hydronautics Ship Model Basin, HSMB, in the period between May and September of 1974. The HSMB is 308 ft long, 24 ft wide and 12 ft deep with a wedge type wavemaker at one end and a beach at the other. The wavemaker, which has an electric hydraulic drive system, can generate both regular and irregular waves. It has sufficient power to generate breaking regular waves with lengths between about 3 and 20 ft. The tests were conducted with the large (No. 1) towing carriage. This carriage is about 40 ft long with an open bay type construction. The speed is variable between 0 and 20 ft/sec in either direction. The carriage provided a platform for all of the power supply, recording and photographic equipment.

The four models were constructed with

wood hulls and with aluminium decks and bulwarks. The model lengths ranged between 6.9 and 10.2 ft. Each of the models was self-propelled and remotely controlled. The power and control signals were transmitted to the models, and instrumentation readings were returned from the models through an umbilical cable system. The propeller RPM and rudder angle were continuously variable within the range of the equipment. The rudders turned at a constant rate which was selected to represent typical full scale conditions. The model rudder angle was controlled by the test engineer on the towing carriage. Figure 1 presents a typical arrangement drawing for one of the models.

Table 1 presents a list of the measurements made, the type of sensor used and the location of the sensor.

Time histories of all these data, except propeller RPM, were recorded on magnetic tape. Final time histories of each run were then recorded on paper strip charts from the magnetic tape. In addition, all of the runs were recorded on video tape. Time sequenced still photographs and motion pictures were taken of selected runs.

Test Procedures

During the test programme, six different types of tests were carried out on one or more of the models. The procedures used are described in the following paragraphs:

Self-Tripping This test was intended to simulate the case in which a towing vessel trips itself by use of power and rudder action. The test was conducted in calm water at zero carriage speed with the model pulling against a towline which was attached to the carriage. The bollard pull or propeller RPM was set and the model was oriented by use of the rudders with the towline leading over the stern or the aft quarter. The maximum possible change in rudder angle was then introduced causing the model to pivot under the towline and to "walk" across the tank with the towline leading over the beam. The model either reached a steady heel angle or capsized in a short time.

Tow Tripping This test was intended to simulate the case in which a towing vessel is capsized by the actions of its tow. Two conditions were investigated. The first was a pure tow tripping in which the model was at rest with the towline leading directly over the beam to the towing carriage. The carriage was then

accelerated slowly to a predetermined speed and the model was pulled sideways. The model would reach a steady heel angle or capsize. In variations of this procedure, power and rudder actions were applied to the model, as the carriage started to move, in an attempt to swing the stern under the towline. The second condition was a combination of self-tripping and tow-tripping. This test was conducted with the model and towing carriage underway at a speed-length ratio of about 0.5, with the model pulling against the towline which was attached to a winch fixed on the towing carriage. The winch could haul in the towline at a predetermined speed. The model was oriented, using its rudders, with the towline leading over the aft quarter to the winch on the carriage. This simulated a towing vessel with its tow yawed off to one side. The towline was then hauled in at a known speed using the winch. This simulated the tow shearing off further to one side. In both types of tow-tripping tests the spring constant of the towline was adjusted to a realistic value.

Free Running in Following Waves

Seakeeping tests were conducted with the model free running in regular following waves. These tests were conducted with the model under the open bay of the carriage. Safety lines were attached to the model to restrain it before the start and at the end of each run or to right it after a capsize had occurred. The wavemaker controls were set and the test runs were started, from the wavemaker end of the tank, after the first several waves had passed. The carriage was accelerated slowly up to a constant speed and the safety lines were slacked off. The average speed of the model was adjusted to the carriage speed by adjusting propeller RPM. The model was steered by the test engineer on the towing carriage. This required considerable skill in steep following waves. Fortunately the models were ruggedly built. Tests were made with the model running straight down the tank or zig-zagging to simulate quartering seas. The run continued until the model capsized or the end of the tank was reached.

Towing in Following Waves A test procedure similar to the free running case was used for towing in following waves except that the towline from the model was attached to the carriage. The propeller RPM was held constant at a value which would produce the properly scaled

bollard pull in calm water. The spring constant of the model towline was adjusted to a reasonable value.

Free Running in Head Waves

The same procedures were used in these tests as were used in the following wave tests. However, the tests were started from the beach end of the tank.

Zero Speed in Regular Beam Waves

These tests were conducted with the model at zero speed and oriented beam to the waves. The position of the carriage was adjusted to stay over the model as it drifted down the tank due to the action of the waves. The run was continued until it was clear that the model would not capsize or until capsize occurred.

4. DESCRIPTION OF MODELS

The models used in the test programme were selected from the sample of 51 vessels compiled in Task 1 of the study. The vessels chosen for modelling were representative of typical vessel types and hull form proportions found in the fleet. They represent fairly modern vessels in that all were built between 1966 and 1969. The characteristics of the models and prototype vessels are given in Tables 2, 3, 4, and 5. The stability characteristics of the models for typical test conditions are given in Figures 2, 3, 4, and 5. These figures also include a profile sketch of each model showing the load waterlines and the extent of superstructure included on the model. All of the models were fitted with bulwarks which contained freeing ports equal to the IMCO recommended area.

5. RESULTS OF SEAKEEPING TESTS

Selected results of the seakeeping tests are presented and discussed in the following paragraphs:-

Free Running in Following Waves

Tests were conducted in following regular waves with the models running free. Typically, tests were conducted with two values of GM for each displacement. In general, the GM's were chosen so that in one case the model would be near to capsizing only in the most extreme conditions and in the other capsizing would occur in more moderate conditions. Typical test results are presented in Figures 6, 7, 8, and 9 in the form of boundaries for wave con-

ditions which caused capsizes. No data are presented for Model T-14 since it would not capsize in following seas for the two GM conditions tested.

In all cases, the capsize and extreme rolls were due either to a direct loss of stability with the model poised on the wave crest or to unstable rolling with a period twice the wave encounter period. These phenomena have been described in detail in Reference 2. In order to capsize by these methods, it was necessary to have both a long encounter period and a long effective roll period. In no case was a model capsized in following waves while running free at a low speed length ratio. It was necessary to have waves long enough ($\lambda/L > 1.5$) and the model speed high enough ($V/\sqrt{L} \sim 1$) for the model to surf for some time with the wave crest near amidship. Because of this, running in direct following seas was more hazardous than zig-zagging or running in quartering seas.

For capsize to occur, it was also necessary to have the apparent stability reduced and roll period increased by water on deck. It is of interest to compare the case of Model S-04, load I-C, with Model T-14, load I-B. For these conditions, the two models have very similar curves of GZ/B vs heel angle with a range of stability of about 42 degrees. Model S-04, as shown in Figure 7, would capsize easily but Model T-14 would not capsize for any of the wave conditions. The difference was that S-04 shipped large amounts of water and T-14 did not. In several cases, tests were run with the freeing port area doubled, but this had no significant effect.

It is also of interest to note that in no case did a model capsize by broaching. All of the models were sufficiently controllable to avoid serious broaching even in the steepest following seas at high speeds.

Towing in Following Waves Tests were conducted in following regular waves with the models towing at a speed length ratio of 0.5. The propeller RPM was set to produce the estimated scale bollard thrust of the prototype. In the case of F-34, a bollard thrust typical of a towing vessel of the same size was selected. The stability levels were the same as were used in the free running tests. The test results for Model S-04 and F-34 are presented in Figures 10 and 11. Model T-14 would not capsize

towing in following seas for the two GM values tested. Only limited following seas towing tests were conducted with Model T-24 in load condition I-A. In this case, with a wave length/model length of 2.1 and a wave height/model length of 0.225 the model did not capsize.

The capsize mechanism which occurred while towing in following seas, was completely different from that which occurred while free running at high speed in following waves. In the towing case, water tended to pile up on deck from waves coming over the stern and sides. This caused a slowly increasing heel which would reach an equilibrium value or result in capsize. The towline was an important contributing factor since it would cause an initial heel and roll moment when lead over the aft quarter. Also, the towline seemed to reduce the surging of the model and, possibly, the pitching. This tended to promote the buildup of water on deck. It may be noted that capsize when towing occurred in shorter waves than capsize when running free at higher speeds. This was because water piled up on deck when the encounter period was short and relative motions were large. It should also be noted that capsize will not occur running free at the same speed and in the same wave conditions which would cause a capsize when towing. However, for each model a stability condition which would result in a capsize when free running at high speed would also result in a capsize when towing at low speeds. The converse was also true, i.e., a stability condition which would not result in a capsize when free-running at high speed would not result in a capsize when towing.

Beam Seas Limited beam sea tests were conducted at zero speed and in certain cases capsize did occur. These capsize cases included Model T-24 at load I-B and Model F-34 at load 3-B. The capsize mechanism was the pile up of water on deck in short steep waves which broke on deck. This resulted in a slowly increasing heel into the waves with little rolling. Eight to twelve wave encounters were required for a capsize. Model T-24 would capsize only if it were held beam to the sea with the safety lines since it had a strong tendency to head bow into the waves.

Model S-04 in load condition I-C would assume a large steady heel into the waves in some beam sea conditions but it did not capsize. Variations in the freeing port area were tried

on Models F-34 and S-04. In each case the freeing port area was increased to twice the IMCO required value. In both cases the larger freeing port area made it easier for water to get on deck and, if anything, made the steady heel larger or caused a capsize to occur sooner.

Head Seas Head sea tests were conducted with Model T-14 running free and towing. The results are presented in Figure 12. Although this model in load condition I-B would not capsize in following or beam seas, capsize in head seas was possible. The capsize mechanism was the rapid buildup of water on deck due to waves breaking over the bow or forward quarter in rapid succession. This would result in a rapidly increasing heel angle which could result in a capsize. In the cases in which a capsize occurred as many as 30 to 50 waves were encountered although the final large increase in heel which resulted in a capsize would occur in about three wave encounters. In general, the towline would assist in the capsize by providing an initial heel and roll moment. However, for the shortest waves the towline tended to right the model. In many respects the capsize mechanism in head seas is similar to the capsize mechanism when towing in following waves and when in beam seas.

A limited number of head sea tests were conducted with Model T-24 in load I-A and Model S-04 in load I-C. In these cases there was no tendency to capsize.

6. RESULTS OF TRIPPING TESTS

The general results of the tripping tests are presented in Table 6. The data for the self tripping tests apply to the case of a rapid application of maximum rudder angle (about 45 degrees) under bollard conditions. Table 6 lists the loading condition, incidence of capsize, and the scaled bollard thrust for the capsize condition. In the event that no capsize occurred, the maximum scaled bollard thrust that could be generated by the model propulsion system is given. The installed horsepower assuming 75 HP per ton of bollard thrust, is also given.

Models T-24, load I-A, F-34, load 3-A and T-14, load I-B, exactly satisfy the Murphy criteria (3) used in the past by the U.S. Coast Guard as a measure of the GM required to prevent self tripping. The prototype powers

used in evaluating the Murphy criteria are 5635 SHP for T-24, 1200 SHP for F-34 and 1960 SHP for T-14. For this test, F-34 was treated as a towing vessel with the fleet average power for this size vessel. It is evident that power in excess of the actual installed power is required to cause a self tripping capsize. However, the margin is small for Model F-34.

The results of the tow-tripping tests conducted by pulling the models laterally from rest are also presented in Table 6. In all cases capsize would occur and the required speeds are listed. These data apply to the conditions with the tow post at the nominal design position. Some tests were conducted with the tow posts moved aft. This showed that, with the tow post further aft, a tow-induced tripping capsize could be prevented.

Some tests were conducted in which power and rudder action were used to swing the stern under the towline. In all cases this did not prevent a capsize and, even, seemed to cause a capsize more quickly.

The author knows of only one criterion for the initial stability of towing vessels which is intended to prevent a tow induced tripping capsize. This criteria, proposed in Norway by Getz and Bakke (4), is based on the assumption that a towing vessel should be able to resist a lateral towing speed between 4 and 5 knots. This assumption may be compared with the actual lateral towing speed required to capsize the models in this study. With a GM defined by the Murphy criteria (3), F-34 will not satisfy a 4 knot requirement and T-14 will not satisfy a 5 knot requirement. A great deal of quantitative model data now exists in the time history records concerning the forces acting on a towing vessel during a tripping capsize. An analysis of these data is beyond the scope of this paper.

7. CONCLUDING REMARKS

The model test programme described has provided considerable insight into the various capsize phenomena which can occur with towing and fishing vessels. It has also provided a base of qualitative and quantitative data for use in the development and evaluation of improved intact stability criteria for these types of vessels.

Efforts are now underway in Task 3 of this study to develop improved criteria using the detailed test data and other tools developed as part of the U.S. Coast Guard research on vessel stability.

A number of specific conclusions can be drawn from the test results presented. They include:-

- a) Some towing and fishing vessels with stability levels which may realistically occur are vulnerable to a capsize by wave induced forces. The offshore supply boat type is a particular example.
- b) Capsize when running free in following seas will occur only if the speed length ratio is high, i.e., about 1.
- c) Vessels which tow at sea will require more stability than those which do not.
- d) The pile up of water on deck is a primary or contributing factor in all of the seakeeping capsize observed in the test programme. This is a dynamic phenomena and is not simply eliminated by lowering bulwark heights or increasing freeing port area. Any improved intact stability criteria should recognise this phenomenon and, as a result, include freeboard requirements as well as some direct measure of the stability.
- e) The current criteria which are intended to prevent self tripping capsize of towing vessels may be conservative for this type of casualty but may not be conservative for the tow induced tripping capsize.

8. ACKNOWLEDGEMENTS

Mr. M Pepper of the Ship Performance Department at Hydronautics Incorporated was the engineer in charge of the actual conduct of the tests described. Cdr. D. Folsom, Mr. J. Amy, Mr. R. Johnson and Mr. W. Cleary of the U.S. Coast Guard made significant contributions to the selection of the models and the guidance of the test programme.

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- 2) Oakley, Paulling and Wood
"Ship Motions and Capsizing in Astern Seas"

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- 3) Murphy
Discussion to "Tug Boat Design"
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- 4) Getz and Bakke
"Stability of Tugs, the Effect of Athwartship Towrope Pull"

The Ship Research Institute of Norway, 1959

Table 1
Instrumentation List

<u>Measurement</u>	<u>Sensor</u>	<u>Location</u>
Roll Angle	GYRO	Model
Pitch Angle	GYRO	Model
Wave Height	Capacitance Wave Probe	Towing Carriage
Rudder Angle	Potentiometer	Rudder Actuator in Model
Towline Tension	2-inch Variable Reluctance Block Gage	Model
Towline Angle Relative to Model	Potentiometer	Towing Bit on Model
Towing Carriage Speed	Carriage Control System	Towing Carriage
Relative Motion Between Model and Wave at Midships	Capacitance Wave Probe	Model
Propeller RPM	Magnetic Tachometer	Model
Towline Haul in Speed	Pulse Counter	Towing Carriage

Table 2
Characteristics of Model T-24

Item	Prototype	Model
Vessel Type	Atlantic Coast Twin Screw Ocean Tug, 1969	
Hull Form	Round Bilge with Ship Stern	
Scale Ratio		17.39
Properties at IMCO Standard Draft		
LWL, ft	116.02	6.67
Beam at Section of Max Area, ft	31.548	1.81
Displacement Tons/lbs	841.3	358.3
BHP	5,750	--
Depth, ft	18.671	1.07
Draft, ft	15.871	0.91
C_B	0.5069	
C_P	0.6182	
C_W	0.8229	
L/B	3.678	
B/T	1.988	
Propeller Dia ft	10.00	0.575
Total Rudder Area ft ²	89.12	0.294
Load Condition 1		
Displacement Tons/lbs	884.89	376.9
Trim, ft	1.859	0.106
Draft, ft	16.286	0.936
Minimum Freeboard, ft	1.435	0.083

Table 4
Characteristics of Model F-34

Item	Prototype	Model
Vessel Type	Pacific Coast Single Screw Crab Boat, 1969	
Hull Form	Single Chine Hull with Transom Stern	
Scale Ratio		11.16
Properties at Load Condition 1		
LWL, ft	86.65	7.76
Beam, ft	25.71	2.30
Displacement Tons/lbs	367	660.9
Trim, ft	1.50	0.14
Depth, ft	14.18	1.27
Draft, ft	12.80	1.15
Minimum Freeboard, ft	1.38	0.12
C_B	0.450	
C_P	0.662	
C_W	0.825	
L/B	3.37	
B/T	2.01	
Propeller Dia ft	6.25	0.560
Total Rudder Area ft ²	32.8	0.263
Properties at Load Condition 2		
Displacement Tons/lbs	267.2	430.6
Trim, ft	1.35	0.121
Draft, ft	10.81	0.969
Minimum Freeboard, ft	2.96	0.265

Table 3
Characteristics of Model S-04

Item	Prototype	Model
Vessel Type	Gulf Coast Twin Screw Supply Vessel, 1969	
Hull Form	2 Chine Hull with Transom Stern	
Scale Ratio		17.47
Properties at IMCO Standard Draft		
LWL, ft	171.07	9.79
Beam at Section of Max Area, ft	38.00	2.18
Displacement Tons/lbs	1529.4	642.52
BHP	--	--
Depth, ft	15.00	0.859
Draft, ft	12.75	0.730
C_B	0.6458	
C_P	0.7277	
C_W	0.8685	
L/B	4.502	
B/T	2.980	
Propeller Dia ft	8.00	0.457
Total Rudder Area ft ²	82.5	0.270
Load Condition 1		
Displacement Tons/lbs	1507	633.1
Trim, ft	1.19	0.068
Draft, ft	12.11	0.693
Minimum Freeboard, ft	1.83	0.105

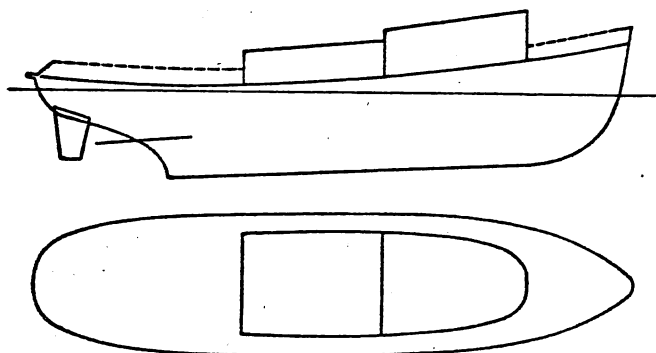
Table 5
Characteristics of Model T-14

Item	Prototype	Model
Vessel Type	Pacific Coast Twin Screw Harbor Tug, 1966	
Hull Form	2 Chine Hull with Transom Stern	
Scale Ratio		12
Properties at IMCO Standard Draft		
LWL, ft	85.62	7.135
Beam at Section of Max Area ft	28.46	2.371
Displacement Tons/lbs	375.9	487.3
BHP	2,000	--
Depth, ft	12.96	1.08
Draft, ft	11.02	0.918
C_B	0.490	
C_P	0.592	
C_W	0.794	
L/B	3.01	
B/T	2.58	
Propeller Dia ft	7.25	0.604
Total Rudder Area ft ²	80.58	0.559
Load Condition 1		
Displacement Tons/lbs	308.1	399.3
Trim, ft	0.0	0.0
Draft, ft	9.76	0.813
Minimum Freeboard, ft	2.922	0.243

Table 6
Full Scale Tripping Characteristics

Self Tripping						
Model	Load Condition	Displ. Tons	GM ft	Capsize	Bollard Thrust Tons	Approximate SHP
T-24	1-A	889.9	3.30	Yes	99	7,425
	1-C	889.9	5.70	No	112	8,400
	2-B	630	2.17	No	120	9,000
S-04	1-A	1507	6.00	No	78	5,850
F-34	3-A	367	2.96	Yes	17.9	1,342
	3-B	367	1.97	Yes	10.7	800
T-14	1-B	308	2.65	No	33	2,475
Tow Tripping						
Model	Load Condition	Displ. Tons	GM ft	Capsize	Sideways Towing Speed at Capsize, Knots	
T-24	1-A	889.9	3.30	Yes	6.2	
	1-C	889.9	5.7	Yes	7.4	
	2-B	630	2.17	Yes	7.4	
S-04	1-A	1507	6.00	Yes	4.9	
F-34	3-A	367	2.96	Yes	3.9	
	3-B	367	1.97	Yes	2.9	
T-14	1-B	308	2.65	Yes	4.75	

Miller



DISPLACEMENT = 889.9 TONS (Full Scale)	
INITIAL STABILITY (Full Scale)	
CONDITION	GM - FT
DESIGN	3.181
TEST 1-A	3.30
TEST 1-B	2.17

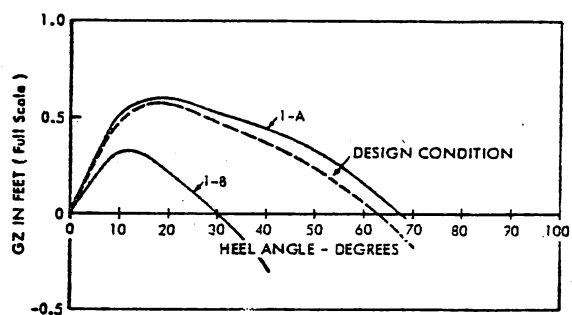
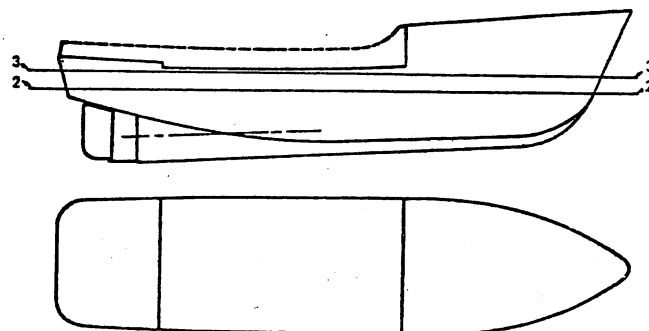


FIGURE 2 - STABILITY CHARACTERISTICS OF MODEL T-24



LOAD 3		LOAD 2	
DISPLACEMENT = 367.0 TONS		DISPLACEMENT = 267.2 TONS (Full Scale)	
INITIAL STABILITY (Full Scale)		INITIAL STABILITY (Full Scale)	
CONDITION	GM - FT	CONDITION	GM - FT
3-A	2.96	DESIGN & 2-A	2.11
3-B	1.97	2-B	1.54

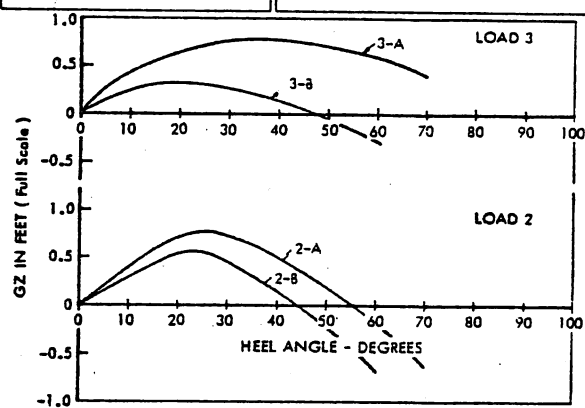
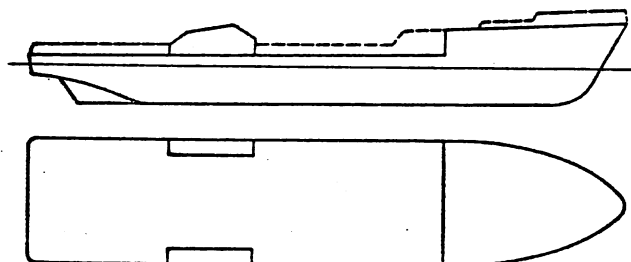


FIGURE 4 - STABILITY CHARACTERISTICS OF MODEL F-34



DISPLACEMENT = 1507.8 TONS (Full Scale)	
INITIAL STABILITY (Full Scale)	
CONDITION	GM - FT
TEST 1-B	7.80
DESIGN & TEST 1-C	6.00

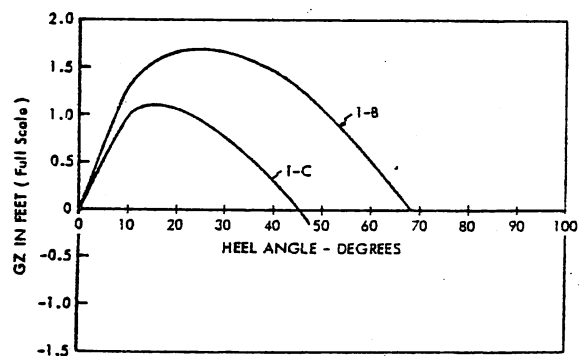
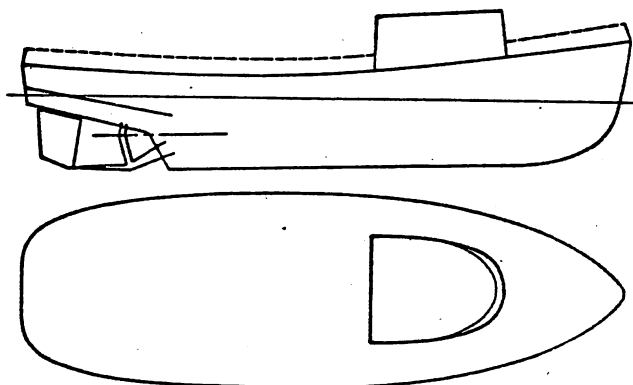


FIGURE 3 - STABILITY CHARACTERISTICS OF MODEL S-04



DISPLACEMENT = 398.1 TONS (Full Scale)	
INITIAL STABILITY (Full Scale)	
CONDITION	GM - FT
DESIGN	5.661
1-B	2.65
1-C	4.0

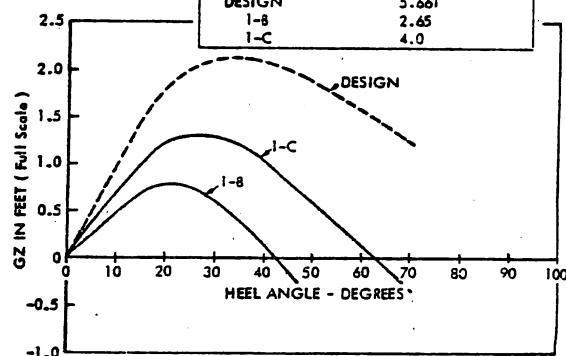


FIGURE 5 - STABILITY CHARACTERISTICS OF MODEL T-14

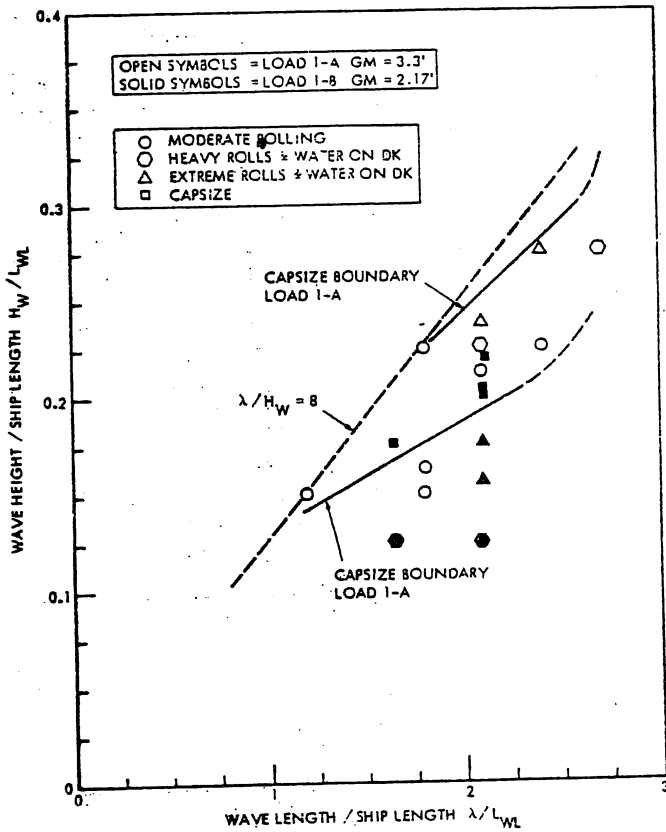


FIGURE 6 - CAPSIZE BOUNDARY FOR T-24 FREE RUNNING AT $V_K/V_L = 1.0$ IN FOLLOWING WAVES

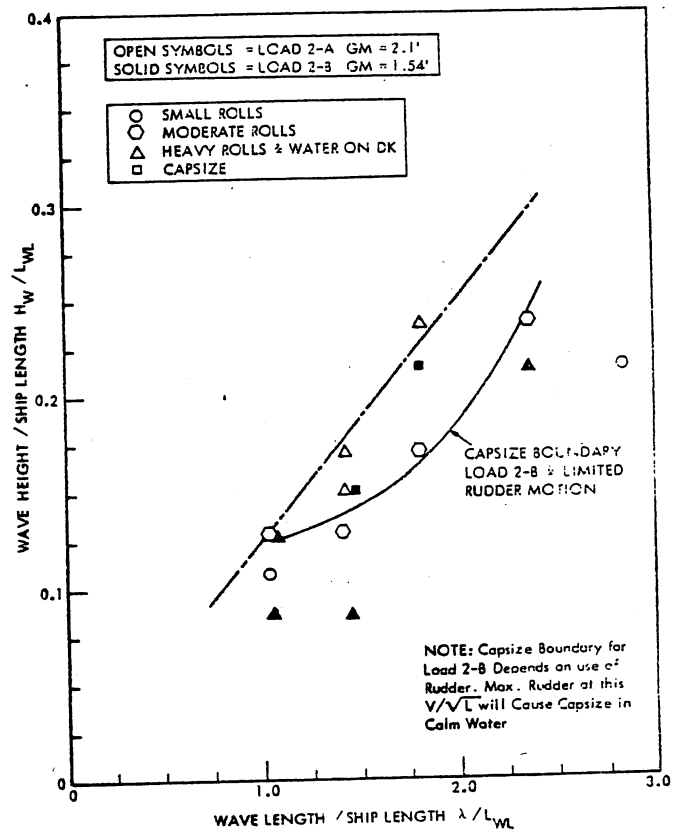


FIGURE 8 - CAPSIZE BOUNDARY FOR F-34 FREE RUNNING AT $V_K/V_L = 1.0$ IN FOLLOWING WAVES - CONDITION 2

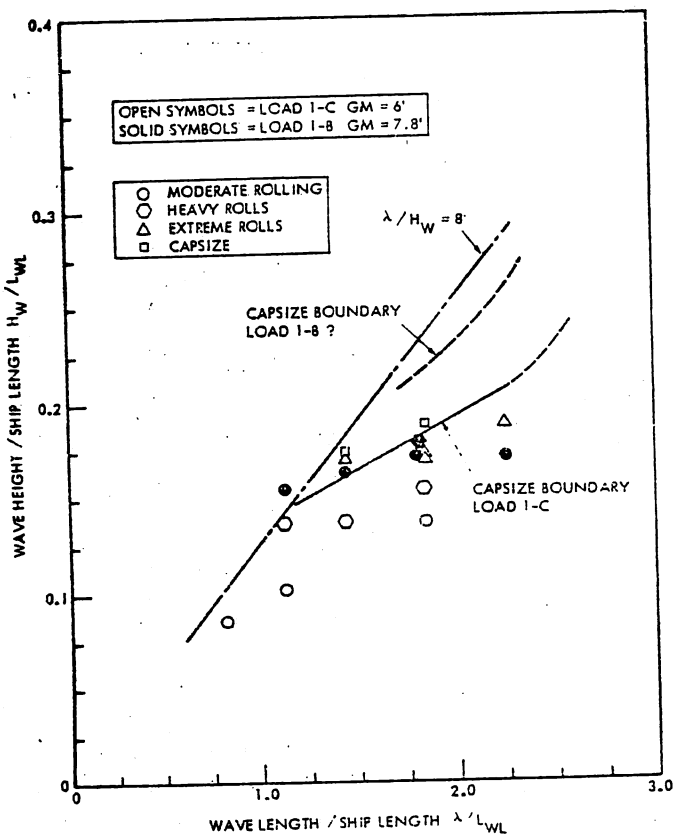


FIGURE 7 - CAPSIZE BOUNDARY FOR S-04 FREE RUNNING AT $V/V_L = 0.9$ IN FOLLOWING SEAS

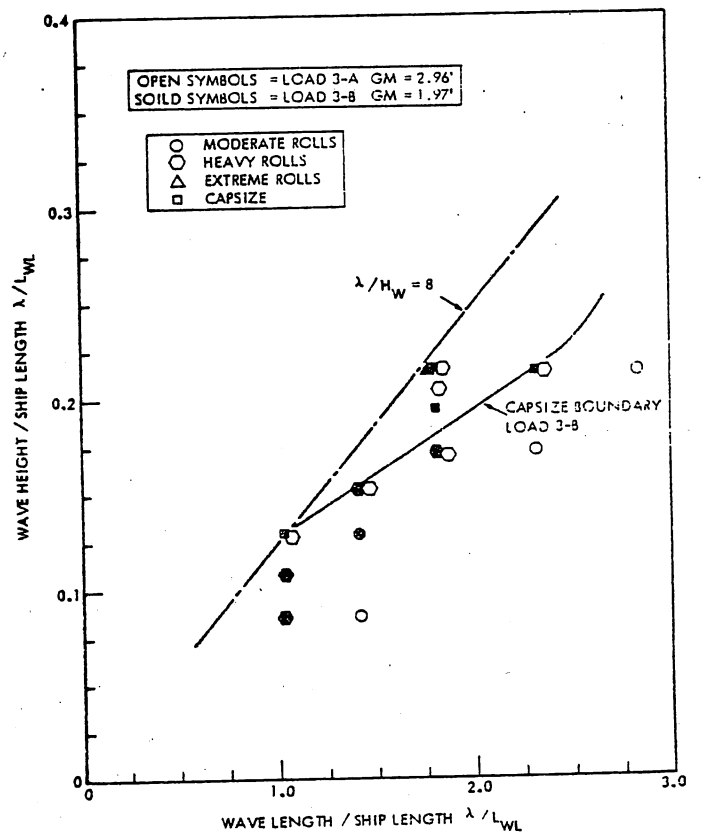


FIGURE 9 - CAPSIZE BOUNDARY FOR F-34 FREE RUNNING AT $V/V_L = 1.0$ IN FOLLOWING WAVES - CONDITION 3

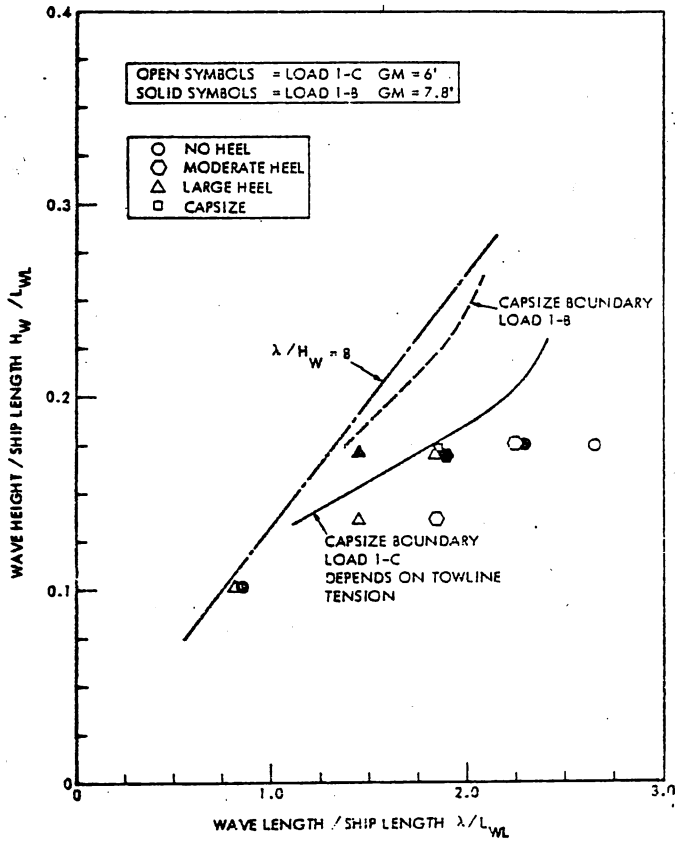


FIGURE 10 - CAPSIZE BOUNDARY FOR S-04 TOWING IN FOLLOWING SEAS

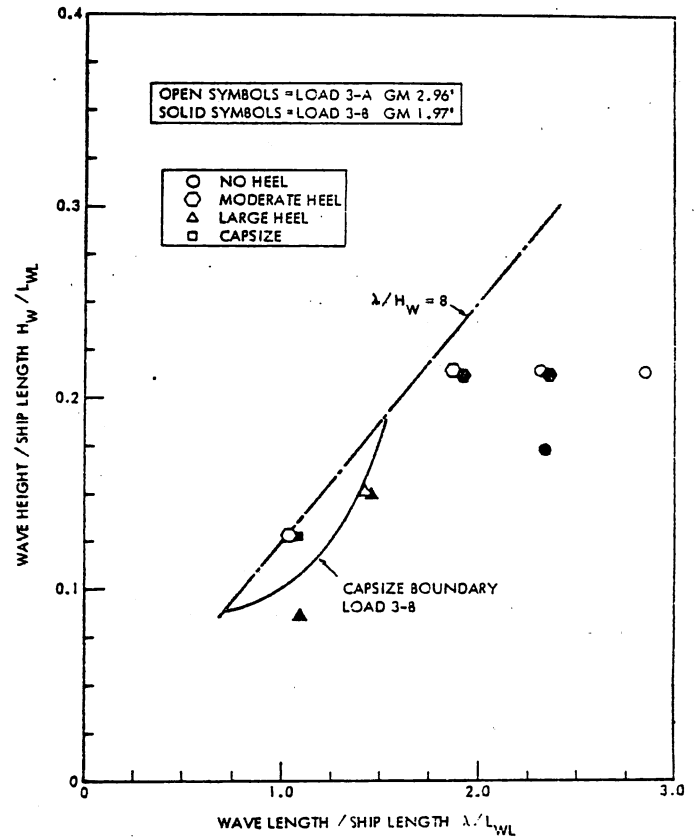
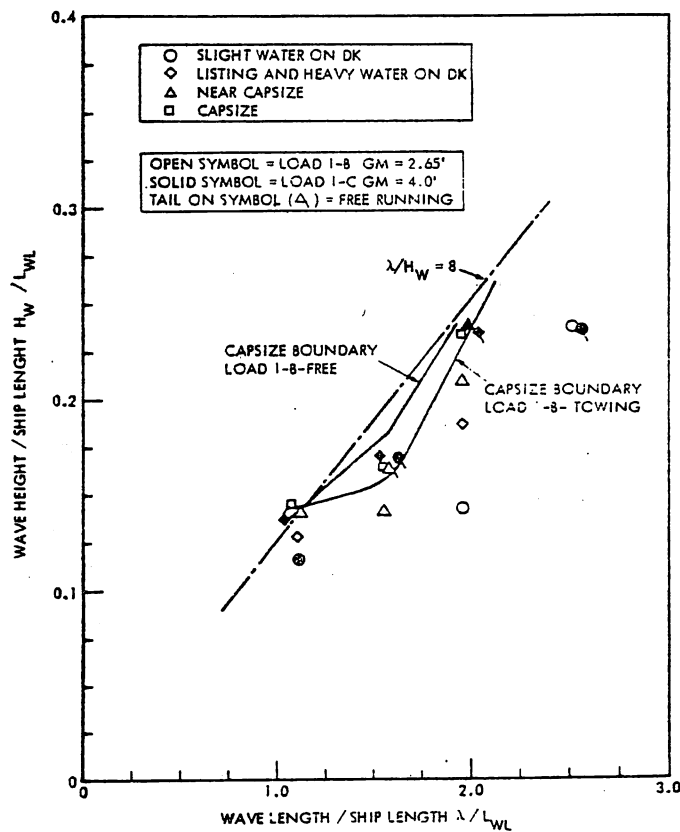


FIGURE 11 - CAPSIZE BOUNDARY FOR F-34 TOWING IN FOLLOWING WAVES

FIGURE 12 - CAPSIZE BOUNDARY FOR T-14 AT $V_K \lambda / L = 0.25$ IN HEAD SEAS

SAFETY OF A VESSEL IN BEAM SEA

by

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1. INTRODUCTION

To estimate ship safety in waves, as many conditions as are likely to be encountered in service must be taken into consideration. In heavy seas all ships, in particular, the small and medium sized should try and avoid running in quartering, following and beam waves which may endanger their safety. But it can happen, due to damage of the rudder or propulsion system that the master loses control of the ship and she begins to drift in beam waves. Such a situation causes heavy rolling motions, at least, around a certain mean value of the roll angle due to the wind forces acting on the ship. For ships with a low freeboard this phenomenon is generally combined with the shipping of water on the deck which can be particularly dangerous. It is rather difficult to describe and investigate such a case by means of mathematical methods. Therefore in the Ship Research Institute of the Gdansk Technical University a programme of experiments for studying this phenomenon was planned and model tests were carried out both in regular and irregular waves. The tests in regular waves were conducted in the model tank, and tests with irregular waves in a lake under the action of natural wind waves. In the present paper the model test results of the behaviour of low freeboard fishing vessels in regular waves are presented.

2. MODELS' PARTICULARS AND TEST PROGRAMME

Two models were tested, one of a side trawler (model A) and the second of a stern cutter (model B). The models' particulars are given in Table 1. Figure (1) shows the body and profile plans of the models. Those parts of the models above the water were made according to the full scale form of the hull with the forecastle and the poop, deckhouses and bulwarks with freeing ports.

The tests were made for a constant displacement. On both models the ordinate of the centre of gravity \overline{KG} was changed to obtain the near critical or lower values of the parameters characterizing the righting arm curves.

Besides this the tests with model A were carried out in the following conditions:

- (a) For two freeboard heights;
- (b) With and without bulwark (the freeing ports were open or closed);
- (c) For two lengths of the deckhouse.

For the tests in regular waves the wave parameters were varied within the following ranges:

wave length $\lambda = 0.75$ to 4.50 m
wave height $\lambda_w = 4$ to 12 (16) cm

The following quantities were measured:

- (i) Wave parameters,
- (ii) Rolling angles,
- iii) Heaving and swaying accelerations,
- (iv) Relative water motions on the windward and leeward sides amidships.

The models' parameters varied during the tests are given in Table 2. Figures (2) and (3) show the righting arm curves of the models tested.

3. MODEL BEHAVIOUR IN REGULAR WAVES

The experiment was carried out in such a way that an approximate constant wave height was maintained as the wave length was varied. Behaviour of the model at a given construction condition (freeboard height, bulwark, length of the deckhouse) and given loading conditions (ordinate of the centre of gravity) depended on the length and height of the wave. It will be illustrated in details by means of tests with model A provided with the low freeboard ($F = 22.5$ mm), bulwark with freeing ports and long deckhouse.

In Figures (4) and (5) are shown the tests results of the amplitudes of the relative motions of water at amidships of the windward side for two positions of the centre of gravity: $KG = 147.0$ mm and $KG = 157.0$ mm. The amplitude curves show two distinct maxima; the first one near the rolling natural frequency and the second one near the heaving natural frequency. Also at frequencies higher than the rolling natural frequency, an additional magnification appears. It is observed both on the windward and leeward sides and always at the same wave frequency although it is independent of the KG value. In the case of $KG = 147$ mm the magnification appears at frequencies a little higher than the natural frequency. It is not very prominent for low wave heights although it increases with increasing wave height.

At first it gives a distinct additional maximum ($S_w = 8$ cm) and further, begins to exceed the maximum caused by rolling motions ($S_w = 10$ and 12 cm). When $KG = 157$ mm the additional magnification appears at frequencies much higher than the natural rolling frequencies and does not reach very significant values.

When the wave frequency is approaching the heaving natural frequency, the relative motion amplitude quickly increases. Due to this the deck is seriously flooded over the edge of the bulwark. The amount of water on the deck and the frequency of its shipping are so high that it cannot flow down off the deck and remains first of all between the bulwark and the deckhouse on the windward side. It creates a heeling moment on the windward side which causes motions of the model near the angle called the "pseudostatic angle of heel". The pseudostatic angle of heel exists for sufficiently high wave frequencies. The frequency changes with wave height and the ordinate of the centre of gravity as other construction parameters remain constant. Above the limit frequency as the frequency increases the pseudostatic angle of heel rapidly increases approaching a limit which depends first of all on the model stability and to a lesser extent on the wave height.

In particular experiments the fixed pseudostatic heeling angle appears relatively soon: after two waves in the case of $KG = 157$ mm, and four to five waves in the case of $KG = 147$ mm. The maximum values of the pseudostatic angle of heel, even when there is sufficient stability (when $KG = 147$ mm), are so high that they constitute a serious danger to the ship's safety. Rolling motions of the model for this range of frequency, at which the pseudostatic angle of heel appears, are usually very low. Only in the case of $KG = 157$ mm, at the lower range of the frequencies at which the pseudostatic angle of heel begins to appear, were intense rolling motions corresponding to subharmonic resonance observed.

Further increases of the ordinate of the centre of gravity ($KG = 168$ mm) caused the model to capsize even in waves which at lower values of KG only induced a pseudostatic angle of heel. Capsizing of the model which had poor stability was always connected with the presence of the water on the deck and capsizing occurred quickly after the encounter of two waves on the windward side.

On the basis of the experiments carried out it was possible to establish for the model considered the dangerous range of beam waves. It is shown in Figure (6).

Effects of Bulwark

To explain the role of the bulwark in establishing the pseudostatic angle of heel additional tests, were conducted with the model having the same freeboard and the deckhouse but without bulwark. Figure (7) shows the relation between amplitudes of relative motions at the windward side and the wave frequency for two different wave heights at $KG = 147$ mm. In comparison with the curves for the model with the bulwark there is no maximum in the vicinity of the rolling natural frequency. This is connected with the increased damping caused by submergence of the deck edge. With $KG = 147$ mm the pseudostatic angles of heel were not observed. It can be concluded that they are to a large extent caused by the bulwark. The tests made on the model without the bulwark at $KG = 168.5$ mm show that waves, which in the case of the model with the bulwark, caused the model to capsize now induced only a pseudostatic angle of heel dependent on the wave height (approximately linear). In this case the bulwark was a cause in the capsizing of the model.

Description of the above phenomena by means of approximate theory is rather difficult. The magnitudes of pseudostatic angles of heel cannot be directly related to the static heeling angle, which appears when the freeing ports are closed and the deck well is filled with water. In Table 3 the maximum values of pseudostatic angles of heel and the maximum values of static heeling angles caused by water on the deck are given. The pseudostatic angles of heel cannot be explained by rolling in the nonlinear part of the righting arm curves because rolling motions at the maximum angles of heel are rather small.

Effects of Freeboard

Similar experiments at different KG values with and without the bulwark were carried out on the model with the increased freeboard ($F = 42.5$ mm). The purpose of the experiments was to explain the influence of freeboard on model behaviour in beam waves. As it was expected the range of the dangerous waves, causing large pseudostatic angles of heel and capsizing the model, decreased to a large extent (see Figure (6), in such a way that the

dangerous wave length also decreased. In the case of very poor stability ($KG = 168$ mm) and without bulwark the pseudostatic angles of heel, which with the model of low freeboard ($F = 22.5$ mm) reached maximum values of the order of 30 deg., with the higher freeboard ($F = 42.5$ mm) were rather small and did not exceed 6 deg. It is shown in Figure (8). In developing further this phenomenon by changing the location of the centre of gravity the pseudostatic angles of heel or capsizing of model always appeared on the windward side.

Effects of Deckhouse Length

Because, in the experiments carried out, the influence of water between the deckhouse and the bulwark on the pseudostatic angles of heel and on the capsizing of the model was significant, additional experiments were carried out with the shorter deckhouse and low freeboard and bulwark. The experiment showed that in the condition of intense deck flooding the influence of the deckhouse length is significant. It was observed for the case of a shorter deckhouse and sufficient stability ($KG = 146.4$ mm) that in spite of intense flooding and large amount of water on the deck, there were no pseudostatic angles of heel on the windward side and only small angles on the leeward side. This situation was also observed at smaller stability ($KG = 158.0$ mm). The comparison of the pseudostatic angles of heel of the models with long and short deckhouses is shown in Figure (9). At very poor stability ($KG = 163$ mm and $KG = 168.5$ mm) the model capsized due to flooding, but the mechanism of capsizing was different to that of the model having a longer deckhouse. Waves of higher frequencies than the heaving natural frequency caused the model to capsize always on the leeward side and they were higher than waves capsizing the model with long deckhouse. But waves of frequency below the heaving natural frequency caused the model to capsize on the windward side in spite of the fact that at lower wave heights small pseudostatic angles of heel appeared on the leeward side, (see Figure (10).)

The tests with the model B were carried out in a narrower range. This model in comparison with the model A had different forms of transverse sections ("broken frames") and

a different form for the above water part of the hull (long forecastle). It was observed that this form of transverse sections effectively protected the deck against flooding, (see Figure (11)), even for very steep waves ($\lambda = 1.0$ m, $\zeta_w = 16$ cm). The model was not flooded over the bulwark edge but only through the bulwark openings. Such small amounts of water on the deck without a deckhouse did not have much effect on the model behaviour and even at poor stability ($KG = 183$ mm) the model did not capsize. It capsized only under additional external impulse.

4. CONCLUSIONS

The experiments carried out demonstrated the complexity of the phenomena associated with the behaviour of low-freeboard models in regular beam waves, with intense flooding over the bulwark edge.

The phenomena which appeared with intense flooding can be described with sufficient accuracy by means of theoretical methods taking into account the nonlinear damping moments of the rolling motions and the wave caused by relative motions of the ship (in particular heaving and swaying motions). Once the deck experiences intense flooding, the pseudostatic angles of heel appear or the model capsizes. These phenomena cannot be described by known mathematical models. The proper mathematical model should take into account such elements as the bulwark and eventually the deckhouse.

The experiments showed that the intense flooding of the deck in beam waves appears at frequencies higher than the natural rolling frequency of the ship. The flooding depends on, among other things, the form of the transverse sections. The bulwark as long as it protects the deck against flooding increases the safety of the ship. But when water is shipped on the deck with high frequency over the bulwark it traps water on the deck and endangers the safety of the ship. In the condition of intense flooding the bulwark and deckhouse together reduces further the ship's safety. Therefore the long deckhouse on the main deck with low freeboard should be avoided and the ships should be provided with superstructures.

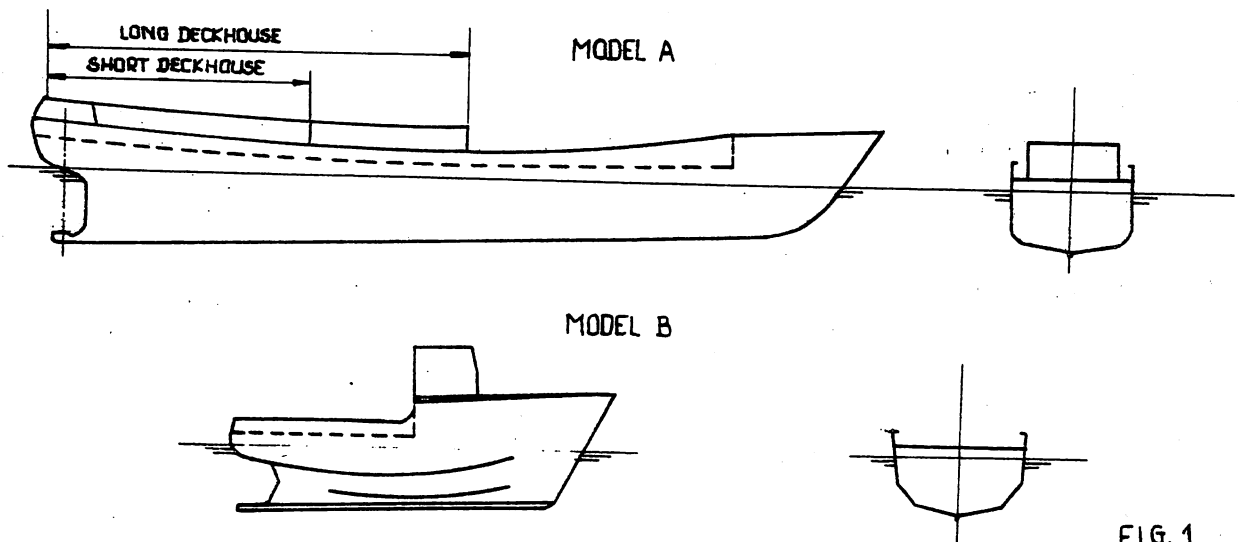


FIG. 1

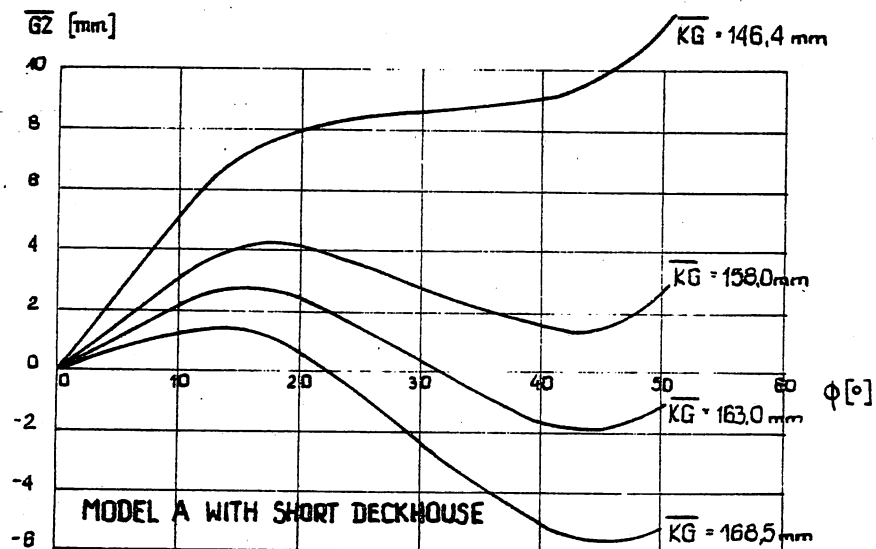
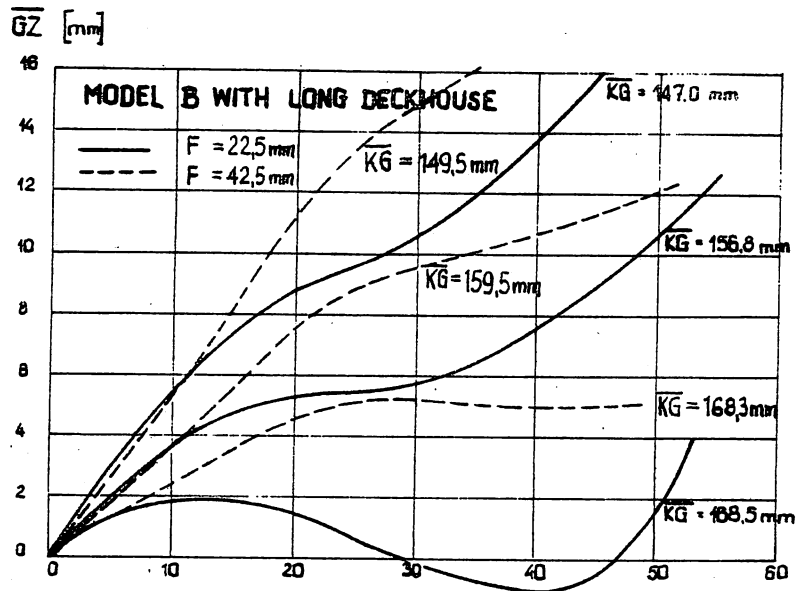


FIG. 2

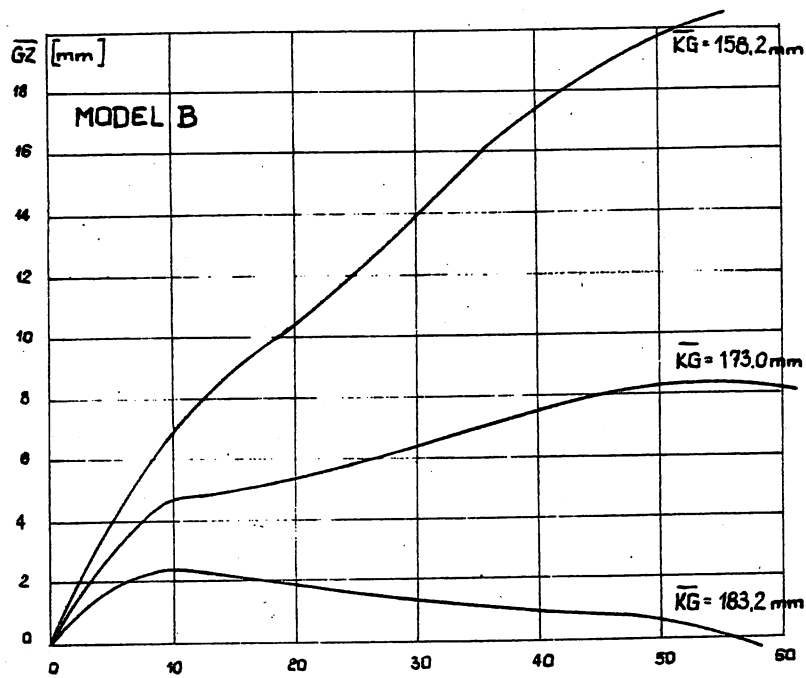


FIG. 3

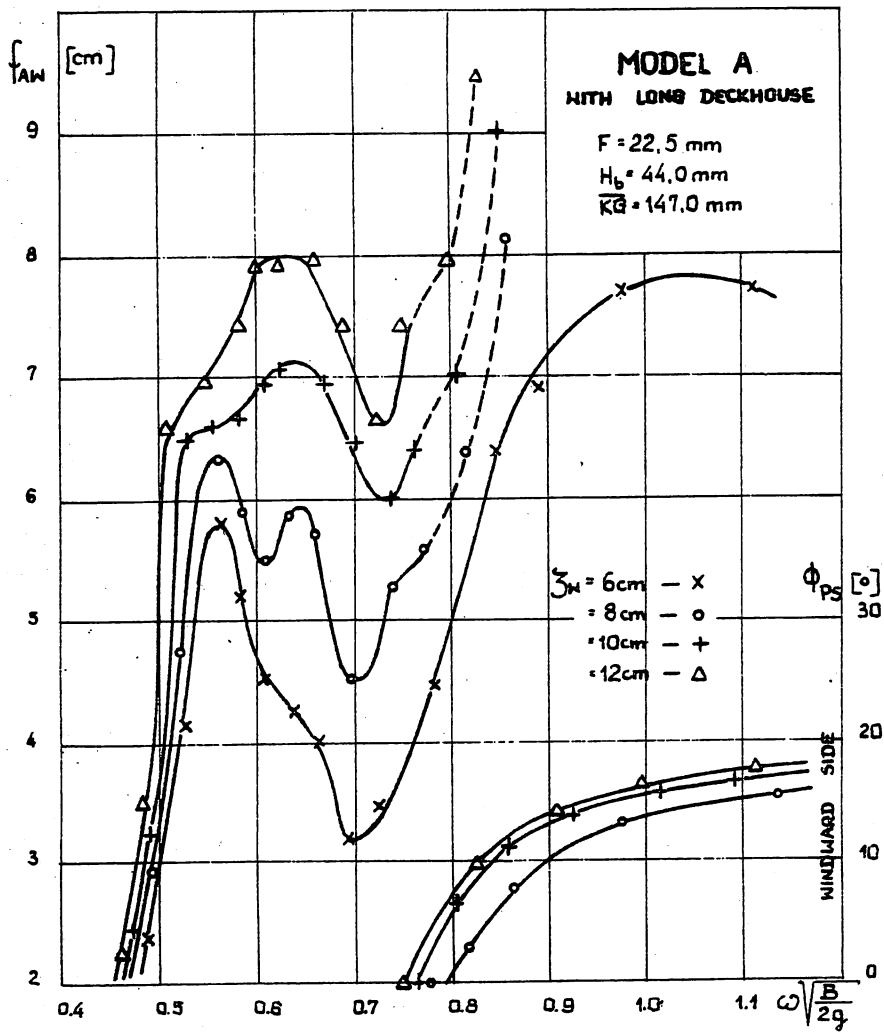


FIG. 4

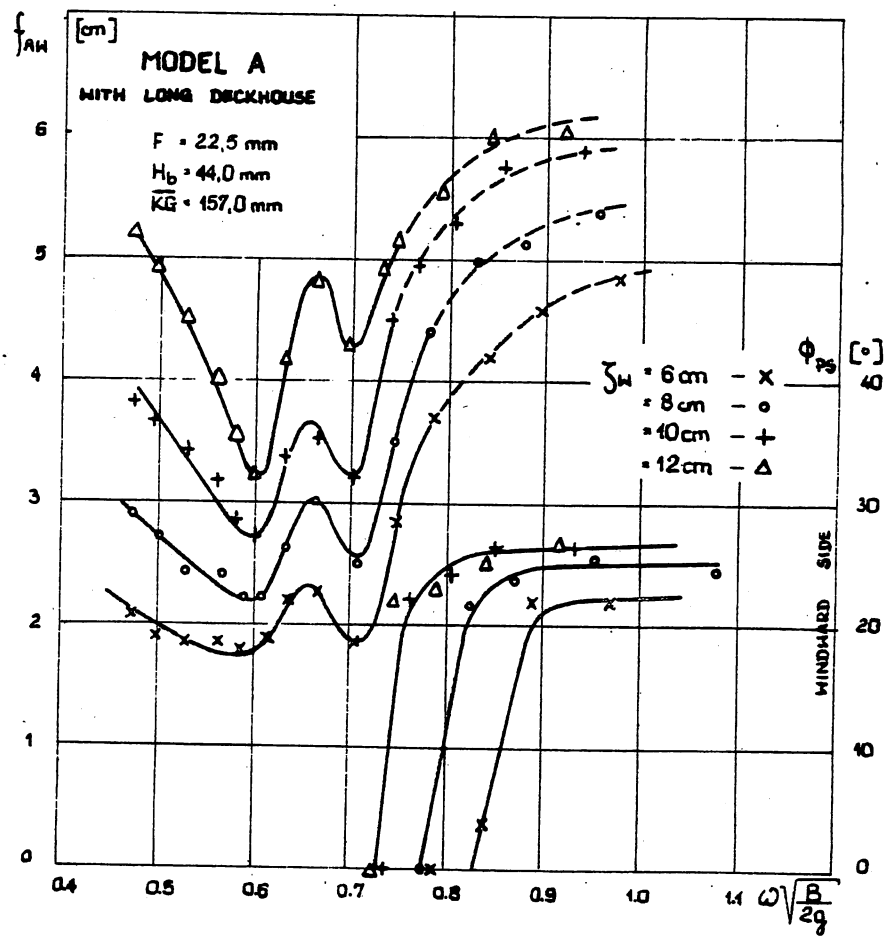


FIG. 5

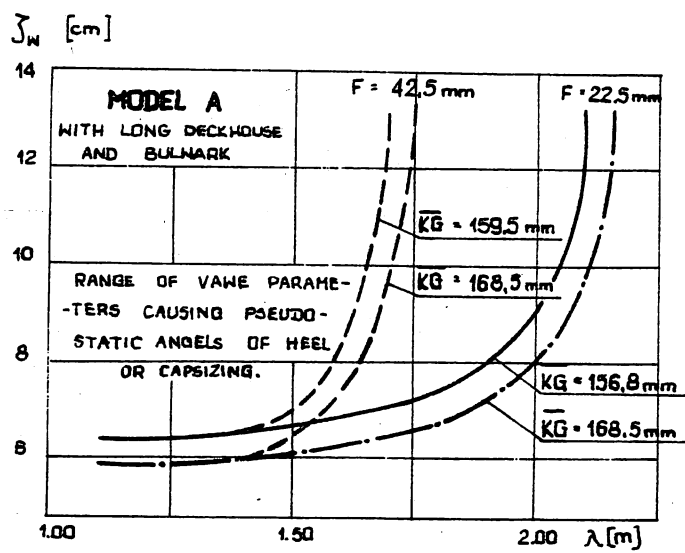


FIG. 6

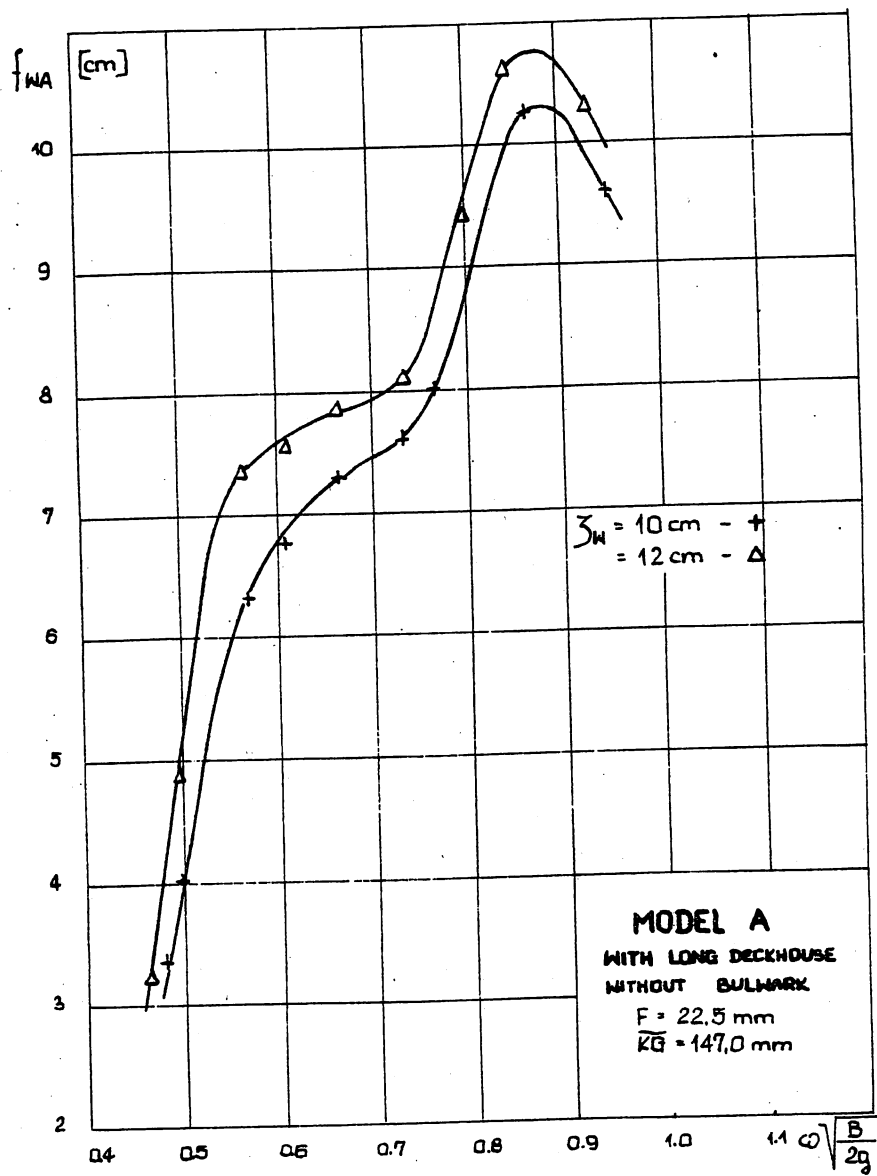


FIG. 7

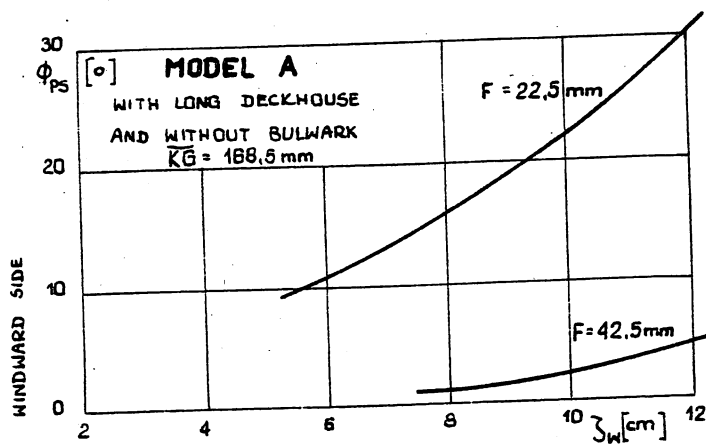


FIG. 8

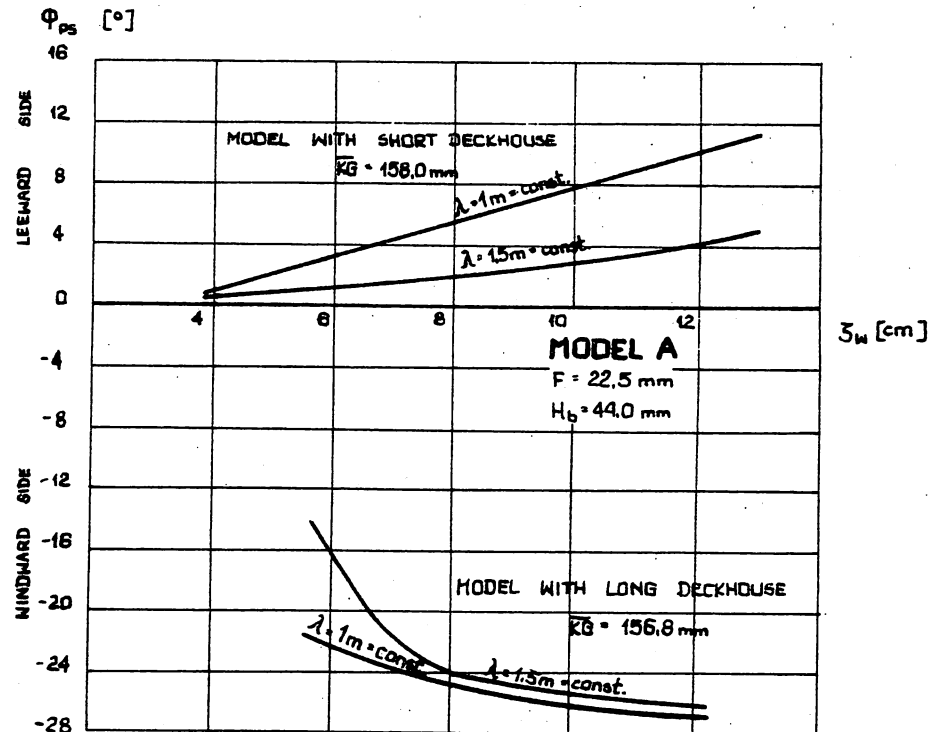


FIG. 9

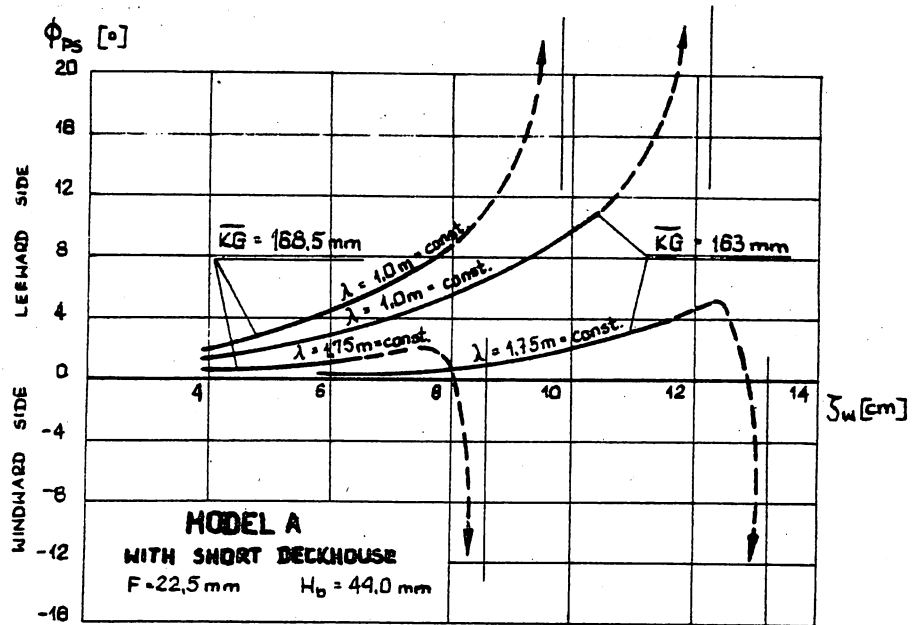


FIG. 10

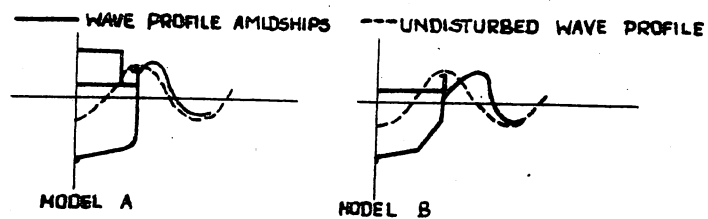


FIG. 11

TABLE 1

Model		A	B
Scale		1 : 25	1 : 14
L _{WL}	m	2.154	1.158
B	m	0.360	0.393
D	m	0.198/0.218	0.214
Δ	Kg	76.0	41.7
C _B	-	0.560	0.485
C _{WL}	-	0.800	0.777
KM	m	0.177	0.191
F	m	0.0225/0.0425	0.0410
H _D	m	0.044	0.065

TABLE 2

MODEL A					MODEL B	
F mm	deckhouse, long 1/L = 0.42		short 1/L=0.22		F mm	with bulwark
	with bulwark		without bulwark	with bul- wark freeing ports open		
	freeing open	ports closed				
22.5	KG=147.0mm =157.0mm =168.5mm	KG=147.0 mm	KG=147.0 mm =168.5 mm	KG=146.4mm =158.0mm =163.0mm =168.5mm	41.0	KG=0.158mm =0.173mm =0.183mm
42.5	KG=149.5mm =159.5mm =168.3mm	KG=149.5 mm	KG=168.5 mm			

TABLE 3

	Model with bulwark		Model without bulwark
\overline{KG} mm	147	157	147
Max pseudo-static angles of heel	15/17*	26	0
Max static heeling angles	7.5	12.5	0

* with open and closed freeing ports

SESSION 3
THEORETICAL STUDIES

<u>Paper No.</u>	<u>Author</u>	<u>Title</u>
3.1	Wellicome, J. (U.K.)	"An Analytical Study of the Mechanism of Capsizing"
3.2	Haddara, M.R. (U.A.R.)	"A Study of the Stability of the Mean and Variance of Rolling Motion in Regular Waves"
3.3	Abicht, W. (Germany)	"On Capsizing of Ships in Regular and Irregular Seas"
3.4	Kastner, S. (Germany)	"On the Statistical Precision of Determining the Probability of Capsizing in Random Seas"
3.5	Boroday, I.K., Nikolaev, E.P. (U.S.S.R.)	"Methods for Estimating the Ship's Stability in Irregular Seas"

AN ANALYTICAL STUDY OF THE MECHANISM

OF CAPSIZING

by

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1. INTRODUCTION

A ship can capsize either as a result of some accident, such as a shift of cargo or flooding following structural damage, or it can occur as a consequence of the ship's roll response to extreme sea conditions. Preventing the first kind of capsize is largely a matter of providing a sufficient margin of statical stability to cover all reasonably foreseeable damage situations and this aspect of stability is fairly well understood by Naval Architects. As yet the mechanisms which lead to a capsize due to wave action alone are not well understood and there is still a lack of either an adequate analytic model or of adequate experimental information upon which to base criteria for judging the survivability of a particular design of ship from this view point.

This paper describes some rather limited investigations of the problem of ship response to extreme wave conditions carried out at Southampton under the author's supervision. The investigations concerned comprised the solution of a plausible equation for the roll motion, using an analogue computer and a digital computer to provide time histories of roll, and the comparison of such solutions with an approximate analytic one. The broad conclusions from these comparisons were that the analytic solution gave a very good estimate of the steady state response to wave excitation in roll, but that it failed to describe the situations in which the computed solutions produced a capsize.

This paper discusses the formulation of the equation of motion used to describe a rolling motion in these studies. The approximate analysis of the steady state response to

regular wave excitation using perturbation methods is given together with a similar treatment of the variational equations defining the stability of this steady state response and the comparison of these analytical solutions with those derived from the computer studies.

2. FORMULATION OF THE EQUATION OF MOTION FOR ROLLING.

Of the six modes of motion for a ship, the rolling motion is the most difficult to model mathematically. Like the pitch and heave modes the major part of the forcing function and the hull stiffness is due to buoyancy forces acting on the hull, but unfortunately in the case of rolling the relation between the righting moment due to buoyancy is far from being a linear function of roll angle. Indeed any study of capsize must acknowledge that at a certain roll angle the static stability will vanish. The damping moments acting on the hull are also non-linear especially at low forward speeds since a large part of the damping comes from the formation of eddies behind bilge keels and hard bilge corners. Moments caused by these eddy formations will be proportional to the square of the roll velocity.

It would seem natural to express the moments acting on the hull as functions of roll angle relative to the wave surface ($\phi(t)$) and the relative roll velocity ($\dot{\phi}(t)$) etc. rather than as functions of absolute motion measured from the vertical ($\theta(t)$). These are related to the wave slope ($\alpha(t)$) by the equation.

$$\theta = \phi + \alpha$$

Thus the roll restoring moment may be written as $M(\phi)$ and the roll damping moment as $D(\dot{\phi})$. There will also be a virtual inertia term of the form $I\ddot{\phi}$. In choosing to represent the moments in this way it is assumed that buoyancy forces act perpendicular to the constant pressure free surface and that wave orbit velocities are small compared to the fluid velocities generated by the rolling hull. Both assumptions are reasonable for long waves.

The equation of motion now takes the form

$$mk^2\ddot{\theta} = -M(\phi) - D(\dot{\phi}) - I\ddot{\phi}$$

where mk^2 is the roll inertia of the hull.

Thus, since $\ddot{\theta} = \ddot{\phi} + \ddot{\alpha}$

$$(mk^2 + I)\ddot{\phi} + D(\dot{\phi}) + M(\phi) = -mk^2\ddot{\alpha} \quad (2.1)$$

This equation describes the relative roll motion ϕ . By choosing different functions D and M a wide range of roll characteristics can be reproduced. The simplest model which contains the essential non-linear features mentioned so far would take the form

$$\ddot{\phi} + k_1\dot{\phi} + k_2\phi|\dot{\phi}| + \omega_0^2\phi - \alpha\phi^3 = -\lambda\ddot{\alpha} \quad (2.2)$$

This form includes a linear damping term $k_1\dot{\phi}$ notionally associated with waves produced by the hull, a quadratic damping term $k_2\phi|\dot{\phi}|$ notionally associated with eddy making and a static restoring moment of cubic form which vanishes at

$$\phi = \pm \omega_0 / \sqrt{\alpha}$$

It is common in the literature on rolling to meet a similar equation to describe the absolute roll motion which takes the form:

$$\ddot{\theta} + k_1\dot{\theta} + k_2\dot{\theta}|\dot{\theta}| + \omega_0^2\theta - \alpha\theta^3 = \omega_0^2\alpha \quad (2.3)$$

Although these two equations are of similar form they are by no means equivalent, as a straight forward substitution will show. There is room for discussion as to which of these two equations best represents the physical process of rolling and although the author has a preference for the relative roll equation (2.2) it is possible to see aspects of the physical problem that may be more suitably expressed in terms of the absolute motion.

So far the discussion has been formulated in a way which implies that the parameters involved could be regarded as constant (at any rate at a fixed

encounter frequency) as would normally be the case in, for instance, pitch and heave motions. However, this would imply the neglecting of important coupling effects between roll and the other modes of motion, notably heave and sway.

The heave coupling effect is interesting in that it can lead directly to unstable rolling in head and following seas in which direct roll excitation by the waves is absent. The effect arises because of changes in the height of the transverse metacentre associated with changes of draught.

In the usual notation of hydrostatics:

$$KM = KB + I_T / \nabla$$

$$GM = GM_0 + \left[\frac{dKB}{d\nabla} + \frac{d}{d\nabla} \left(\frac{I_T}{\nabla} \right) \right] z \quad (2.4)$$

where z is the instantaneous heave relative to the wave surface. Since the linear stiffness term is directly proportional to the GM the effect of the heave coupling implied by equation (7.4) can be introduced by modifying (2.2) to read

$$\ddot{\phi} + k_1\dot{\phi} + k_2\phi|\dot{\phi}| + \omega_0^2(1 + \beta P(t))\phi - \alpha\phi^3 = -\lambda\ddot{\alpha} \quad (2.5)$$

In this form an estimate of the time dependent stiffness term $\omega_0^2\beta P(t)$ can be obtained by considering the heave response of the hull and the corresponding changes in both displaced volume ∇ and transverse second moment of waterplane area I_T .

A final physical effect to be taken into account is the difference between the pressure distribution within a wave and that which would be expected on a hydrostatic basis. Because the disturbance caused by the wave decays exponentially with depth the effective wave slope measured, for example, in terms of the pressure changes occurring at the mean draught of the hull is somewhat less than the wave surface slope at the free surface, especially in the case of waves of shorter lengths. This can be taken into account in an equation of the form of (2.5) if α is regarded as being the effective wave slope and ϕ as the roll motion relative to the effective wave slope. The relation between effective and true wave slopes will be detailed when considering the formulation of the wave spectrum used in predicting motions in an irregular sea.

3. GENERAL COMMENTS ON STEADY STATE RESPONSES OF NON-LINEAR SYSTEMS

With a non-linear system such as that

represented by equation (2.5) the response of the system to a periodic wave excitation can be expected to be some form of periodic roll motion. It is a characteristic of such a system however that it can have several possible steady state responses to the same basic wave excitation. For instance, taking the case $k_1, k_2 = 0$ and $\beta = 0$ in equation (2.5), the roll equation reduces to

$$\ddot{\phi} + \omega_0^2 \phi - 2\phi^3 = -\lambda \ddot{\alpha} \quad (3.1)$$

On substituting a response $\phi = A \cos \omega t$ we have

$$(\omega_0^2 - \omega^2) A \cos \omega t - 2A^3 \cos^3 \omega t = -\lambda \ddot{\alpha}$$

$$\text{but } \cos^3 \omega t = \frac{3}{4} \cos \omega t + \frac{1}{4} \cos 3\omega t$$

$$\text{so that } -\lambda \ddot{\alpha} = (\omega_0^2 - \omega^2 - \frac{3}{4} 2A^2) A \cos \omega t - \frac{1}{4} 2A^3 \cos 3\omega t$$

$$\text{whence } \alpha = \frac{1}{\lambda \omega^2} (\omega_0^2 - \omega^2 - \frac{3}{4} 2A^2) A \cos \omega t + \frac{2A^3}{36\omega^2 \lambda} \cos 3\omega t \quad (3.2)$$

This effective wave slope would produce the response $\phi = A \cos \omega t$. Now in the particular case where

$$\omega_0^2 - \omega^2 - \frac{3}{4} 2A^2 = 0 \text{ equation (3.2)}$$

reduces to $\alpha = \frac{2A^3}{36\omega^2 \lambda} \cos 3\omega t$ and in this case the roll response at frequency ω would be taking place in response to a regular wave at an encounter frequency of 3ω . This kind of response is called a subharmonic response and is a kind of response not met with in a linear system. In this case the hull is rolling with a frequency ω which is close to its own natural roll frequency in response to the relatively high wave encounter frequency of 3ω . Effective wave slopes at this frequency are in most cases small.

A second particular case arises when the roll amplitude A is small so that the term proportional to $\cos 3\omega t$ is small and the roll response at frequency ω is a result of a nearly regular wave excitation at that frequency. In this case we have

$$\alpha = \alpha_0 \cos 3\omega t = \frac{1}{\lambda \omega^2} (\omega_0^2 - \omega^2 - \frac{3}{4} 2A^2) A \cos \omega t$$

Thus the amplitude of the roll response for a given maximum wave slope α_0 is found from the cubic equation

$$\frac{3}{4} 2A^3 + (\omega_0^2 - \omega^2) A + \lambda \omega^2 \alpha_0 = 0 \quad (3.3)$$

It follows from equation (2.3) that there can be three possible amplitudes of roll motion at the given encounter frequency for any given wave slope. Appendix I develops an analysis of the roll response in regular waves from (2.5) for the case where the response is at encounter frequency and Figure (1) shows the form of the response curves obtained from this solution together with a subharmonic response curve. The existence of three solutions at low frequencies can clearly be seen. Again this situation is not met in a linear system in which there is only one response to a given excitation.

4. THE STABILITY OF STEADY STATE MOTIONS

It is necessary to consider the stability of each steady state motion in a truly dynamic sense. This can be done by considering what happens in circumstances where the motion is slightly disturbed from the steady state either because the roll amplitude or the phase difference between the roll motion and the wave is different from the steady state value. If in the subsequent motion the differences between the actual motion and the steady state become smaller then the steady state is clearly dynamically stable, whereas if in the subsequent motion the differences increase and the actual solution departs further and further from the steady state then this steady state is unstable. This view of dynamical stability is one which is probably unfamiliar to most Naval Architects.

Let us suppose that the actual response in roll is

$$\phi = f(t) + \psi(t)$$

where $f(t)$ is a steady state response and $\psi(t)$ is a small disturbance to the steady state. We need only consider what happens when ψ is sufficiently small for terms in ψ^2, ψ^3, ψ^4 etc. to be neglected. Then we have to the lowest order of approximation

$$\dot{\phi} \dot{\phi} = \dot{f} \dot{f} + 2 \dot{f} \dot{\psi}$$

$$\text{and } \phi^3 = f^3 + 3f^2\psi$$

Substituting into equation (2.5) we have

$$\ddot{f} + \ddot{\psi} + k_1(\dot{f} + \dot{\psi}) + k_2(\dot{f}|\dot{f}| + 2\dot{f}|\dot{\psi}|) + \omega_0^2(1 + \beta P(t))(f + \psi) - 3a(f^3 + 3f^2\psi) = -\lambda\ddot{\alpha} \quad (4.1)$$

but since $f(t)$ is actually a solution to equation (2.5) we find that equation (4.1) reduces to the equation

$$\ddot{\psi} + (k_1 + 2k_2|f(t)|)\dot{\psi} + [\omega_0^2(1 + \beta P(t)) - 3af(t)]\psi = 0 \quad (4.2)$$

Equation (3.2) is called the variational equation for the stability of the solution $\phi = f(t)$. If on solving this variational equation it is found that the disturbance $\psi(t)$ grows with time then $f(t)$ is an unstable solution. If $\psi(t)$ approaches zero as time passes or simply approaches a steady state solution, so that a small disturbance remains small, then $f(t)$ is a stable solution.

In the particular case where $k_1 = k_2 = \beta = 0$

$$\text{and } f(t) = A \cos \omega t$$

equation (4.2) reduces to

$$\ddot{\psi} + (\omega_0^2 - 3aA^2 \cos^2 \omega t)\psi = 0$$

$$\text{or, since } \cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

$$\ddot{\psi} + (n^2 - \gamma \cos 2\omega t)\psi = 0 \quad (4.3)$$

$$\text{where } n^2 = \omega_0^2 - \frac{3}{2}aA^2$$

$$\text{and } \gamma = -\frac{3}{2}aA^2$$

Equation (3.3) is known as the Mathieu equation. This equation has a large number of unstable solutions centred about encounter frequencies

$$\omega = n, \frac{1}{2}n, \frac{1}{4}n, \dots$$

... etc. These instabilities are discussed in most textbooks on non-linear oscillations and Figure (2), reproduced from one such source, shows the regions in which particular combinations

of ω and γ result in an unstable motion.

Each region of instability of the Mathieu equation exhibits the same general characteristic. Within an unstable region the response $\psi(t)$ contains terms with frequencies related to the encounter frequency. For example, in the first unstable region ψ contains terms with frequencies $\omega, 2\omega, 5\omega$ etc. whilst in the second region ψ contains terms with frequencies $\omega, 3\omega, 5\omega$ etc.

The amplitudes of these various harmonic terms grow exponentially with time except on the boundaries of the unstable regions where the solution containing these frequencies becomes a steady state motion of fixed amplitude. Outside the unstable regions ψ contains frequencies which are not harmonically related and the motion is steady state but not periodic. Evidently if $\psi(t)$ is initially small it cannot grow except within the unstable regions.

Appendix II discusses the derivation of boundary curves along which the variational equation (4.2) exhibits steady state solutions consisting of harmonics of the exciting frequency ω . These boundary curves divide the roll response diagram into regions of stable and unstable steady state response as shown in Figure (3). In this diagram boundary curves are shown for the first two regions of instability. It seems likely that the boundary curves for higher order of instability lie within the unstable region defined by these first two boundary curves. The analysis of Appendix II is restricted to the case where the roll response corresponds to that met in regular or nearly regular waves.

5. COMMENTS ON THE OCCURRENCE OF CAPSIZING IN REGULAR WAVES

It is possible to speculate on the existence of two distinct situations which could lead to a capsize in regular waves. Normally, if for any particular effective wave slope and frequency of encounter there exists at least one stable steady state motion it is reasonable to suppose that the roll motion would settle down to that motion and no capsize would occur. On the other hand if none of the

possible steady state motions is stable then a capsize is the only possible roll response. This is the first possible mechanism for capsizing. The second possible situation in which a capsize can occur can best be visualized by referring to Figure (4). Suppose the hull is rolling at point P close to the point of vertical tangency of the response curve. Next suppose that the effective wave slope increases slightly so that the response curve moves to the left of P. It is now necessary for the roll amplitude to increase to correspond to point Q. Point Q itself will represent a stable steady state, but it is possible that in the transitional period roll angles will be achieved in excess of the steady state roll at Q so that a capsize could occur in this transitional period if the overshoot is large enough. It should be noted that this second mechanism has nothing directly connecting it to the stability of the steady state motions at P and Q in the sense described in the last section.

To sum up, capsizing can occur

- a) If a combination of wave slope and encounter frequency is met at which all the possible steady state motions are unstable, or
- b) In a transitional situation between a low amplitude response mode and a high amplitude response mode both of which may be stable steady state modes.

In the next section it will be seen that in the experiments with the analogue computer it was possible to simulate both types of capsize.

6. ANALOGUE COMPUTER SIMULATION OF ROLLING

The paper so far has been concerned with a theoretical description of roll behaviour and of possible capsize mechanisms. This section describes some analogue computer studies of the rolling behaviour of a ship, particularly in large wave slopes, on the assumption that equation (2.5) provides a reasonable mathematical model of the process. In the analogue studies the effect of heave coupling was ignored (i.e. it was assumed that $\beta = 0$ in equation (2.5)). The stiffness

and damping parameters were chosen to be reasonably representative of the results of ship model experiments on roll behaviour and one set of parameters was adhered to throughout as follows:-

$$\begin{aligned} k_1 &= 1/30 & \text{sec}^{-1} \\ k_2 &= 1/10 & \text{rad sec}^{-1} \\ \omega_0 &= 0.500 & \text{rad sec}^{-1} \\ a &= 0.25 & \text{rad sec}^{-2} \end{aligned}$$

These parameters correspond to a ship with a natural roll period of 12.6 secs and a range of stability of 1 radian (57.3°). The levels of damping result in a roll amplification factor (roll amplitude/wave slope) of 7 in waves of 5° wave slope.

The purposes of the exercise were three-fold:-

- i) To compare the lowest order approximate analytic solution (as described in Appendix I) with observed roll responses obtained from the analogue simulation.
- ii) To investigate the conditions under which capsize occurred in regular waves.
- iii) To compare results in regular waves with results obtained in a simulated irregular sea representative of severe storm conditions.

6.b) REGULAR WAVE ANALOGUE RESULTS

The predicted and measured roll responses in regular waves are compared in Figures (5) and (6). The curves drawn are those derived from equation (A8) of Appendix I, whilst the data spots are analogue computer measurements. The agreement between the two is remarkable good even in wave slopes of nearly 30° - a slope beyond anything likely in the real world at sea. Looking at Figure (6) it can be seen that a number of points have been obtained along the upper response curve at frequencies where the upper and lower response curves overlap. These points were obtained by a combination of techniques such as starting the simulation with a high initial roll angle, starting at a frequency outside

the overlap region or starting with a high wave slope and gradually reducing the wave slope to the required value. Whichever technique was employed in a particular instance, there tended to be large transient motions whilst bringing the model to the desired combination of wave slope and encounter frequency values. These transient motions were controlled by using a comparator switch to raise the level of damping by a factor of 100 whenever the absolute roll angle exceeded a preset value.

Despite the use of the comparator, it became increasingly more difficult to establish a steady state response at large values of wave slope and eventually it proved impossible to do so at frequencies which lay outside the regions of instability derived in the appendix II. Figure (6) has superimposed on it the analytical stability boundaries derived from equations (A17) and (A19) of appendix II and also a tentative boundary along which the analogue simulation appeared to become unstable. The simulator boundary and the analytic boundaries are markedly different in character. The nature of the capsize which occurred from a point along the upper boundary is illustrated in Figure (7) which shows a regular roll motion about a steadily increasing mean roll angle up until point x at which the comparator switched in to prevent a run away increase of roll angle and a saturation of the operational amplifiers in the computer.

An examination of the disturbance ψ as derived from the variational equation (4.2) shows that entering one of the analytic regions of instability would result in a build up of roll motion about a zero mean. Thus, not only does the analysis not indicate the correct region of capsize but it also fails to predict the way in which the capsize takes place. There would appear to be some omission in the analysis of a fairly fundamental nature.

Figure (8) is an extract from Figure (6) illustrating the gap in which the analogue computer response shows no steady state response. This gap first appears in wave slopes of 0.28 radians.

Trials were conducted to examine the transient motion which took place following a shift of excitation frequency to cause a jump from the lower response curve to the

to the upper response curve. A typical transient response is shown in Figure (9). It can be seen that an overshoot occurs with the roll motion temporarily exceeding the final steady response. With wave slopes above 0.15 radians (8.5°) this overshoot was sufficient to cause capsize.

6.b) ANALOGUE SIMULATION OF IRREGULAR WAVE RESPONSES

Having demonstrated the capsize mechanisms for regular waves the analogue simulation was rebuilt to include a seven component representation of the Pierson-Moskovitz wave spectrum for a significant wave height of 16.2m. To develop such a sea would require a 50 kt wind to blow for 48 hours over a fetch of 1000km. This sea represents quite severe conditions which observer data suggests might occur once per year in the North Atlantic area. The particular wave periods chosen for the irregular wave implied a repetition of the wave record approximately every 20 rolls. An overall gain control was used to multiply the wave slopes by a factor referred to in Figure (10) as the sea attenuation factor. Circuits were provided to generate values of rms roll by integration over a preset period (1000 secs.)

The wave slope used throughout was the effective wave slope obtained by multiplying each frequency component by the factor

$$e^{-\frac{\omega^2 T}{2g}} \quad \text{where } \omega = \text{frequency of component wave}$$

and T = ship draught (taken as 6.0m).

This same relationship was also built into the regular wave simulation. The response curves of Figure (6) are for constant effective wave slope and the rms wave slopes of Figure (10) refer to effective wave slope also. An analytical relation between rms roll angle ($\sigma_{\phi N}$) and rms wave slope (σ_{ϕ}) can be obtained by the method given by Vassilopoulos (Reference (1)). This method assumes that the roll response is a narrow band-pass process and this is true of the absolute motion only. However, if the absolute motion is narrow band in character the rms absolute roll angle $\sigma_{\phi N}$ is related to σ_{ϕ} by and equation of the form

$$\sigma_{\phi N}^2 = \sigma_{\theta N}^2 + \sigma_{\alpha}^2$$

A predicted curve of rms roll angle derived on this basis and plotted in Figure (10) produces moderately good agreement with the measured rms response up to the point at which the analogue simulated a capsize. In irregular waves the analogue produced a capsize when the rms wave slope reached 0.14 rads corresponding to a significant wave slope of 0.28 rads. It is interesting that this significant wave slope agrees exactly with the effective wave slope at which instability first occurs in the regular wave case.

7. THE EFFECT OF HEAVE COUPLING ON CAPSIZE

Equation (2.5) gives the roll equation in the form

$$\ddot{\phi} + k_1 \dot{\phi} + k_2 \phi + \omega_0^2 (1 + \beta P(t)) \phi - a \phi^3 = -\lambda \ddot{z}$$

where the term $\omega_0^2 \beta P(t)$ represents the heave coupling effect. Returning to basic principles the complete linear stiffness term can be written as

$$\omega_0^2 (1 + \beta P(t)) \phi = \frac{gGM}{\lambda} + \frac{g}{\lambda} \frac{dGM}{dT} z(t)$$

where $z(t)$ is the heave motion relative to the wave which arises as a response to the wave $\alpha(t)$. By computing the heave response in regular waves in the normal way we can determine $P(t)$ for the regular wave case as

$$\omega_0^2 \beta P(t) = \bar{\alpha} \frac{g}{\lambda} \frac{dGM}{dT} z_0 \cos(\omega t + \delta + \theta)$$

corresponding to a wave slope of

$$\alpha = \bar{\alpha} \cos(\omega t + \theta)$$

z_0 is the heave response per unit wave slope and θ is the phase of z relative to α . Using the principle of linear superposition we can extend these equations to an irregular

sea by writing

$$\omega_0^2 \beta P(t) = \sum_n \bar{\alpha}_n \beta(\omega_n) \cos(\omega_n t + \theta_n + \theta'_n)$$

corresponding to a wave slope of

$$\alpha(t) = \sum_n \bar{\alpha}_n \cos(\omega_n t + \theta'_n)$$

In the above equations

$$\beta(\omega_n) = \frac{g}{GM \lambda} \frac{dGM}{dT} z_0(\omega_n)$$

The effect of heave coupling in this form was investigated on a digital computer using a step-by-step method to integrate the equation of roll response. The numerical method used was the Gill version of the Runge-Kutta technique.

There were two reasons for transferring the calculations to a digital computer at this stage. In the first place the analogue computer which was available did not have enough operational amplifiers and in the second place it gave a chance to check the accuracy of the analogue simulation. Broadly speaking the two types of solution were in agreement in predicting a capsize outside the limits of the unstable regions defined in the perturbation analysis of appendices I and II. There were, however, differences in the frequencies at which capsize occurred in a given wave slope and the digital model capsized more abruptly with less tendency for a drift of mean roll angle prior to the actual capsize.

Returning to the heave coupling effect, Figure (11) shows the relation between rms roll angle and rms wave slope for a range of values. Clearly in an actual case the rate of change of GM with draught ($\frac{dGM}{dT}$) may be either positive

or negative. On Figure (1.1) curves are drawn corresponding to a value of β calculated for a typical cargo vessel in beam seas and to values of β of ± 10 times this calculated value. There is also a curve corresponding to $\beta = 0$ which should compare directly with Figure (10) derived from the analogue simulator, since the same set of wave components were used in each case. In fact the detailed shape of the curve is distinctly different near the point of capsize and the digital model capsized in a slightly lower rms wave slope.

It is interesting to note that heave coupling does influence the point of capsize, but not by a very large amount in view of the large range of B values chosen in Figure (11).

8. COMMENTS AND CONCLUSIONS

The material covered in this paper is based on work done by a number of students as final year project investigations. It is inevitable in these circumstances that the work is incomplete in a number of ways. In particular there is a need to compare carefully the analogue and digital computer simulations with a view to resolving the differences between them. There is also a need for a much more careful investigation of the stability boundaries in regular waves. With this reservation in mind the following points emerge from these investigations.

- a) The perturbation analysis technique has been shown to give an extremely accurate prediction of the non-linear roll response in regular waves.
- b) The related equivalent linearization technique for predicting roll variance in irregular waves is also good, although it is not as accurate as the prediction of regular sea response.
- c) In regular waves the analogue exhibits a gap between the stable parts of the roll response curves above a certain critical wave slope in which no stable response occurs. The value of this critical wave slope compares well with the significant wave slope at which capsize occurs in irregular waves.
- d) Where slight changes are occurring in effective wave slope or encounter frequency in nearly regular waves transition from a low amplitude response to a high amplitude response can occur and this provides an alternative capsize mechanism in nearly regular waves which has nothing to do with the analytical instability of the two steady state modes. It is not known whether a mechanism similar to this could arise in fully random waves.

e) The investigations of the stability of the steady state roll motion via the variational method of Appendix II does not account for the instability found in the analogue simulation. On an intuitive basis the analogue result seems more reasonable than the analytical, which predicts stable rolling with relative motions in excess of the angle of vanishing static stability. If the analogue result is indeed correct then there is something fundamental missing in the application of conventional variational analyses to the prediction of capsize as outlined in this paper.

f) It would be nice to be able to demonstrate for a particular ship that for the anticipated characteristics of Gz curve and roll damping factors the rms wave slope that causes a capsize is so large that it would be met sufficiently rarely to be considered acceptable from a safety angle. To be met perhaps once in 100 ship years for instance. There could be argument about choice of wave spectrum and level of probability that would be acceptable, of course, but in principle this would provide a single rational criterion to judge whether the intact ship has adequate stability. However, at this stage, it is the author's opinion that the conditions leading to a capsize are not well enough understood to formulate regulations in these terms.

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APPENDIX I : STEADY STATE RESPONSE
- PERTURBATION ANALYSIS

The following analysis of roll response will be restricted to the case where heave coupling is absent (i.e. $\beta = 0$) and for the case where the wave excitation is of the form $\alpha \cos(\omega t + \delta)$ so that the equation for the relative roll motion is

$$\ddot{\phi} + k_1 \dot{\phi} + k_2 \phi = \omega^2 \lambda \alpha \cos(\omega t + \delta) \quad (A1)$$

The essence of the perturbation method is to assume that the parameters in (A1) are of the order of magnitude of a small quantity ϵ . Thus we write $k_1 = \epsilon k_1$, $k_2 = \epsilon k_2$, $\alpha = \epsilon \bar{\alpha}$. We next assume that the solution can be expanded in a series of successively smaller terms according to the scheme:

$$\phi = \phi_0 + \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots$$

and that

$$\omega = \omega_0 + \epsilon \omega_1 + \epsilon^2 \omega_2 + \epsilon^3 \omega_3 + \dots \quad (A2)$$

We can now expand the non-linear terms in equation (A1) as power series in ϵ , the first few terms of which are

$$\begin{aligned} \phi^2 &= \phi_0^2 + 2\epsilon \phi_0 \phi_1 + \epsilon^2 (\phi_0^2 \phi_1^2 + 2\phi_0 \phi_1^2) + \dots \\ \phi^3 &= \phi_0^3 + 3\epsilon \phi_0^2 \phi_1 + 3\epsilon^2 (\phi_0^2 \phi_1^2 + \phi_0 \phi_1^3) + \dots \end{aligned}$$

where $s_g(\phi_0) = \pm 1$ according as $\phi_0 \geq 0$

If we now expect the expansion scheme to hold for all small values of ϵ we can form a sequence of equations between the coefficients of ϵ^n on the LHS equation (A1) and the coefficients of ϵ^n on the RHS as follows:

$$\text{terms in } \epsilon^0: \ddot{\phi}_0 + \omega^2 \phi_0 = 0$$

$$\text{terms in } \epsilon^1: \ddot{\phi}_1 + \omega^2 \phi_1 = \omega^2 \lambda \bar{\alpha} \cos(\omega t + \delta) - k_1 \dot{\phi}_0 - k_2 \phi_0 |\dot{\phi}_0| + \omega_1 \dot{\phi}_0 + \bar{\alpha} \phi_0^3$$

$$\begin{aligned} \text{terms in } \epsilon^2: \ddot{\phi}_2 + \omega^2 \phi_2 &= -k_1 \dot{\phi}_1 - 2k_2 \phi_0 \dot{\phi}_1 \phi_1 \\ &+ \omega_1 \dot{\phi}_1 + \omega_2 \phi_1 + 3\bar{\alpha} \phi_0^2 \phi_1 \end{aligned}$$

terms in ϵ^3 :

$$\ddot{\phi}_3 + \omega^2 \phi_3 = -k_1 \dot{\phi}_2 + \dots \quad (A3)$$

In principle each one of these equations can be solved in turn to provide progressively more accurate approximations to the solution of equation (A1).

Looking first at the terms in ϵ^1 , ϕ_1 will contain a particular solution corresponding to each term on the RHS of this equation. We need a solution to the equation

$$\ddot{x} + \omega^2 x = \omega^2 \lambda \bar{\alpha} \cos(\omega t + \delta)$$

The solution concerned is

$$x = -\omega^2 \lambda \bar{\alpha} \cdot (t/2\omega) \sin(\omega t + \delta)$$

This is a term whose amplitude is increasing linearly with time t and thus cannot form part of a steady state solution. Such a term in the solution is called a 'secular' term and care must be taken to remove terms in $\cos \omega t$ and $\sin \omega t$ from the RHS of each equation in turn by proper choice of the quantities $\omega_1, \omega_2, \omega_3$ etc so as to avoid the appearance of secular terms in the solution. In fact the apparently superfluous introduction of these coefficients in equation (A2) was made expressly for this purpose.

The solution to our sequence of equations (A3) now proceeds as follows:

$$\phi_0 = A \cos \omega t \quad (A4)$$

giving

$$\phi_0^3 = A^3 \cos^3 \omega t = \frac{A^3}{4} [3 \cos \omega t + \cos 3\omega t]$$

$$\text{and } \dot{\phi}_0 |\dot{\phi}_0| = -A^2 \omega^2 \sin \omega t |\sin \omega t|$$

Which can be expanded as a Fourier series to give

$$\dot{\phi}_0 |\dot{\phi}_0| = -\frac{8A^2 \omega^2}{3\pi} \left(\sin \omega t - \frac{1}{5} \sin 3\omega t - \frac{1}{35} \sin 5\omega t + \dots \right)$$

We now find that the equation defining becomes

$$\begin{aligned} \ddot{\phi}_1 + \omega^2 \phi_1 &= \omega^2 \lambda \bar{\alpha} \cos(\omega t + \delta) + A k_1 \omega \sin \omega t \\ &+ \frac{8k_2 A^2 \omega^2}{3\pi} \left(\sin \omega t - \frac{1}{5} \sin 3\omega t - \frac{1}{35} \sin 5\omega t + \dots \right) \\ &+ \omega_1 A \cos \omega t + \frac{\bar{\alpha} A^3}{4} [3 \cos \omega t + \cos 3\omega t] \end{aligned}$$

(A5)

On expanding $\cos(\omega t + \delta) = \cos \omega t \cos \delta - \sin \omega t \sin \delta$ the coefficients of $\cos \omega t$ and $\sin \omega t$ in (A5) are respectively

$$\omega^2 \lambda \bar{\alpha} \cos \delta + \omega_1 A + \frac{3}{4} \bar{\alpha} A^3$$

and

$$-\omega^2 \lambda \bar{\alpha} \sin \delta + A k_1 \omega + \frac{8 k_2 A^2 \omega^2}{3 \pi}$$

Both these coefficients must be set to zero to avoid the appearance of secular terms in ϕ_1 and this provides equations for ω_1 and for the phase angle δ between wave slope and roll response.

$$\sin \delta = \frac{1}{\omega^2 \lambda \bar{\alpha}} \left[A k_1 \omega + \frac{8 k_2}{3 \pi} A^2 \omega^2 \right] \quad (A6)$$

$$\text{and } \omega_1 = -\frac{1}{A} \left[\omega^2 \lambda \bar{\alpha} \cos \delta + \frac{3}{4} \bar{\alpha} A^3 \right] \quad (A7)$$

The remaining terms in equation (A5) now yield the result

$$\ddot{\phi}_1 + \omega^2 \phi_1 = -\frac{8 k_2 A^2 \omega^2}{3 \pi} \left[\frac{1}{5} \sin 3 \omega t + \frac{1}{35} \sin 5 \omega t \dots \right] + \frac{\bar{\alpha} A^3}{4} \cos 3 \omega t$$

It can be seen that the solution for ϕ_1 will contain a sequence of terms which are odd order harmonics of ωt . The expression for ϕ_1 will contain a complementary solution $\mu_1 \cos \omega t + \mu_2 \sin \omega t$ and the coefficients μ_1, μ_2 and ω_1 will need to be chosen so that there are no secular terms in ϕ_2 . Although in principle we can proceed to find ϕ_1 , and then in turn ϕ_2, ϕ_3 etc, in fact the algebra rapidly becomes too complex to handle. As it happens, the solution we have obtained for ϕ_0 is of itself an extremely good approximation to the exact solution to equation (A1) as can be seen by comparison with the analogue computer solution (see fig (5) and fig (6) and there is little to be gained in proceeding further.

The solution thus far is

$$\phi = \phi_0 = A \cos(\omega t)$$

with $\sin \delta = \frac{1}{\omega^2 \lambda \alpha_0} \left[A k_1 \omega + \frac{8 k_2}{3 \pi} A^2 \omega^2 \right]$

and

$$\omega^2 - \omega_0^2 = \omega_1 = -\frac{1}{A} \left[\omega^2 \lambda \alpha_0 \cos \delta + \frac{3}{4} \bar{\alpha} A^3 \right]$$

On eliminating the phase angle δ we find the following relation between roll amplitude A , encounter frequency ω and wave slope α_0 :

$$\frac{\omega^2 \lambda \alpha_0}{A} = \left\{ (\omega^2 - \omega_0^2 + \frac{3}{4} \bar{\alpha} A^2)^2 + k_1 \omega + \frac{8}{3 \pi} k_2 A \omega^2 \right\}^{1/2} \quad (A8)$$

It can be seen that on reducing the problem to a linear case by setting $k_2 = a = 0$ we obtain the solution

$$\frac{A}{\lambda \alpha_0} = \frac{\omega^2}{V(\omega^2 - \omega_0^2)^2 + k_1^2 \omega^2} \quad (A9)$$

This is an exact solution to the linear problem as obtained by ordinary methods.

APPENDIX II : STABILITY OF STEADY STATE RESPONSE

The perturbation analysis method discussed in some detail in Appendix I can be used to find a stable state solution to the variational equation (4.2). It is possible to obtain the first order stability boundary to equation (A1) without difficulty as, in order to retain terms of order ϵ , the variational equation only requires the approximation $\phi = \phi_0$. However, in order to obtain the second order boundary, it becomes necessary to retain terms of order ϵ^2 and it then becomes necessary to use the approximation $\phi = \phi_0 + \epsilon \phi_1$. Moreover it is also necessary to carry through two stages of the perturbation solution for the disturbance ψ of equation (4.2). Algebraically this becomes very complex. In order to make the problem more manageable two simplifications will be introduced:

- i) The quadratic damping term $k_2 \phi^2$ will be replaced by a cubic term $k_3 \phi^3$ to avoid the need for Fourier expansions.
- ii) The effective wave slope will be allowed to be slightly irregular so that the exact solution to the steady state roll response is $\phi = A \cos \omega t$.

Thus, the stability analysis will relate to the roll equation

$$\ddot{\phi} + k_1 \dot{\phi} + k_3 \phi^3 + \omega_0^2 \phi - 2\phi^3 = F(t) \quad (A10)$$

where $F(t)$ is chosen so that $\phi = A \cos \omega t$

Straightforward substitution yields the result

$$F(t) = A(\omega_0^2 - \omega^2 - \frac{3}{4} A^2 a^2) \cos \omega t - A\omega \left[k_1 + \frac{3}{4} A^2 k_3 \omega^2 \right] \sin \omega t - \frac{A^3 a^2}{4} \cos 3\omega t + \frac{A^3 k_3 \omega^3}{4} \sin 3\omega t.$$

Provided the roll amplitude A is small $F(t)$ corresponds to a nearly regular wave $\alpha = \alpha_0 \cos(\omega t + \delta)$ where

$$\lambda \omega^2 \alpha_0 \cos \delta = A(\omega_0^2 - \omega^2 - \frac{3}{4} A^2 a^2) \quad (A11)$$

$$\text{and } \lambda \omega^2 \alpha_0 \sin \delta = A\omega \left(k_1 + \frac{3}{4} A^2 k_3 \omega^2 \right) \quad (A12)$$

These equations provide a steady state response equation

$$\frac{\omega^2 \lambda \alpha_0}{A} = \sqrt{(\omega^2 - \omega_0^2 + \frac{3}{4} A^2 a^2)^2 + (k_1 \omega + \frac{3}{4} k_3 A^2 \omega^3)^2} \quad (A13)$$

Equation (A3) can be compared with equation (A8) of Appendix I to see that there is agreement with the regular wave solution to this order of approximation.

With this solution for ϕ the variational equation corresponding to equation (A10) becomes

$$\ddot{\psi} + (k_1 + 3k_3 \phi^2) \dot{\psi} + (\omega_0^2 - 3a\phi^2) \psi = 0$$

and on substituting $\phi = A \cos \omega t$ we have

$$\ddot{\psi} + (k_1 + \mu - \mu \cos 2\omega t) \dot{\psi} + (\omega_0^2 - \nu - \nu \cos 2\omega t) \psi = 0 \quad (A14)$$

where

$$\mu = \frac{3}{2} k_3 A^2 \omega^2 \quad \text{and} \quad \nu = \frac{3}{2} a A^2$$

We now require to investigate the steady state solutions of equation (A14) for the sequence of cases $\omega = \omega_0, \frac{1}{2}\omega_0, \frac{1}{3}\omega_0$ etc. In this paper we will only deal with the first two cases.

First order Stability Boundary

In terms of a small quantity ϵ we are looking for a steady state solution to (A14) such that

$$\omega^2 = \omega_0^2 + \epsilon \omega_1 + \epsilon^2 \omega_2 \dots$$

and

$$\psi = \psi_0 + \epsilon \psi_1 + \epsilon^2 \psi_2 \dots$$

and it will be assumed that $k_1 = \epsilon k_1$, $\mu = \epsilon \bar{\mu}$, $\nu = \epsilon \bar{\nu}$ so that these parameters are themselves small. We now obtain a sequence of equations

$$\epsilon^0: \ddot{\psi}_0 + \omega_0^2 \psi_0 = 0$$

$$\epsilon^1: \ddot{\psi}_1 + \omega_0^2 \psi_1 = (\omega_1 + \bar{\nu} + \bar{\nu} \cos 2\omega t) \psi_0 - (k_1 + \bar{\mu} - \bar{\mu} \cos 2\omega t) \dot{\psi}_0$$

$$\varepsilon^2: \ddot{\psi}_2 + \omega^2 \psi_2 = (W_1 + \bar{v} + \bar{v} \cos 2\omega t) \psi_1 - (k_1 + \bar{\mu} - \bar{\mu} \cos 2\omega t) \dot{\psi}_1 + W_2 \psi_0$$

etc.

Solving these equations in turn we have

$$\psi_0 = \beta \cos(\omega t + \delta')$$

and

$$\ddot{\psi}_1 + \omega^2 \psi_1 = \beta(W_1 + \bar{v} + \bar{v} \cos 2\omega t) \cos(\omega t + \delta') + 2\omega(k_1 + \bar{\mu} - \bar{\mu} \cos 2\omega t) \times \sin(\omega t + \delta')$$

On substituting

$$\cos 2\omega t \cos(\omega t + \delta') = \frac{1}{2} \{ \cos(\omega t - \delta') + \cos(3\omega t + \delta') \}$$

and

$$\cos 2\omega t \sin(\omega t + \delta') = \frac{1}{2} \{ -\sin(\omega t - \delta') + \sin(3\omega t + \delta') \}$$

It can be seen that to avoid secular terms in ψ_1, W_1 and δ' must be chosen so that

$$0 = (W_1 + \bar{v}) \cos \delta' + \frac{1}{2} \bar{v} \cos(\omega t - \delta') + \omega(k_1 + \bar{\mu}) \sin(\omega t + \delta') + \frac{1}{2} \bar{\mu} \sin(\omega t - \delta')$$

This is equivalent to the two equations

$$(W_1 + \frac{3}{2} \bar{v}) \cos \delta' + \omega(k_1 + \frac{3}{2} \bar{\mu}) \sin \delta' = 0$$

and

$$-(W_1 + \frac{1}{2} \bar{v}) \sin \delta' + \omega(k_1 + \frac{1}{2} \bar{\mu}) \cos \delta' = 0$$

Thus for a steady state motion to exist we require

$$\tan \delta' = \frac{\omega(k_1 + \frac{1}{2} \bar{\mu})}{W_1 + \frac{1}{2} \bar{v}} \quad (A15)$$

and

$$(W_1 + \frac{3}{2} \bar{v})(W_1 + \frac{1}{2} \bar{v}) + \omega^2(k_1 + \frac{3}{2} \bar{\mu})(k_1 + \frac{1}{2} \bar{\mu}) = 0$$

(A16)

Carrying the analysis to higher orders would provide further equations defining W_2, W_3 etc. If we only retain the terms of order ε that we have evaluated so far we can unite

$$\varepsilon W_1 = \omega^2 - \omega_0^2$$

and on substituting into equation (A/16) we have

$$(\omega^2 - \omega_0^2 + \frac{9}{4} a A^2)(\omega^2 - \omega_0^2 + \frac{3}{4} a A^2) + \omega^2(k_1 + \frac{3}{4} k_3 A^2 \omega^2)(k_1 + \frac{3}{4} k_3 A^2 \omega^2) = 0$$

(A17)

Equation (A17) defines a boundary curve $\omega = f(A, k_1, k_3, \omega_0)$ delineating the first region of instability for equation (A14).

When damping is zero ($k_1 = k_3 = 0$) these boundary curves are given simply by

$$\omega^2 = \omega_0^2 - \frac{3}{4} a A^2$$

$$\text{and } \omega^2 = \omega_0^2 + \frac{9}{4} a A^2$$

These two curves pass through the peaks of the response curves and through the points of vertical tangency as can be seen from figure (6). Damping coefficients which are typical of ship roll damping values do not cause the stability boundary to be significantly different from these limiting curves except for the region where the undamped curves approach the frequency axis at $\omega = \omega_0$.

Second Order Stability Boundary

At this stage we seek a steady state solution to (A14) such that

$$4\omega^2 = \omega_0^2 + \varepsilon W_1 + \varepsilon^2 W_2 \dots$$

and

$$\psi = \psi_0 + \varepsilon \psi_1 + \varepsilon^2 \psi_2 \dots$$

It will be assumed that $v = \varepsilon \bar{v}$

$$k_1 = \varepsilon^2 k_1 \text{ and } \mu = \varepsilon^2 \bar{\mu}$$

so that the perturbation scheme now becomes

$$\varepsilon^0: \ddot{\psi}_0 + \omega^2 \psi_0 = 0$$

$$\varepsilon^1: \ddot{\psi}_1 + \omega^2 \psi_1 = (W_1 + \bar{v} + \bar{v} \cos 2\omega t) \psi_0$$

$$\epsilon^2: \ddot{\psi} + 4\omega^2 \psi_2 = (W_1 + \bar{v} + \bar{v} \cos 2\omega t) \psi_1 - (k_1 + \bar{\mu} - \bar{\mu} \cos 2\omega t) \dot{\psi}_0 + W_2 \psi_0$$

The solution sequence is

$$\psi_0 = \beta \cos(2\omega t + \delta')$$

To avoid secular terms in ψ_1 we require $W_1 + \bar{v} = 0$ so that

$$\ddot{\psi}_1 + 4\omega^2 \psi_1 = \beta \frac{\bar{v}}{2} \{ \cos(4\omega t + \gamma) + \cos \delta' \}$$

$$\therefore \psi_1 = \beta \frac{\bar{v}}{2} \left[-\frac{1}{12\omega^2} \cos(4\omega t + \gamma) + \frac{\cos \delta'}{4\omega^2} \right]$$

$$+ C \cos(2\omega t + \eta)$$

To avoid secular terms in ψ_2 there must be no terms of frequency 2ω in the equation for ψ_2 . On substituting for ψ_1 and ψ_0 we find the required condition is

$$-\frac{\beta \bar{v}^2}{48\omega^2} \cos(2\omega t + \delta') + \frac{\beta \bar{v}^2}{8\omega^2} \cos \delta' \cos 2\omega t + 2\beta \omega (k_1 + \bar{\mu}) \sin(\omega t + \delta') + \beta W_2 \cos(2\omega t + \delta') = 0$$

This equation is equivalent to the two conditions

$$\left(W_2 + \frac{5\bar{v}^2}{48\omega^2} \right) \cos \delta' + 2\omega (k_1 + \bar{\mu}) \sin \delta' = 0$$

and

$$-\left(W_2 - \frac{\bar{v}^2}{48\omega^2} \right) \sin \delta' + 2\omega (k_1 + \bar{\mu}) \cos \delta' = 0$$

hence

$$\tan \delta' = \frac{2\omega (k_1 + \bar{\mu})}{W_2 - \frac{\bar{v}^2}{48\omega^2}}$$

and

$$\left(W_2 + \frac{5\bar{v}^2}{48\omega^2} \right) \left(W_2 - \frac{\bar{v}^2}{48\omega^2} \right) + 4\omega^2 (k_1 + \bar{\mu})^2 = 0 \quad (A18)$$

The stability boundary of this second order region becomes

$$4\omega^2 = \omega_0^2 + \epsilon W_1 + \epsilon^2 W_2 = \omega_0^2 - \bar{v} + \epsilon^2 W_2$$

or

$$4\omega^2 = \omega_0^2 - \frac{3}{2} \bar{v} A^2 + \epsilon^2 W_2 \quad (A19)$$

The undamped case $k_1 = k_3 = 0$ again yields simple roots of equation (A18) of the form

$$\epsilon^2 W_2 = -\frac{5\epsilon^2 \bar{v}^2}{48} = -\frac{45\bar{v}^2 A^4}{192\omega^4}$$

and

$$\epsilon^2 W_2 = +\frac{\epsilon^2 \bar{v}^2}{48\omega^2} = +\frac{9\bar{v}^2 A^4}{192\omega^4}$$

Thus the undamped stability boundaries are

$$4\omega^2 = \omega_0^2 - \frac{3}{2} \bar{v} A^2 + \frac{9\bar{v}^2 A^4}{192\omega^4}$$

and

$$4\omega^2 = \omega_0^2 - \frac{3}{2} \bar{v} A^2 - \frac{45\bar{v}^2 A^4}{192\omega^4}$$

The effect of damping on these boundaries is small except where the curves approach the frequency axis at $\omega = \frac{1}{2}\omega_0$. The stability

boundary defined by equations (A18) and (A19) is shown on Figure (6).

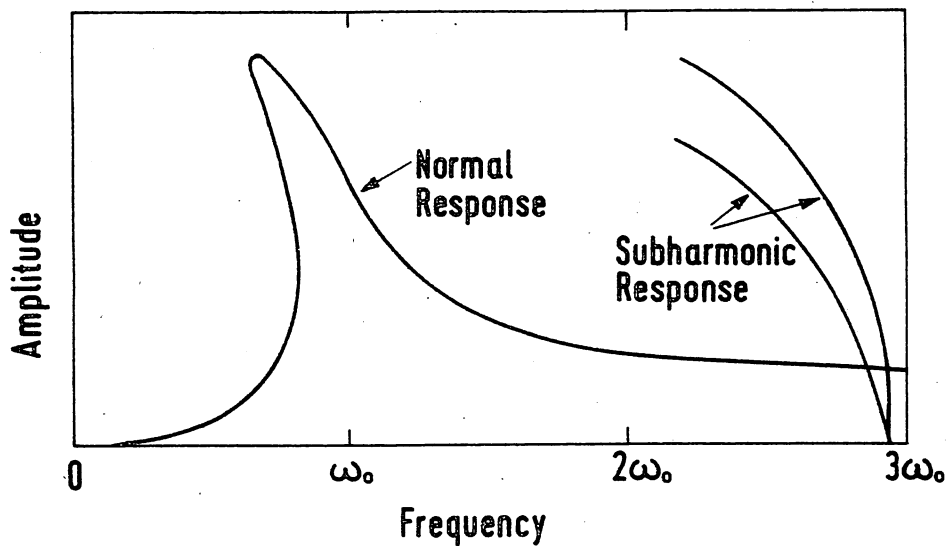


FIG. (1). NONLINEAR ROLL RESPONSE CURVES.

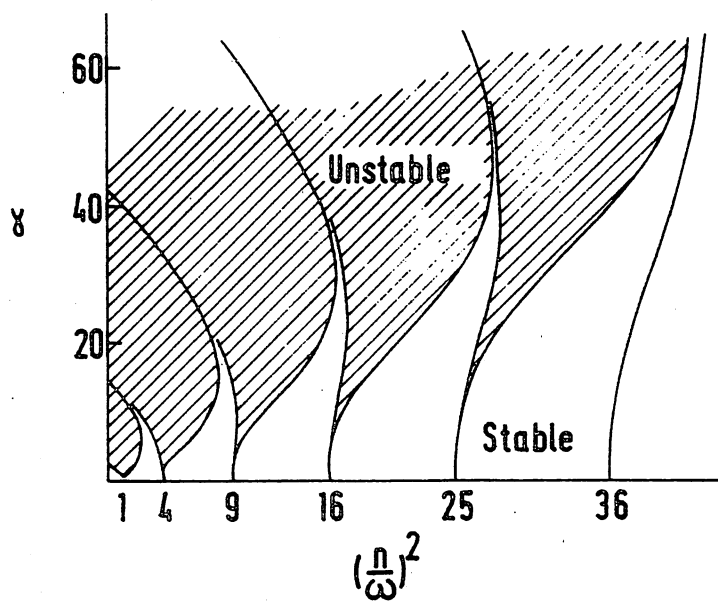


FIG. 2. STABILITY BOUNDARIES OF MATHIEU EQN.

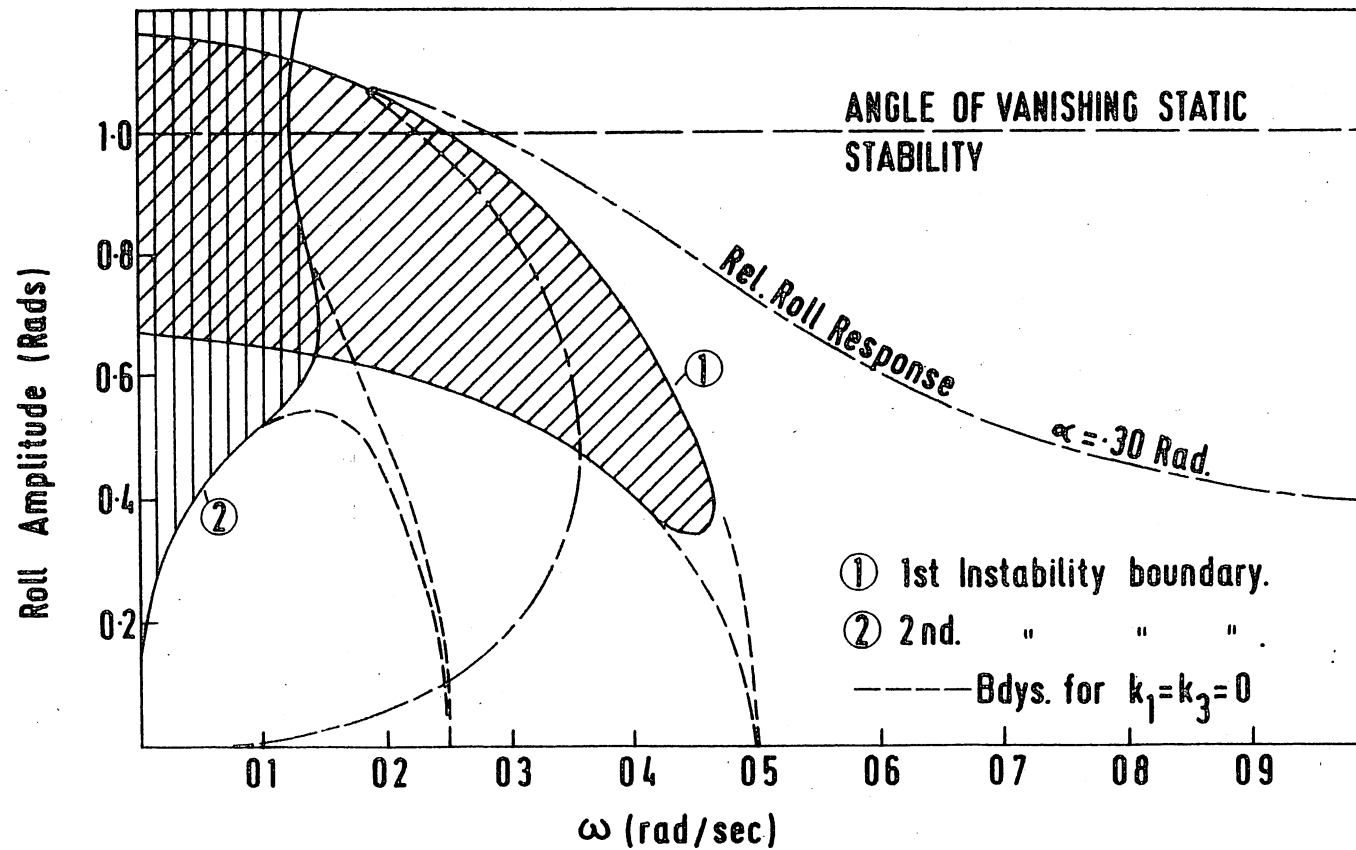


FIG.3. ANALYTIC STABILITY BOUNDARIES FOR ROLLING
IN REGULAR WAVES.

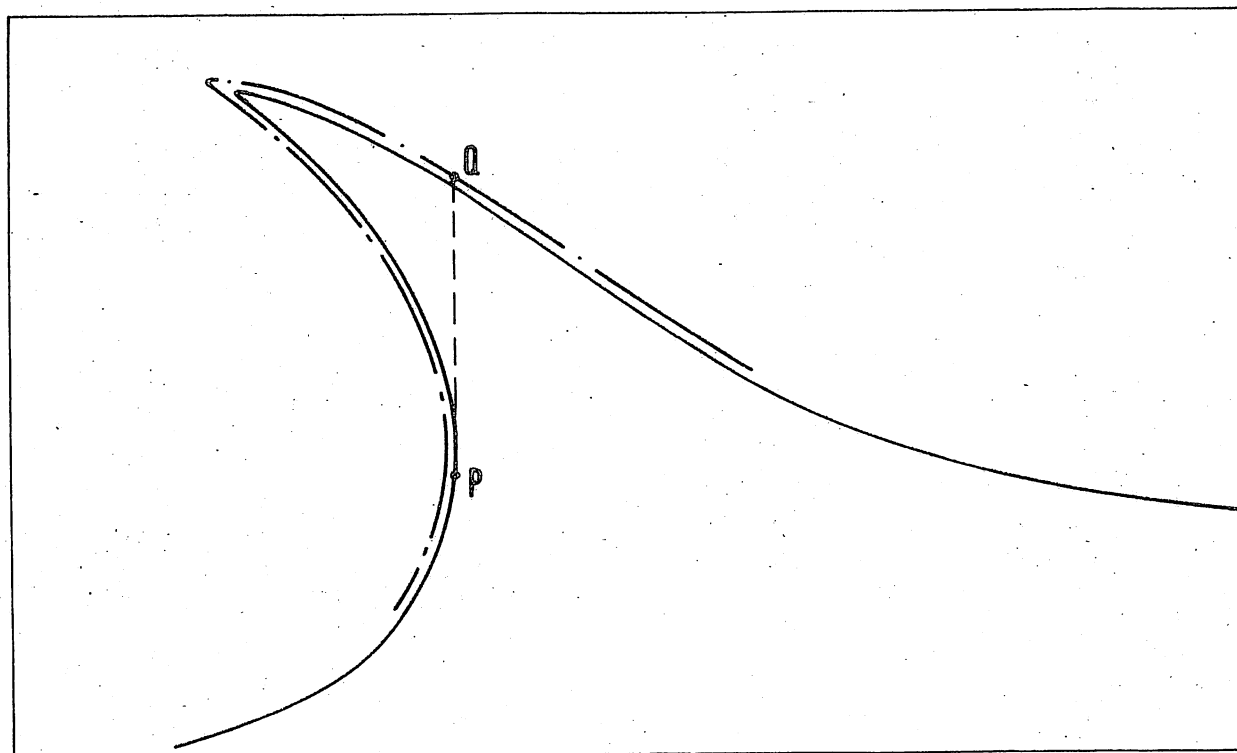


FIG. 4. TRANSITIONAL SITUATION.

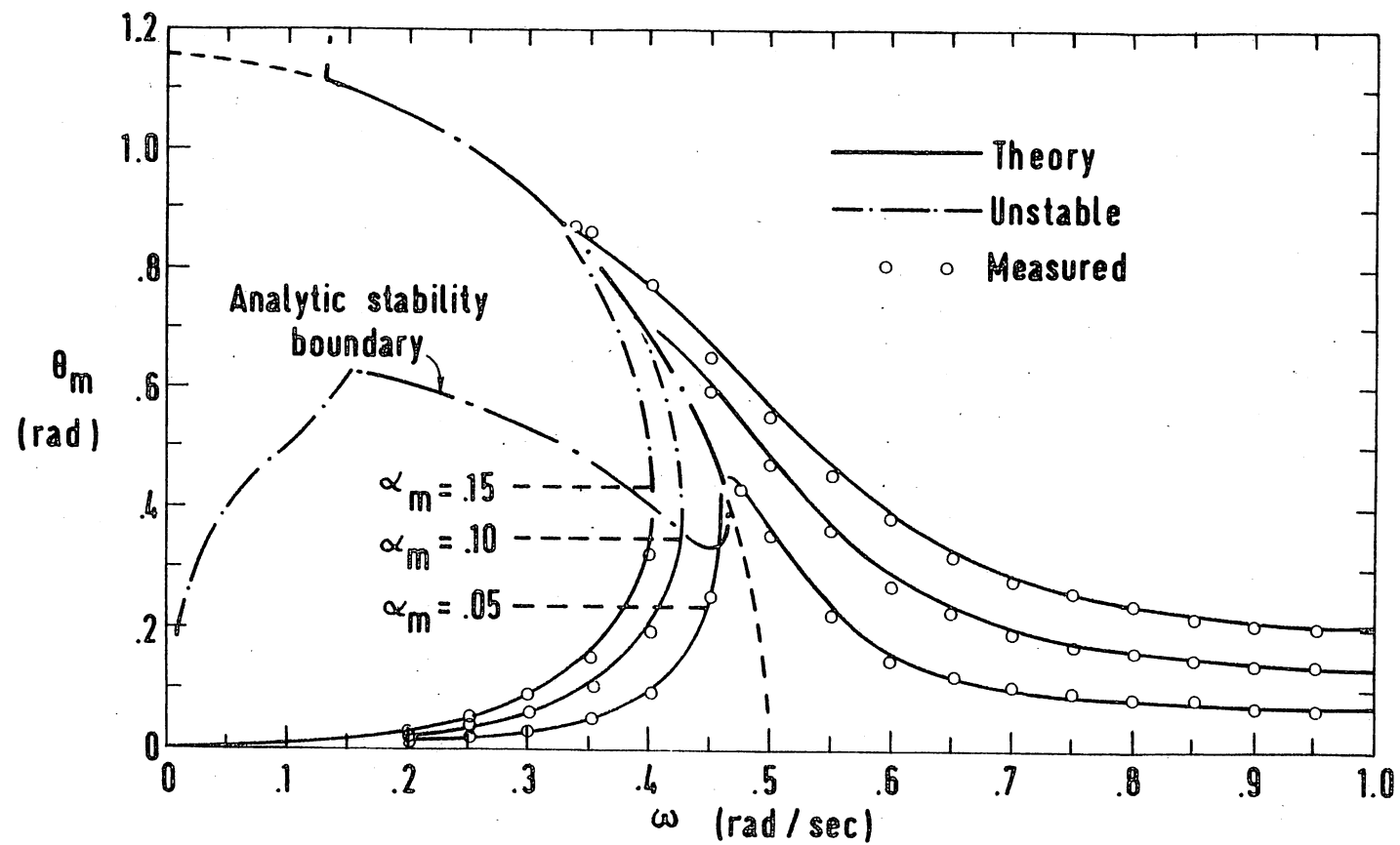


FIG. 5. REGULAR RESPONSE CURVES.

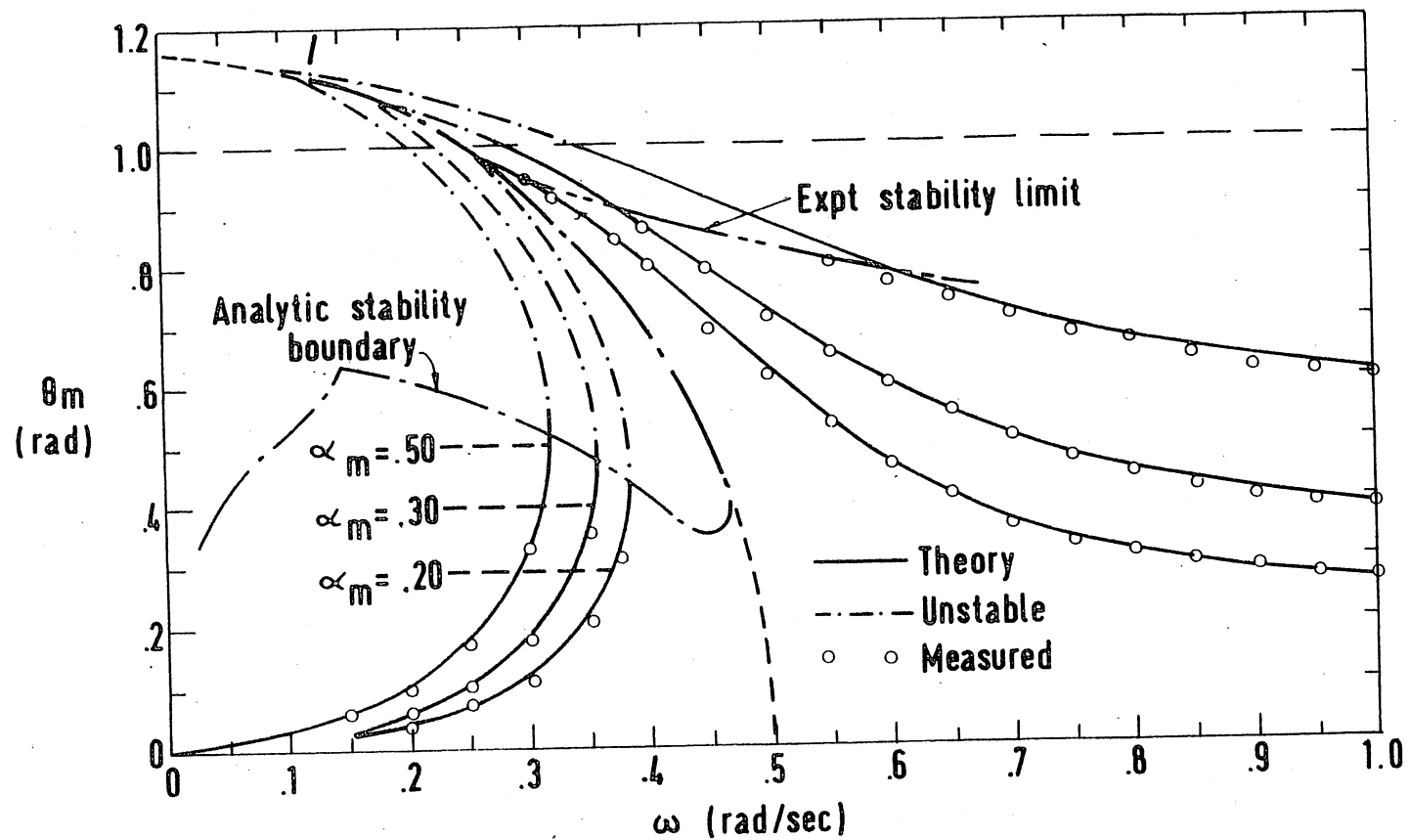
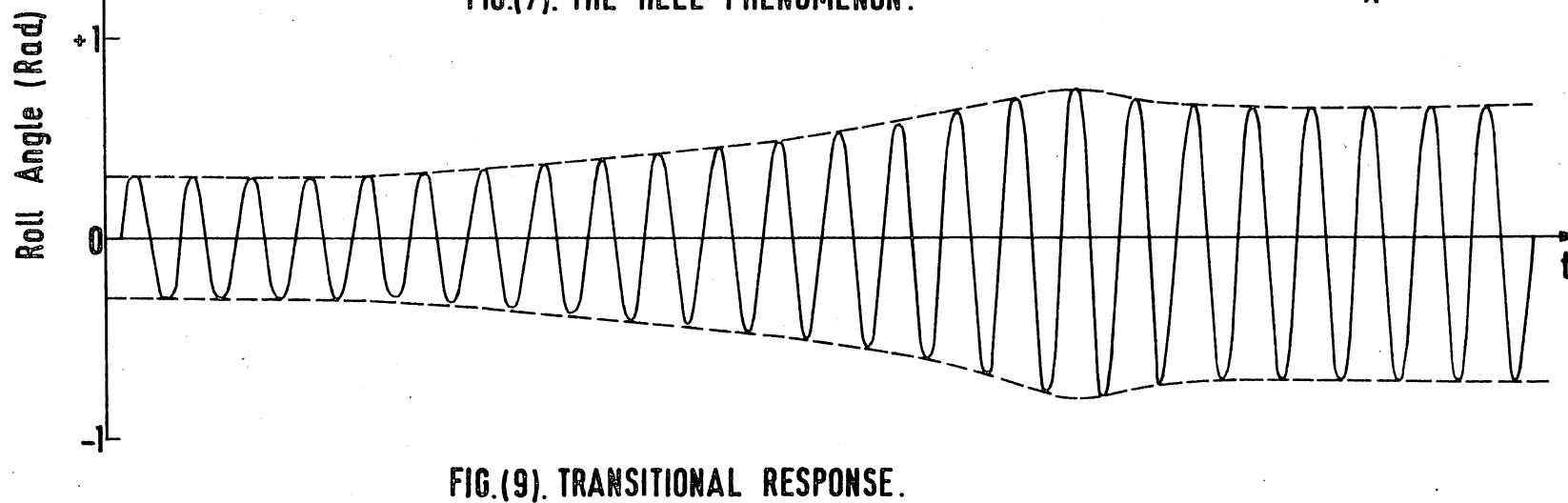
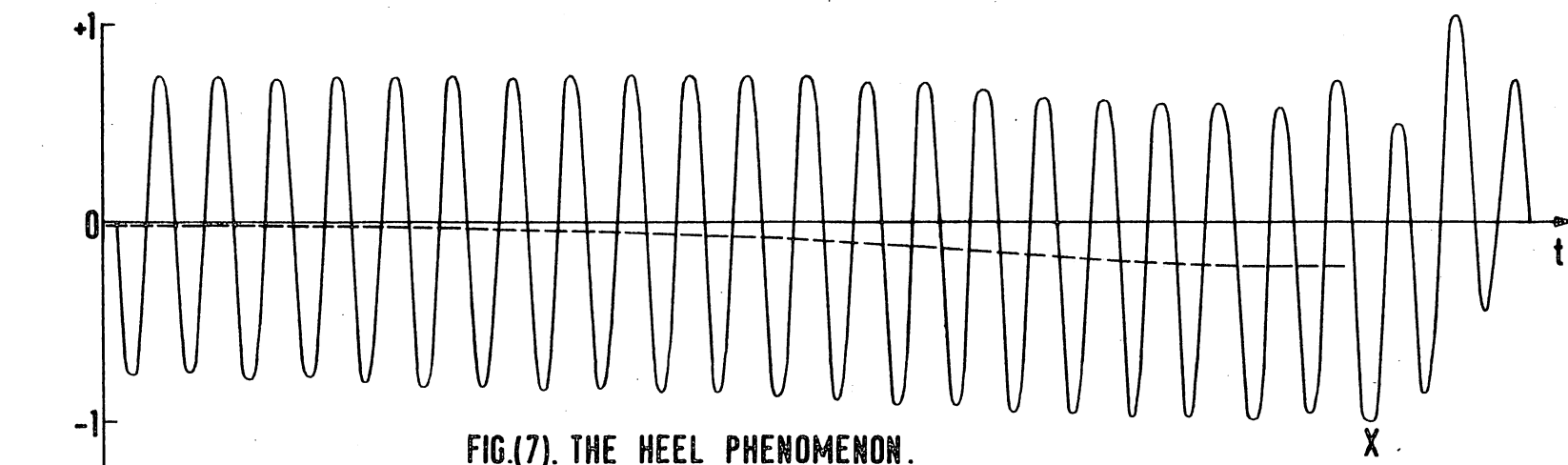


FIG. 6. REGULAR RESPONSE CURVES.



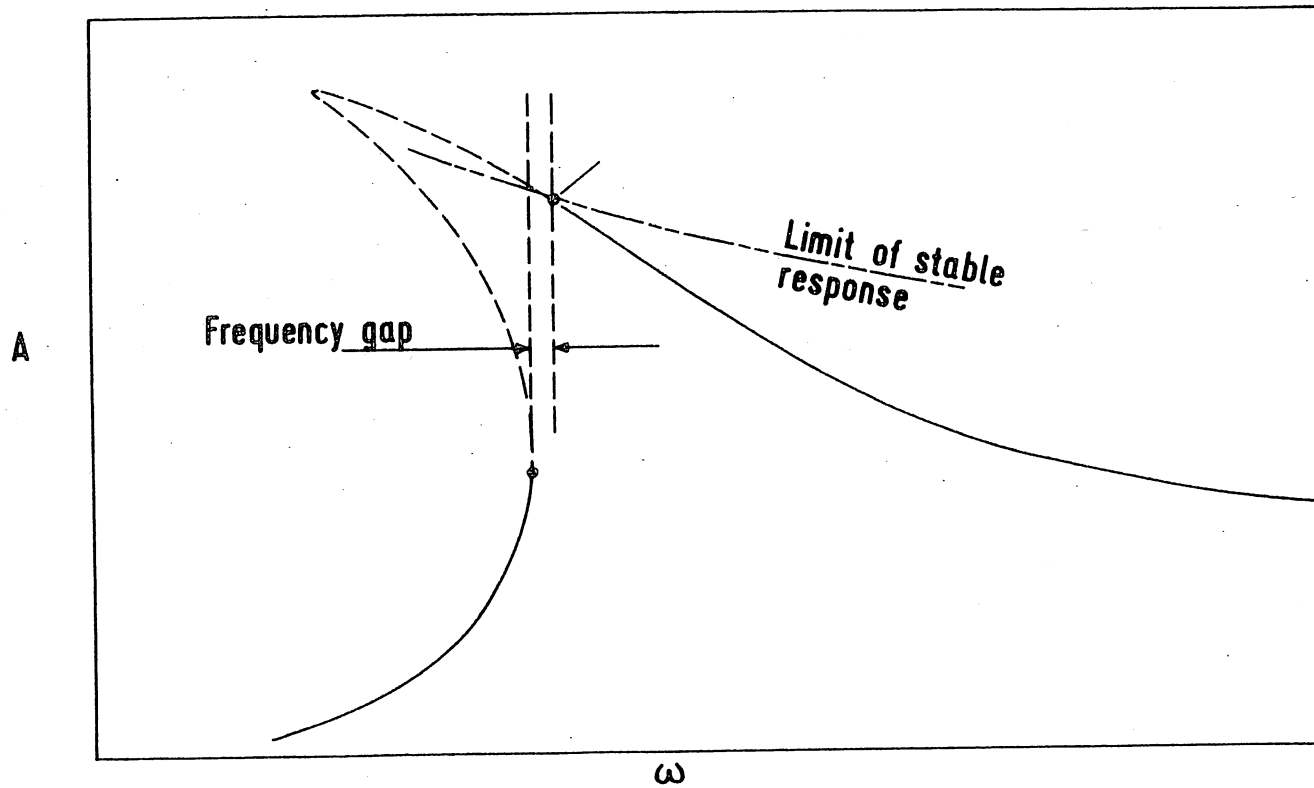


FIG.(8). GAP IN STABLE RESPONSE OF ANALOGUE SIMULATION.

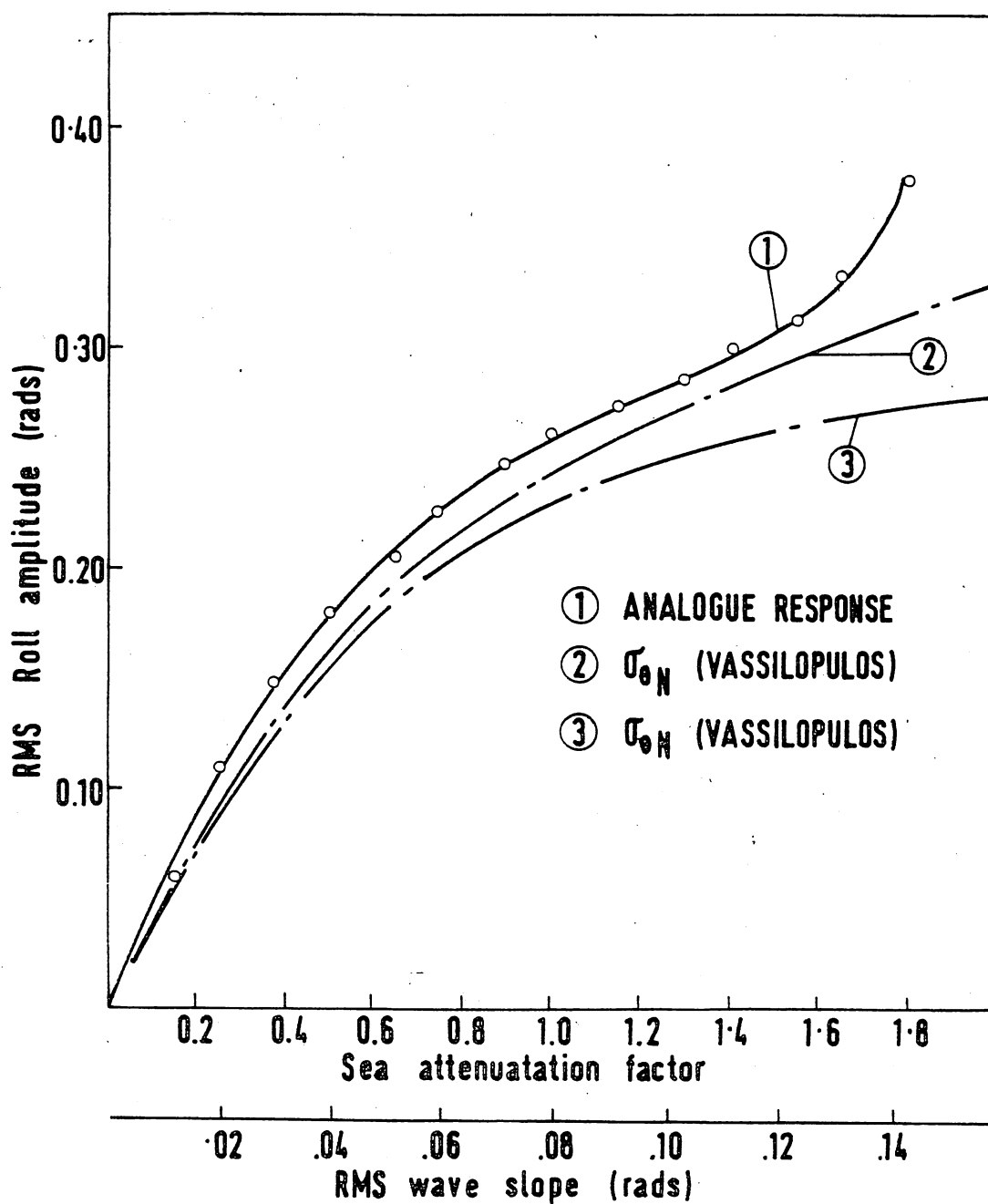


FIG.10. ANALOGUE IRREGULAR SEA RESPONSE.

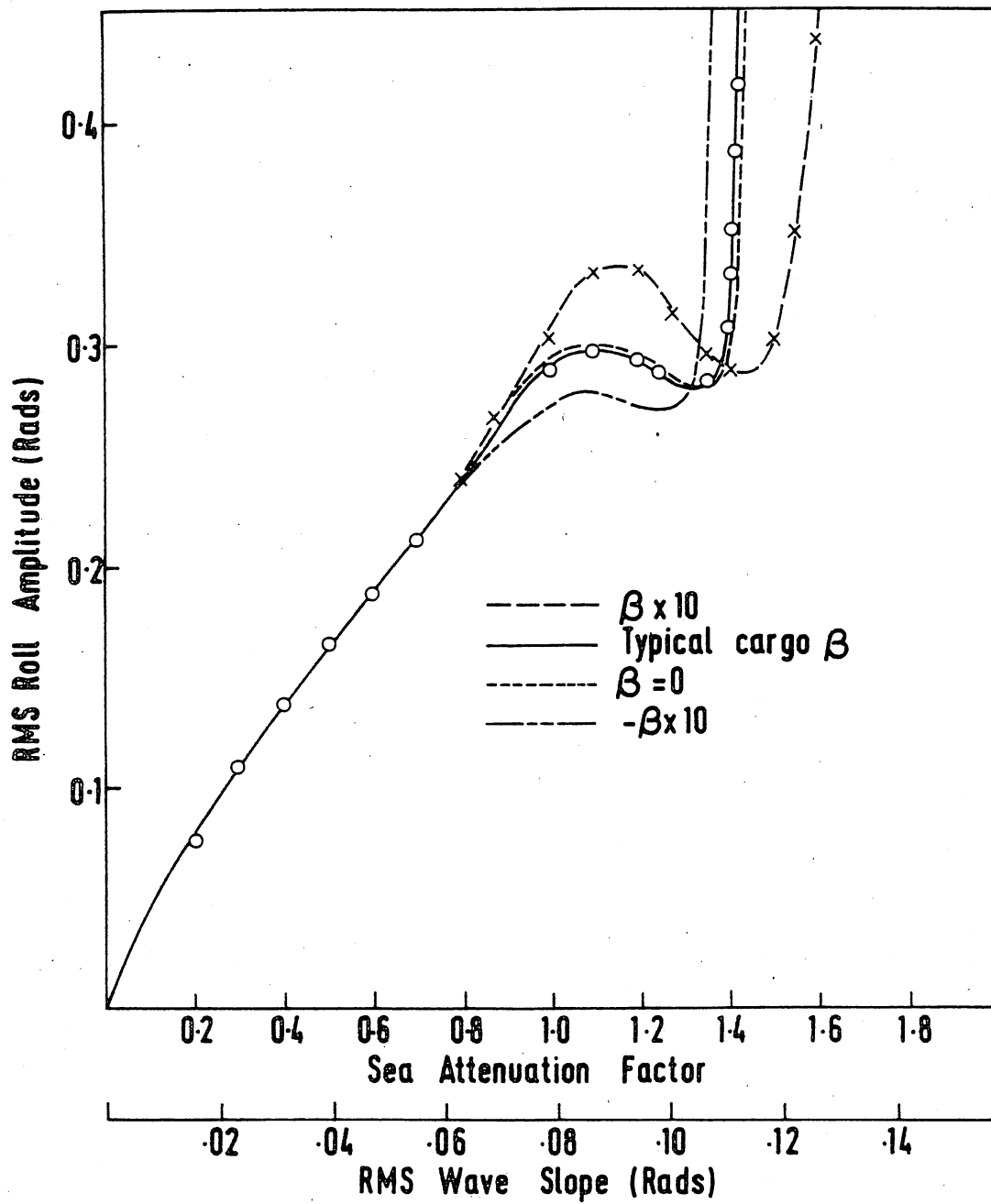


FIG.11. HEAVE COUPLING EFFECT ON RESPONSE IN IRREGULAR WAVES.

A STUDY OF THE STABILITY OF THE MEAN AND VARIANCE
OF ROLLING MOTION IN RANDOM WAVES

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SUMMARY

The Fokker-Planck equation of the rolling motion of a ship moving in random oblique waves is derived. This is a partial differential equation which describes the conditional probability density function of the ship motion. The wave excitation is assumed to be white noise and the ship is allowed to have a time varying restoring moment.

The Fokker-Planck equation is used to obtain the ordinary differential equations which govern the behaviour of the motion's mean and variance as functions of time. The stability of the solutions of these equations is then investigated using an analytical technique.

Results obtained for the cases of following and quartering seas indicate the existence of certain instabilities. The conditions under which such instabilities occur are obtained.

1. INTRODUCTION

Ship hull nonlinearities and transverse stability variations have been known to affect the rolling motion stability in regular waves. It was believed that this type of instability is not of particular consequence in a real irregular sea. However, it has been shown in Reference [1] that ships in a random seaway may experience unstable rolling motion caused by hull nonlinearities. The question of the effect of transverse stability variations is still open.

Few techniques are available for the calculation of the nonlinear stationary response of ships in random waves. Among these, one finds, the perturbation methods, [2], the equivalent linearization methods, [3], and the Fokker-Planck equation method, [4]. Although the last technique is best suited to the study of transient response of random processes, it is very difficult to apply in practice. A modification to this method has been presented in a previous work, [5], and used to calculate the stationary response of nonlinear roll. Comparison with the results obtained using the existing methods shows that the modified technique has the advantage of being both highly accurate and flexible.

The object of this work is twofold. First, to illustrate the use of the modified technique to study the stability of ship motion in random seas. Second, to discuss the effect of transverse stability variations on rolling motion stability.

The approach consists of deriving the Fokker-Planck equation which governs the conditional probability density function of rolling motion in random oblique seas. This equation is used to derive the differential equations which describe the propagation of the roll motion mean and variance. Stability of these equations is then investigated. The results of this investigation indicate that ships may experience unstable roll in a quartering sea. Conditions under which this unstable motion occurs are derived and the instability causing mechanism is explained.

2. EQUATION OF MOTION

Consider a ship travelling with a constant average forward velocity in an oblique sea. The forward velocity of the ship makes an angle α , with the general direction of wave propagation. Assuming that the ship is allowed only to roll, one can use the laws of rigid body dynamics to write the equation of motion in the following form:-

$$\ddot{\theta} + 2\gamma n \dot{\theta} + n^2 \theta \{1 + Q(t)\} = K(t) \quad (1)$$

where

- θ is the angle of roll,
- γ is the non-dimensional damping coefficient,
- n is the natural frequency,
- $Q(t)$ is the time variations in the transverse stability of the ship,
- $K(t)$ is the wave exciting moment,
- t is the time.

A dot over the variable indicate differentiation with respect to time.

The sea is considered to consist of an infinite sum of sinusoidal waves which have closely spaced frequencies and uniformly distributed random phases. The wave excitation moment is then, a random process. This process is assumed to have a uniform spectral density (white noise). The use of such an ideal process greatly simplifies the analysis, yet it provides physically meaningful results. This is rendered possible by the highly resonant nature of rolling motion.

This assumption of wave exciting moment may be formulated as follows:-

$$\begin{aligned} \langle K(t) \rangle &= 0, \\ \langle K(t_1)K(t_2) \rangle &= R\delta(t_1 - t_2) \end{aligned} \quad (2)$$

where $\langle \rangle$ means the ensemble average, R is the exciting moment variance and δ is the Dirac delta function. Furthermore, we are going to assume that the exciting moment is a gaussian process.

The dependence of the restoration on time reflects the influence which the wave motion has on the righting moment of a ship in an oblique sea. The restoring moment in roll is known

to be a nonlinear function of rolling angle. Here, we are going to consider only the case in which the restoring moment is a linear function of the angle. This way we can study the effect of the transverse stability variations alone. The analysis can be easily extended to cover the nonlinear case. We can thus, rewrite equation (1) as

$$\ddot{\theta} + 2\gamma n \dot{\theta} + n^2 \theta \left\{1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)\right\} = K(t) \quad (3)$$

where

- ϵ is a small parameter and $|\epsilon| < 1$.
- A_i is the amplitude of the i th component of the transverse stability variation.

Using the following transformation:-

$$\begin{aligned} y_1 &= \dot{\theta}, \\ y_2 &= \theta, \end{aligned}$$

and substituting into equation (3), we obtain

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -2\gamma n y_1 - n^2 y_2 \left\{1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)\right\} + K(t) \end{aligned} \quad (4)$$

3. FOKKER-PLANCK EQUATION

Consider the Brownian motion of a particle in a field of force, in which the force is not only proportional to the displacement of the particle but also varies with time as given by equation (3). The motion of this particle can be described by equations exactly similar to (4). It is well known that Brownian motion can be idealised as a Markov process.

A vector stochastic process $Y(t)$, is said to be Markov if the conditional probability density function that Y lies in the interval $(Y_n, Y_n + dY_n)$ at time t_n , given that Y is equal to Y_1 at t_1 , Y_2 at time t_2 , ..., and Y_{n-1} at time t_{n-1} (where $t_n > t_{n-1} > \dots > t_1$) depends only on the value of Y at time t_{n-1} . Thus, for

a Markov process, we have

$$P_n(Y_n, t_n | Y_1, t_1; Y_2, t_2; \dots; Y_{n-1}, t_{n-1}) = P_2(Y_n, t_n | Y_{n-1}, t_{n-1}) \quad (5)$$

where $P_2(Y_2, t_2 | Y_1, t_1) dY$ is the probability that Y lies in the interval $(Y_2, Y_2 + dY_2)$ at time t_2 given that $Y = Y_1$ at time t_1 . Then the conditional probability density function P_2 , completely describes a Markov process. The dimension of the process is determined by the number of the components of the process $Y(t)$. The conditional probability density function of a Markov process satisfies a partial differential equation which is called the Fokker-Planck equation.

We derive now the Fokker-Planck equation for the rolling motion of a ship which is described by the differential equations (4). The approach which we are going to follow, consists in treating ship motion as a limiting case to discrete random walk in two dimensions. The one dimensional version of this approach was used by Kac, [6], in treating the problem of a particle in a field of constant force and an elastically bound particle. Originally, the approach was suggested by Smoluchowski.

Consider a particle which can move in a plane (y_1, y_2) , under the action of two forces, f_1 and f_2 in the y_1 and y_2 directions respectively. These forces are expressed as follows:-

$$f_i = C_i h_i(y_1, y_2, t), \quad i = 1, 2$$

where $h_i(y_1, y_2, t)$, $i = 1, 2$, are normalised functions of the position of the particle in the plane and C_i ,

$i = 1, 2$ are constants. The motion of the particle consists of a series of steps, such that it can move either to the right, to the left, forward or backward each step. The step lengths are d_1 and d_2 in the y_1 and y_2 directions respectively and the step duration is T .

Instead of the probability density $P_2(y_1, y_2; t | y_{10}, y_{20})$ we consider the discrete probability density $P(m_1 d_1, m_2 d_2; sT | i_1 d_1, i_2 d_2)$ which is

the probability that the particle is at the point $(m_1 d_1, m_2 d_2)$ at time sT , given that it was at point $(i_1 d_1, i_2 d_2)$ at time zero. We write Smoluchowski equation for the two-dimensional case as follows:-

$$P\{m_1 d_1, m_2 d_2; (s+1)T | i_1 d_1, i_2 d_2\} = \sum_{k_1, k_2} P(k_1 d_1, k_2 d_2; sT | i_1 d_1, i_2 d_2) \times P(m_1 d_1, m_2 d_2; T | k_1 d_1, k_2 d_2) \quad (6)$$

The transition probability $P(m_1 d_1, m_2 d_2; T | k_1 d_1, k_2 d_2)$ is given by

$$P(m_1, m_2; 1 | k_1, k_2) = \frac{1}{4} \{1 + h_2(k_1, k_2)\} \delta(m_1, k_1) \delta(m_2, k_2 - 1) + \frac{1}{4} \{1 + h_1(k_1, k_2)\} \delta(m_1, k_1 - 1) \delta(m_2, k_2) + \frac{1}{4} \{1 - h_2(k_1, k_2)\} \delta(m_1, k_1) \delta(m_2, k_2 + 1) + \frac{1}{4} \{1 - h_1(k_1, k_2)\} \delta(m_1, k_1 + 1) \delta(m_2, k_2)$$

where we have dropped $d_1, d_2; T$.

Substituting into equation (6), we obtain

$$P(m_1, m_2; s+1 | i_1, i_2) = \frac{1}{4} (1+h_2) P(m_1, m_2+1; s | i_1, i_2) + \frac{1}{4} (1-h_2) P(m_1, m_2-1; s | i_1, i_2) + \frac{1}{4} (1-h_1) P(m_1-1, m_2; s | i_1, i_2) + \frac{1}{4} (1+h_1) P(m_1+1, m_2; s | i_1, i_2)$$

Since the probability density function, P , has continuous derivatives, we can expand the terms according to Taylor's theorem. By retaining terms up to first order on the left hand side and up to second order to the right hand side and taking the limits as

$$T \rightarrow 0, \quad d_1, d_2 \rightarrow 0, \quad (d_1^2/2T) = \langle \Delta y_1^2 \rangle; \quad (d_2^2/2T) = \langle \Delta y_2^2 \rangle$$

$$(d_1 h_1 / 2T) = \langle \Delta y_1 \rangle, (d_2 h_2 / 2T) = \langle \Delta y_2 \rangle.$$

We have

$$\begin{aligned} \frac{\partial P}{\partial t} = & - \frac{\partial}{\partial y_1} (y_2 P) + \\ & + \frac{\partial}{\partial y_2} [2 \int n y_2 + n^2 y_1 \{1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)\}] P \\ & + \frac{1}{2} R \frac{\partial^2 P}{\partial y_2^2} \end{aligned} \quad (7)$$

This is the required Fokker-Planck equation. The solution of this equation subject to the initial condition

$$P(y_1, y_2; t / y_{10}, y_{20}) =$$

$$\delta(y_1 - y_{10}) \delta(y_2 - y_{20}) \text{ as } t \rightarrow 0,$$

yields the conditional probability density function which describes the random process (y_1, y_2) .

Instead of actually solving equation (7), we are going to use it in deriving the ordinary differential equations which govern the behaviour of the mean and variance of the process as functions of time.

Expected Value Equations:-

We multiply the two sides of equation (7) by y_1 and integrate the equation with respect to y_1 and y_2 from $-\infty$ to ∞ . We then, repeat the same process using y_2 , and obtain

$$\begin{aligned} \dot{u}_1 &= u_2 \\ \dot{u}_2 &= - \langle 2 \int n y_2 + n^2 y_1 \{1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)\} \rangle \end{aligned} \quad (8)$$

Variance Equations:-

We repeat the same process described in obtaining equations (8) using $(y_1 - u_1)^2$, $(y_2 - u_2)^2$ and $(y_1 - u_1)(y_2 - u_2)$ instead of the variables y_1 and y_2 . We then have

$$\begin{aligned} \dot{v}_1 - 2 \dot{u}_1 u_1 &= 2 v_{12} - 2 u_1 u_2 \\ \dot{v}_2 - 2 \dot{u}_2 u_2 &= - 4 \int n v_2 - 2 n^2 (v_{12} - u_1 u_2) (1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)) \\ &+ 4 \int n u_2^2 + R \\ \dot{v}_{12} - \dot{u}_1 u_2 - u_1 \dot{u}_2 &= v_2 - u_2^2 - 2 \int n v_{12} + 2 \int n u_1 u_2 \\ &+ n^2 (u_1^2 - v_1) (1 + \epsilon \sum_i A_i \sin(\omega_i t + \theta_i)) \end{aligned}$$

In deriving equations (8) and (9) have used the following boundary conditions:-

$$P \left| \begin{array}{l} y_i = \infty \\ y_i = -\infty \end{array} \right| = y_i^3 P \left| \begin{array}{l} y_i = \infty \\ y_i = -\infty \end{array} \right| = y_i \frac{\partial P}{\partial y_i} \times$$

$$\left| \begin{array}{l} y_i = \infty \\ y_i = -\infty \end{array} \right| = 0$$

4. STABILITY ANALYSIS

In the previous section, we derived the equations which govern the behaviour of the expected values of the roll angle and velocity. These are two coupled linear differential equations with time dependent coefficients. The literature on such equations is abundant. One of the methods which can be used to study the stability of these equations, was introduced by Struble [7]. The method has been extended to the case of n-coupled equations by Hsu [8]. The method combines the slowly varying parameters technique and the classical perturbation method. Hsu's analysis is quite suitable for our purpose.

Before we start our stability investigation, we would like to rewrite equations (8) in the following form:

$$\begin{aligned} \dot{u}_1 &= u_2 \\ \dot{u}_2 &+ n^2 u_1 \end{aligned}$$

$$= -\xi \{ b u_2 + n^2 u_1 \sum_i A_i \sin(\omega_i t + \theta_i) \} \quad (10)$$

where we have made the substitution

$$\xi b = 2 \int n.$$

This is justified, since it has been shown experimentally that linear rolling damping is a small quantity.

We now consider a solution to equations (10) in the following form:-

$$\begin{aligned} u_1 &= C(t) \cos nt + D(t) \sin nt + \sum_m \xi^m x_m \\ u_2 &= -n C(t) \sin nt + n D(t) \cos nt + \sum_m \xi^m \dot{x}_m \end{aligned} \quad (11)$$

where $C(t)$ and $D(t)$ are slowly varying parameters with time. Substituting (11) into (10), we get

$$\dot{C} \cos nt + \dot{D} \sin nt = 0 \quad (12)$$

and

$$\begin{aligned} &- n \dot{C} \sin nt + n \dot{D} \cos nt + \xi (\ddot{x}_1 + n^2 x_1) \\ &= \xi n b (C \sin nt - D \cos nt) \\ &- \xi n^2 \sum_i (C \cos \omega_i t + D \sin \omega_i t) \times \\ &(B_i \cos \omega_i t + E_i \sin \omega_i t) \end{aligned} \quad (13)$$

where we have made the substitution

$$\begin{aligned} A_i \sin(\omega_i t + \theta_i) \\ = B_i \cos \omega_i t + E_i \sin \omega_i t \end{aligned}$$

In deriving equation (13), we have retained terms up to first order in ξ . Using trigonometric identities, we can rewrite equation (13) as

$$\begin{aligned} &- n \dot{C} \sin nt + n \dot{D} \cos nt + \xi (\ddot{x}_1 + n^2 x_1) \\ &= \xi n b (C \sin nt - D \cos nt) \end{aligned}$$

$$\begin{aligned} &- \frac{1}{2} \xi n^2 \sum_i \{ I_{1i} \cos(n + \omega_i)t + I_{2i} \cos(n - \omega_i)t \\ &+ I_{3i} \sin(n + \omega_i)t + I_{4i} \sin(n - \omega_i)t \} \end{aligned} \quad (14)$$

where

$$I_{1i} = C B_i - D E_i$$

$$I_{2i} = C B_i + D E_i$$

$$I_{3i} = D B_i + C E_i$$

$$I_{4i} = D B_i - C E_i$$

Two cases should be considered. The non-resonance case, in which $2n$ is not nearly equal to any of the ω_i 's. The resonance case, in which $2n$ is nearly equal to one of the ω_i 's, say ω_s .

However, it has been shown that only the resonance case is of significance [1]. Thus, we are going to treat only the resonance case. In this case we divide equation (14) into two equations as follows:-

$$\begin{aligned} &- n \dot{C} \sin nt + n \dot{D} \cos nt \\ &= \xi n b (C \sin nt - D \cos nt) \\ &- \frac{1}{2} \xi n^2 \{ I_{2s} \cos(n - \omega_s)t \\ &+ I_{4s} \sin(n - \omega_s)t \} \end{aligned} \quad (15)$$

and

$$\begin{aligned} &\ddot{x}_1 + n^2 x_1 \\ &= \sum_{i \neq s} \{ I_{1i} \cos(n + \omega_i)t + I_{2i} \cos(n - \omega_i)t \\ &+ I_{3i} \sin(n + \omega_i)t + I_{4i} \sin(n - \omega_i)t \} \\ &+ I_{1s} \cos(n + \omega_s)t + I_{3s} \sin(n + \omega_s)t \end{aligned} \quad (16)$$

The division has been made so that equation (16) would be free of secular terms. Thus, the stability of equations (10) is solely determined by the behaviour of the slowly varying parameters C and D as functions of time. If these variables grow up with time, then solutions of (10) are unstable. If they decay with time, then solutions of (10) are stable.

Let

$$\omega_s = 2n + \lambda \quad \text{and} \quad nt = \beta$$

Then equation (15) becomes

$$-\dot{C} \sin \beta + \dot{D} \cos \beta =$$

$$\epsilon b \{C \sin \beta - D \cos \beta\}$$

$$- \frac{1}{4} \epsilon n^2 \{I_{2s} \cos (\beta + \lambda t) - I_{4s} \sin (\beta + \lambda t)\}$$

(17)

We can use equations (12) and (17) and the assumptions underlying the method of slowly varying parameters to obtain the following two equations:-

$$\dot{C} = -\frac{1}{2} \epsilon b C - \frac{1}{4} \epsilon n^2 (I_{2s} \sin \lambda t + I_{4s} \cos \lambda t) \quad (18)$$

$$\dot{D} = -\frac{1}{2} \epsilon b D - \frac{1}{4} \epsilon n^2 (I_{2s} \cos \lambda t + I_{4s} \sin \lambda t) \quad (19)$$

Using the change of variables

$$F = C - j D \quad G = C + j D$$

we get

$$\dot{G} = -\frac{1}{2} \epsilon b G + \frac{1}{4} \epsilon n^2 F (E_s - j B_s) \exp(-j \lambda t) \quad (20)$$

$$\dot{F} = -\frac{1}{2} \epsilon b F + \frac{1}{4} \epsilon n^2 G (E_s + j B_s) \exp(-j \lambda t) \quad (21)$$

We consider solutions for equations (20) and (21) in the following form:-

$$F = F_0 \exp (qt + \frac{1}{2} \lambda t)$$

$$G = G_0 \exp (qt - \frac{1}{2} \lambda t)$$

Substituting into (20) and (21), we get

$$q^2 + \epsilon b q + \frac{1}{4} (\lambda^2 + \epsilon^2 b^2 - \frac{1}{4} \epsilon^2 A_s^2) = 0$$

This quadratic equation could be solved for the possible values of the exponent, q. These are the values for which there exist nontrivial solutions for equations (20) and (21). We can now write down the conditions necessary for equation (10) to have a stable solution. This is given as follows:-

for $\lambda = 0$, if $A_s < 2b$ the solution is stable

if $A_s > 2b$ the solution is unstable

(22)

We have thus related the magnitude of the amplitude of the resonant component of the transverse stability variation to the magnitude of the linear damping coefficient. The consideration of linear damping only does not limit the applicability of the method. One can calculate the equivalent damping coefficient and add it to the linear damping coefficient. One can also extend the procedure to cover the case of nonlinear damping.

5. DISCUSSION

We have presented in the previous sections an analytical technique for the study of ship motion mean and variance stability in random seas. The example of rolling has been chosen to illustrate the procedure. The study of rolling in oblique random seas indicates that under certain conditions the ship will possess unstable motion. This does not necessarily mean that the ship will capsize. However, excessive rolling should be expected. This is caused by parametric resonance, which occurs when the ship resonates with a wave whose frequency is double the natural frequency of the ship. If the random sea does not possess any

significant energy at such a frequency, then there is no danger of unstable rolling. We would like to show that ordinary ships may travel in random seas which possess significant energy at frequencies equal to double the natural frequencies for such ships.

Consider the Pierson-Moskowitz spectrum, one can easily show that maximum energy is always centred about a frequency, σ_m given by

$$\sigma_m = 2.277 / \sqrt{H}$$

where H is the wave significant height in feet. Then, the worst condition would be when the ship's natural frequency satisfies the following relation:-

$$2n = \sigma_m + \sigma_m^2 U \cos \alpha / g$$

However, the conditions would still be unfavourable when

$$2n = 1.2\sigma_m + (1.2\sigma_m)^2 U \cos \alpha / g \quad (23)$$

because even at this frequency the waves would still have about eighty percent of its maximum energy.

The natural frequency of the ship in rolling can be expressed as

$$n = C_3 / \sqrt{L}$$

where C_3 is a constant and L is the ship length. The wave significant height can also be related to the ship's length as

$$H = \mu L$$

Substituting into equation (23), we get

$$\begin{aligned} U / \sqrt{gL} \\ = (\mu \sqrt{g} / 7.465 \cos \alpha) (2C_3 - 2.732/\sqrt{\mu}) \end{aligned} \quad (24)$$

Equation (24) has been plotted for an

example ship, whose description is given in Table I. The curves shown in figure (2), indicate that unstable motion is expected when

$$\mu = .028, \alpha = 120^\circ, \text{ and}$$

$$F_r = .15$$

A condition like this can be easily met.

stability calculations have also been carried out for the above mentioned ship. A Pierson-Moskowitz spectrum of 3.05 ms. significant height was used. The spectrum of the variations in the transverse stability is shown in Figure (1-a), for a resonant case. It has been found that if this ship travels without bilge keels under the conditions indicated she will suffer unstable motion. The addition of bilge keels will cause this instability to disappear.

We have also performed the calculations for a non-resonant condition at a wave significant height of 4.61 ms. The results appear in Figure (1-b). Comparison between these two curves shows that variations in the transverse stability are more severe in a quartering sea than in a following sea. This is a result of resonance.

A final work remains to be said about the use of Pierson Moskowitz spectrum in performing the numerical calculation, while we have built the mathematical model on a white noise spectrum. It is felt that this is justified because of the narrow band characteristics of rolling motion.

6. CONCLUSIONS

We thus came to the following conclusions:-

- 1) The procedure presented in this work is extremely flexible. It can be used in the calculation of the stationary ship motion as well as in investigating the stability of the motion. Linear and nonlinear motions could be treated. Variations in the transverse stability of the ship can also be accommodated.
- 2) Transverse stability variations may be more significant in a quartering sea than in a following sea. It thus seems appropriate to direct more efforts towards the study of rolling motion in oblique seas.

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LIST OF SYMBOLS

A_i	Amplitude of the i th component of the transverse stability variation
a_i	Amplitude of the i th component of sea wave
b	Damping coefficient
C, D, F, G	Slowly varying parameters with time
d_1, d_2	Displacements of a particle in Brownian motion
f_1, f_2	Forces acting on a particle in Brownian motion
g	Acceleration due to gravity
h_1, h_2	Normalized functions of the position of a particle in Brownian motion
H	Wave significant height
$K(t)$	Wave exciting moment
L	Ship length
m_1, m_2, i_1, i_2, s	Integers
n	Ship's natural frequency in roll
$P(y_2, t_2 y_1, t_1)$	Conditional probability density function
$P_1(x, y, z, t)$	Pressure distribution in i th wave
$Q(t)$	Time variations in the transverse stability of the ship
q	Stability index
R	Variance of wave exciting moment
S	Wave spectral density
T, t	Time
U	Forward velocity of the ship
u_1, u_2	Mean value of the roll angle and velocity respectively
v_1, v_2	Variance of roll angle and velocity respectively
y_1, y_2	Roll angle and velocity respectively
Y	Vector random process
α	Ship's heading
γ_i	Phase angle of i th wave
δ	Dirac delta function
ε	Small parameter
μ	Ratio of wave significant height to ship length
ω_i	Frequency of encounter with the i th wave component
ϕ	Roll angle

Φ	Velocity potential
ρ	Water density
σ_i	Frequency of the i th wave component
θ_i	Phase angle of the i th component of the transverse stability variation
η	Non-dimensional damping coefficient

TABLE IExample Ship Particulars

Length	108	ms
Beam	16	ms
Draft	6.19	ms
Metacentric height	0.35	ms
Displacement	7667.	t
Natural frequency in roll	.34	rad./sec.
Damping coefficient (without bilge keels) η	.005	
Damping coefficient (with bilge keels) η	.03	

APPENDIX A

We shall calculate the wave moment acting on a ship travelling with a constant average forward velocity in random oblique waves. We are going to use the following assumptions:-

- 1) The random wave consists of a large number of infinitesimal deep water waves which have closely spaced frequencies and uniformly distributed phase angle.
- 2) The pressure distribution in the wave is not affected by the presence of the ship.
- 3) The fluid is inviscid.
- 4) The flow is irrotational.

Consequently, second and higher order terms in wave amplitude and ship motion are neglected with the exception of second order terms in which linear coupling between rolling angle and wave amplitude exists. These are the terms which reflect the effect of wave motion on the transverse stability of the ship. We could have adopted a formal second order theory. This, however, would have obscured the phenomenon which we would like to study.

Consider the three sets of coordinate axes given by:-

- a) $\bar{x}\bar{y}\bar{z}$, fixed in space.
- b) $\overset{\wedge}{\wedge}{\wedge}\overset{\wedge}{\wedge}{\wedge}$
 $\overset{\wedge}{\wedge}{\wedge}$ xyz, moving with the average motion of the ship such that $\overset{\wedge}{\wedge}{\wedge}$ oxy lies in the undisturbed free surface when the ship stops.
- c) xyz, fixed in the ship such that the origin coincides with the ship's centre of gravity.

It can be easily shown that the systems $\bar{o}\bar{x}\bar{y}\bar{z}$ and oxyz are related through the following relations:

$$\begin{aligned}\bar{x} &= (x + Ut)\cos\alpha - (y - \phi z)\sin\alpha \\ \bar{y} &= (x + Ut)\sin\alpha + (y - \phi z)\cos\alpha \\ \bar{z} &= (\phi y + z - z_0)\end{aligned}\quad (A-1)$$

where

U is the constant forward velocity of the ship

z_0 is the vertical distance between the ship's centre of gravity and the undisturbed water surface.

α is the angle between ox and $\bar{o}\bar{x}$.

Since only first order terms in the wave amplitude will contribute to the wave moment, then one can use the principle of superposition to calculate this moment. In what follows, we are going to calculate the wave moment corresponding to one wave only.

The velocity potential for the i th component of the wave takes the form [9]

$$\begin{aligned}\bar{\phi}_i(\bar{x}, \bar{y}, \bar{z}, t) \\ = -g \frac{a_i}{\sigma_i} \exp(-k_i \bar{z}) \cos(k_i \bar{x} + \sigma_i t + \gamma_i)\end{aligned}\quad (A-2)$$

where

$\bar{\phi}_i$ is the velocity potential
 a_i is the amplitude of the i th component
 σ_i is the frequency of the i th component
 γ_i is the phase angle of the i th component

Using Bernoulli's law, we get the pressure distribution in the wave as

$$\begin{aligned}p_i(\bar{x}, \bar{y}, \bar{z}, t) \\ = \rho g z - \rho g a_i \exp(-k_i \bar{z}) \sin(k_i \bar{x} + \sigma_i t + \gamma_i)\end{aligned}\quad (A-3)$$

The equation of the free surface of the wave can be obtained as

$$z_{wi} = a_i \sin(k_i \bar{x} + \sigma_i t + \gamma_i) \quad (A-4)$$

Substituting (A-1) into (A-3) and (A-4) we get

$$\begin{aligned}p_i(x, y, z, t) &= \rho g(z - z_0 + \phi y) \\ &- \rho g a_i M_i \phi z \exp(-k_i z + k_i z_0) \\ &\cos(L_i x - M_i y + \omega_i t + \gamma_i)\end{aligned}$$

$$-\rho g a_i (1 - k_i \phi y) \exp(-k_i z + k_i z_0) \sin(L_i x - M_i y + \omega_i t + \gamma_i) \quad (A-5)$$

and

$$z_{wi} = z_0 - \phi y + a_i \sin(L_i x - M_i y + \omega_i t + \gamma_i) \quad (A-6)$$

where k_i is the wave number and

$$L_i = k_i \cos \alpha$$

$$M_i = k_i \sin \alpha$$

$$\omega_i = \sigma_i + U L_i$$

The roll moment is given by

$$RM = \iint_S \{ z p_i \cos(\vec{n}, y) - y p_i \cos(\vec{n}, z) \} dz dx$$

where

\vec{n} is the outward normal to the wetted surface of the ship

S is the wetted surface of the ship

Using Gauss's theorem, we can rewrite the moment as

$$RM = \iiint_V \left\{ z \frac{\partial p_i}{\partial y} - y \frac{\partial p_i}{\partial z} \right\} dx dy dz \quad (A-7)$$

where V is the volume enclosed by the wetted hull and the waterplane. However, contributions from the waterplane integral is zero because water pressure there is zero.

Substituting (A-5) into (A-7) and using (A-6), we get

$$RM = -\Delta GM \phi \left\{ 1 - (a_i / \nabla GM) [I_{i1} \cos(\omega_i t + \gamma_i) + I_{i2} \sin(\omega_i t + \gamma_i)] \right\} + E_i(t) \quad (A-8)$$

where

Δ is the ship displacement

∇ is the immersed volume of the ship

$E_i(t)$ is the wave exciting moment

$$I_{i1} = \iiint \{ y M_i (1 - 2k_i z) \exp(-k_i z + k_i z_0) \sin L_i x \sin M_i y + (k_i z + k_i^2 y^2 - M_i^2 z^2) \exp(-k_i z + k_i z_0) \sin L_i x \cos M_i y \} dx dy dz$$

$$I_{i2} = \iiint \{ (k_i z + k_i^2 y^2 - M_i^2 z^2) \exp(-k_i z + k_i z_0) \cos L_i x \cos M_i y + y M_i (1 - 2k_i z) \exp(-k_i z + k_i z_0) \cos L_i x \sin M_i y \} dx dy dz$$

Summing the individual components, we get the total rolling moment as

TRM =

$$-\Delta GM \phi \left[1 - \sum_i (a_i / \nabla GM) \{ I_{i1} \cos(\omega_i t + \gamma_i) + I_{i2} \sin(\omega_i t + \gamma_i) \} \right] + \sum_i E_i(t)$$

This expression could be written in the following form:-

TRM =

$$-\Delta GM \phi \left[1 + \varepsilon \sum_i A_i \sin(\omega_i t + \theta_i) \right] + K(t) \quad (A-9)$$

where $\varepsilon = \frac{a_i}{\nabla GM} \sqrt{I_{i1}^2 + I_{i2}^2}$, $\theta_i = \gamma_i + \tan^{-1} I_{i2} / I_{i1}$

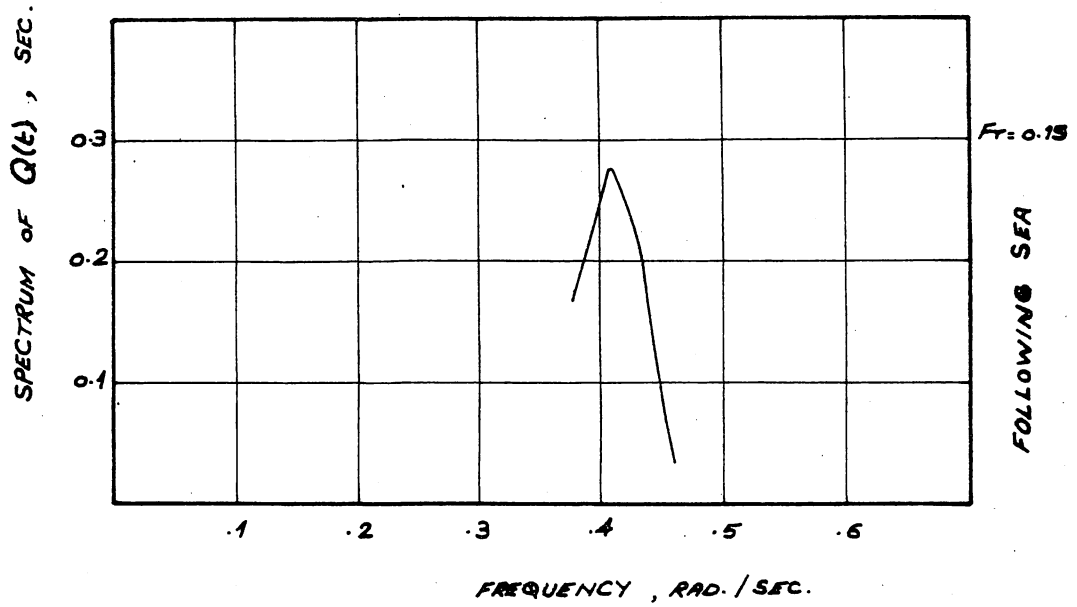


FIG. 1 A

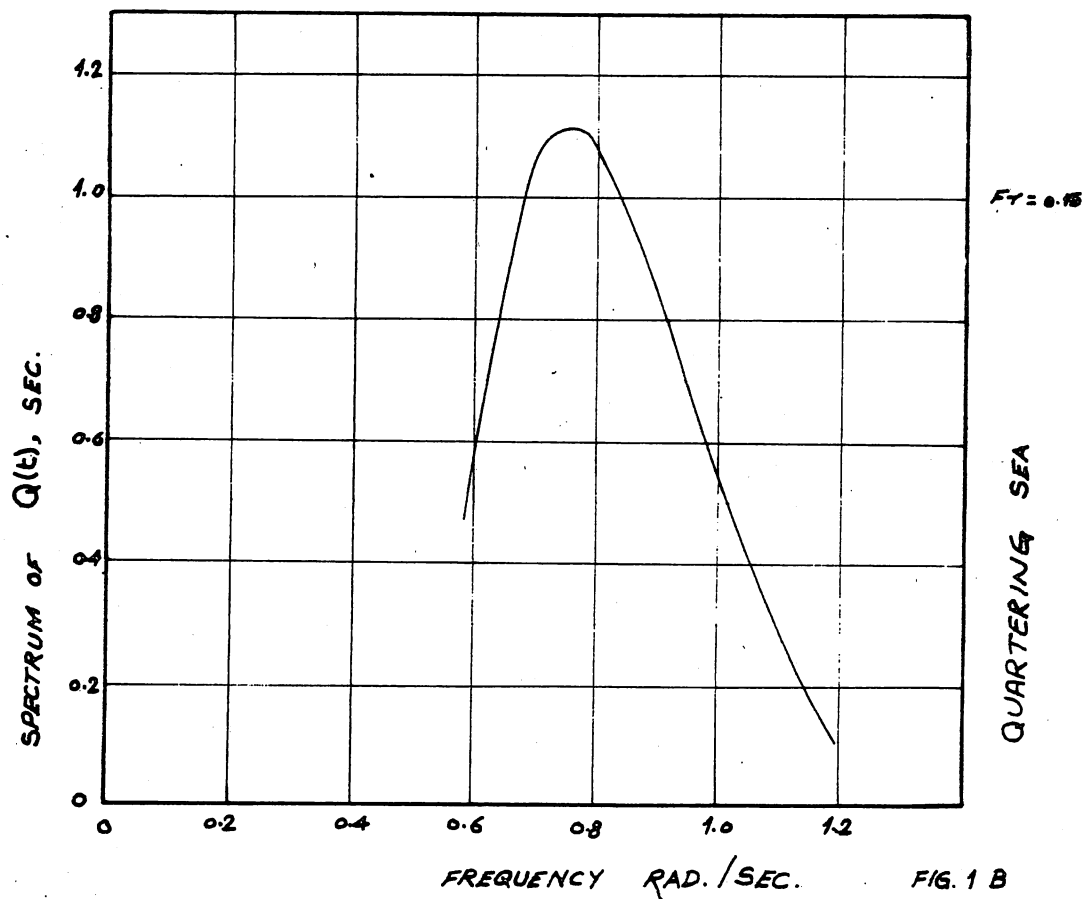


FIG. 1 B

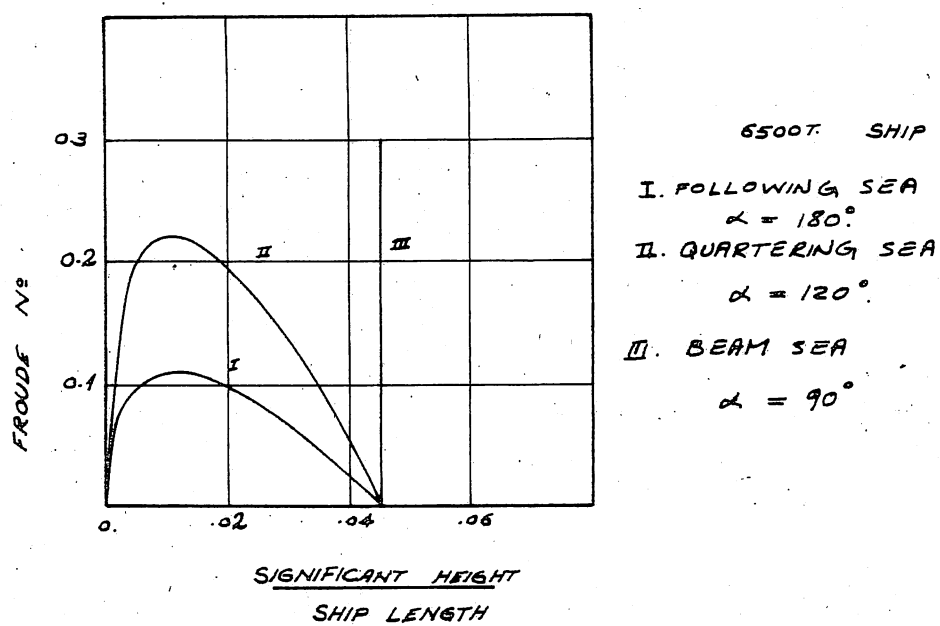


FIG. 2

ON CAPSIZING OF SHIPS IN REGULAR AND IRREGULAR SEAS

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1. INTRODUCTION

Prediction of safety of intact ships in the natural sea is still a problem. Existing stability criteria are not completely dependable because for reasons of practicability too many influences are neglected. From casualties and model experiments it is known that ships may capsize (e.g. in a following sea) even though conventional stability requirements are fulfilled. Up to now such cases have been rare; but an uncritical application of these criteria to modern and unusual types of vessels may prove to be disastrous.

Before trying to get a better standard of valuation for judging the safety of a ship in waves, those parameters which govern the degree of excitation should be pointed out. Therefore a short survey of the theory of rolling in regular and irregular waves will be given in the following two chapters. The special conditions in which a ship may capsize by rolling can then be derived. It would be a great step forward if these influences (among others the shape of the hull, the location of the superstructures, the natural period of roll, the Froude number, and the sea spectrum) were already taken into account in the early stage of design. In particular, special attention should be paid to this point in case anti-rolling devices are not provided.

By this procedure, it can often be found out whether a ship will be endangered by rolling or not and, if necessary, a modification to the projected lines or other particulars of influence should be made in order to reduce the probability of capsizing. In some

cases, however, it will be difficult to assess the excitation by the parameters mentioned above and a closer consideration of the rolling forced by the sea becomes indispensable. Therefore the last section deals with some ideas how a criterion can be set up taking into regard all important forces and moments involved in rolling and expressing the danger of capsizing by a single numerical value.

2. ROLLING IN BEAM SEAS

The calculation of the rolling motion in regular and irregular waves is generally known provided that the usual simplifications are made. It may be allowed to recall the treatment to our mind.

Rolling in Regular Transverse Waves

The slope of the wave causes an inclining moment whose value varies at a harmonic function according to the different positions of the ship in the travelling wave. The equation of the forced motion is:-

$$T' \cdot \frac{d^2 \phi}{dt^2} + R \cdot \frac{d\phi}{dt} + \Delta \cdot h(\phi) \\ = \Delta \cdot \overline{GM} \cdot \alpha_M \cdot \cos \omega_w t$$

where

T' = mass moment of inertia (added moment of inertia of entrained liquid included)

ϕ = angle of inclination

R = damping coefficient (resistance to rolling is assumed to vary linearly with the angular velocity)

Δ = displacement

$h(\phi)$ = righting arm (a function of the angle of heel)

\overline{GM} = metacentric height

α_M = maximum wave slope

ω_w = circular frequency of wave

This equation can be solved nearly with mathematical exactness if the righting arm curve can be expressed by the function

$$h(\phi) = \overline{GM} \cdot (\phi + C\phi^3)$$

The statical stability curves of many ships correspond with this function even up to greater angles of inclination. From the "wall-side formula" it can be derived that C depends on the metacentric radius BM and the metacentric height GM:-

$$C = \frac{1}{2} (\overline{BM}/\overline{GM}) - 1/6$$

Substituting for T' the value $(\Delta/g) \cdot k'^2$ for R the value $2(\Delta/g) \cdot K \cdot k'^2 \cdot \omega_\phi$, and for \overline{GM} the value $k'^2 \cdot \omega_\phi^2/g$ (where k' = radius of gyration of mass, K = dimensionless damping constant, and ω_ϕ = circular frequency of unresisted free rolling) we obtain the differential equation:-

$$\frac{d^2\phi}{dt^2} + 2K \cdot \omega_\phi \frac{d\phi}{dt} + \omega_\phi^2 \cdot (\phi + C\phi^3) = \omega_\phi^2 \cdot \alpha_M \cos \omega_w t$$

From the theory of unresisted rolling in still water it is known circular frequency of roll is only constant if the amplitude of roll ϕ_M is very small. Approximately the ratio of the frequency at greater angles of roll to that at small angles is:-

$$\frac{\omega}{\omega_\phi} = \sqrt{1 + \frac{1}{4} C \cdot \phi_M^2}$$

If we replace the term $\omega_\phi^2 \cdot (\phi + C\phi^3)$ by the term $\omega^2 \cdot \phi$ we get a new differential equation whose solution is almost the same as that of the original equation. After transient motions have died out the oscillation is harmonic:-

$$\phi = \phi_M \cdot \cos(\omega_w t - \epsilon)$$

with

$$\phi_M =$$

$$\frac{\alpha_M}{\sqrt{(1 + \frac{1}{4} C \cdot \phi_M^2 - \left[\frac{\omega_w}{\omega_\phi}\right]^2)^2 + 4K^2 \left[\frac{\omega_w}{\omega_\phi}\right]^2}}$$

and

$$\tan \epsilon =$$

$$\frac{2K \cdot \left[\frac{\omega_w}{\omega_\phi}\right]}{1 + \frac{1}{4} C \cdot \phi_M^2 - \left[\frac{\omega_w}{\omega_\phi}\right]^2}$$

Typical curves of amplitudes of roll versus frequency of excitation are shown in Figure 1. If $C > 0$ (or if a wall-sided vessel $GM < 3 BM$) the maximum amplitude of resisted rolling - depending on the amount of damping - will be reached at a ratio $\omega_w/\omega_\phi > 1$ and if $C < 0$ (or $GM > 3 BM$) at a ratio $\omega_w/\omega_\phi < 1$. Only if $C \approx 0$ (or $GM \approx 3 BM$) resonance is to be expected at $\omega_w/\omega_\phi \approx 1$. The validity of this theory was demonstrated and proven by experiments by Wendel, Reference (1).

Apart from the arrangement of anti-rolling devices there is only little possibility of reduction of roll in beam seas after the ship is built. The best method, of course, is to avoid resonant frequencies at all. If the amplitude of roll should not exceed a given value the range of the ratios ω_w/ω_ϕ which should not be attained can be calculated. As can be seen from Figure 1 the width of this range depends on the shape of the righting arm curve ($C > 0$, $C = 0$ or $C < 0$) and the damping constant K. The practical result of such a consideration is that according to the maximum wave steepness $(H/\lambda)_M$ and the maximum wave length λ_M a certain range of values

\overline{GM} must be avoided. For example, the metacentric height of a ship with a linear restoring moment ($G = 0$) navigating in an area with $(H/\lambda)_M \leq 1/40$ and $\lambda_M \leq 60$ m must be smaller than $\overline{GM} = 0.96$ m if an amplitude of $\phi_M = 6^\circ$ shall not be exceeded ($B = 10$ m; Figure 2).

Fortunately, in practice the amplitudes of roll are generally smaller than the values calculated by the linear theory. This is not only a result of the nonlinear restoring moment. It must also be noted that the actual excitation is less severe because the natural seaway is always irregular. Another reason is the fact that the resistance to rolling is not only a function of $d\phi/dt$ - and as sometimes stated also of $(d\phi/dt)^2$ - but also varies with the angle of roll. A typical curve of the damping constant K versus angle of roll ϕ is shown in Figure 3 (Reference (2)). As we can see the damping coefficient increases considerably at large angles of roll. On the other hand in the range of actual damping coefficients the maximum amplitude of forced rolling decreases rapidly when the damping grows (from Figure 1). Hence, the angles the ship will take in beam seas are of limited magnitude and the danger of capsizing is much lower than may be assumed if these effects are not taken into consideration. This statement is proved by the results of capsizing statistics. In an investigation of the causes of the capsizing of 31 German ships it was found that no ship capsized solely by rolling in beam seas. Rolling in a following or quartering sea, however, turned out to be the only cause for the loss of four ships (Table 5 of Reference (3)).

Rolling in Irregular Beam Seas

Though, as a rule, the probability of capsizing in beam seas is smaller than in following seas the important points concerning the method of getting statistical information about the rolling motion shall be mentioned briefly.

As we know from the linear theory there is a direct proportionality between the amplitude of roll ϕ_M and the amplitude of the regular transverse wave ξ_A . The ratio of these amplitudes can be determined for every ship as a function of the wave frequency ω and is called "frequency response function".

$$Y_{\phi, \xi}(\omega) = \frac{\phi_M}{\xi_A}$$

Similar to the representation of the irregular seaway itself by a superposition of regular harmonic waves with different amplitudes, frequencies, and random phase lags, the response of a ship to an irregular seaway can be represented by the sum of its responses to the several sinusoidal component waves. Therefore, we only have to multiply the ordinates of the sea spectrum $S_\xi(\omega)$ by the square of the corresponding ordinates of the frequency response function in order to obtain the roll amplitude spectrum:-

$$S_\phi(\omega) = Y_{\phi, \xi}^2(\omega) \cdot S_\xi(\omega)$$

From the roll spectrum we can get the same sort of statistical information as from the sea spectrum. Just as the wave amplitudes do, the amplitudes of roll closely follow a Rayleigh distribution. This enables us to calculate, for instance, the average roll amplitude, the average of the 1/3 greatest roll amplitudes, the average of the 1/10 greatest roll amplitudes, etc.

In this way we can compare the rolling motions of different ships in different beam seas in an early stage of design and, as a result of this, it is seen how rolling can be reduced by appropriate design modifications.

3. ROLLING IN FOLLOWING SEAS

Because of the absence of a heeling moment it was assumed for a long time that rolling in beam seas must be heavier than in following seas. Excitation caused by the changing shape of the submerged part of the vessel and its influence on the restoring moment was disregarded until Grim pointed out in 1952 that resonance may occur also in longitudinal waves (Reference (2)). It is a remarkable fact that in spite of the capsizing of several ships in following seas the theory of this kind of excitation was not applied earlier to ship motion.

4. ROLLING IN REGULAR LONGITUDINAL WAVES OF FOLLOWING SEAS

In longitudinal waves the righting-arm curve oscillates between two extremes: a lower curve in the vicinity of the situation "crest amidships" and a higher curve in the vicinity of the situation "trough

amidships". The period of fluctuation is equal to the period of encounter $T_E = 2\pi/\omega_E = \lambda/c - V$

(ship overtaken by the wave; c = wave velocity, V = speed of ship). If the time-dependent restoring moment varies linearly with the angle of inclination the equation of motion can be solved:-

$$T: \frac{d^2\phi}{dt^2} + R: \frac{d\phi}{dt}$$

$$+ \Delta \cdot (\overline{GM}_m + \delta \overline{GM} \cdot \sin \omega_E t) \cdot \phi = 0$$

where \overline{GM}_m is the mean value and $\delta \overline{GM}$ the amplitude of the harmonically varying metacentric height. Assuming that the mean radius of gyration of mass equals the radius of gyration in still water the differential equation can be written as follows:-

$$\frac{d^2\phi}{dt^2} + 2K \cdot \omega_\phi \cdot \frac{d\phi}{dt}$$

$$+ \omega_\phi^2 (1 + \frac{\delta \overline{GM}}{\overline{GM}_m} \cdot \sin \omega_E t) \cdot \phi = 0$$

In order to get the solution the transformation

$$\phi = \phi_1 \cdot e^{-K\omega_\phi t}$$

must be made. We then obtain a Mathieu differential equation:-

$$\frac{d^2\phi_1}{dt^2} + \omega_\phi^2 (1 - K^2 + \frac{\delta \overline{GM}}{\overline{GM}_m} \sin \omega_E t) \cdot \phi_1$$

$$= 0$$

For the judgement of safety in longitudinal waves it is sufficient to know whether the solutions are stable or unstable because without an external exciting moment a sudden rise of the rolling motion is only possible in the case of instability. The stability of the motion depends on three factors: the tuning factor $\Lambda = \omega_E/\omega_\phi$

the metacentric height ratio $\delta \overline{GM}/\overline{GM}_m$, and the damping constant K . In "stability charts" calculated by Ince and Strutt for unresisted motions and by Kotowski for resisted motions the zones of instability are plotted (Reference (4)). Figure 4 shows the corresponding diagrams for resisted and unresisted rolling in longitudinal waves. Every point lying beyond the limit curves of the hatched areas represents a dangerous situation of instability. The range of unfavourable combinations of $\delta \overline{GM}/\overline{GM}_m$ and

ω_ϕ/ω_E - located in the vicinity of

$$\omega_\phi/\omega_E = 0.5, 1.0, 1.5, 2.0, \dots$$

- expands with increasing amplitude of metacentric height fluctuation $\delta \overline{GM}$ and decreasing average initial stability \overline{GM}_m .

Apart from the existence of several resonant frequencies there is another characteristic feature of motions caused by a time-dependent restoring moment (so-called "parametric" excitation). As can be seen from the top diagram in Figure 4 the effect of the damping moment is restricted to a decrease of the zones of instability. Unlike the resisted rolling in transverse waves the amplitude of roll is not reduced. Of course, this is only true if the resistance to rolling varies linearly with the angular velocity of roll. Nevertheless, it is to be assumed that this fact is one of the reasons why more ships capsized in longitudinal than in transverse waves.

There are two important consequences which the designer can draw from the results of this simple linear theory:-

- a) At different positions of the ship in the wave the metacentric heights and righting-arm curves should be nearly the same. This can be realized by an adequate design of shiplines and superstructures. Figure 5 shows two examples: by the flaring sides in the after and forward quarter of the first vessel the water-planes in the "crest" and "trough" condition differ very much and also cause great differences between the respective values \overline{GM} . This effect will be enlarged by a small freeboard, deck sheer, and the arrangement of poop and forecastle (further decrease of the righting arms in the "crest" condition and increase in the "trough" condition). The second vessel, however, is not endangered by fluctuation

of stability in longitudinal waves. δGM and $\delta h(\phi)$ are very small because of the parallel running frames and the arrangement of a bridge (avoidance of small levers in the "crest" condition).

- b) The metacentric height and the speed of the ship shall be such that under normal operating conditions the ratio of frequencies ω/ω_E amounts to values between 0.5 and 1.0, between 1.0 and 1.5, or between 1.5 and 2.0 (greater values are generally unproblematic because the zones of instability vanish rapidly owing to the increasing influence of damping. See the top diagram in Figure 4.)

5. ROLLING IN IRREGULAR FOLLOWING SEAS

The changing righting-arm curves due to the different wave patterns can also cause a parametric excitation in irregular longitudinal waves. The excitation, however, is less severe since there are always some components of the irregular wave whose frequencies of encounter do not fall into a zone of instability.

Contrary to the rolling in irregular beam seas the rolling in irregular following or head seas cannot be estimated by the known statistical methods. The reason for this is that the principle of superposition can only be applied if each of the regular component waves produces a rolling motion with an amplitude which is proportional to the amplitude of wave. But, as explained in Section 4, rolling in a longitudinal wave results from the existence of unstable solutions of the equation of motion and large angles of roll may be attained even though the wave height is small (see Figure 4: the sole effect of a small wave height is a small range of instability resulting from the minor fluctuation of stability δGM).

As a "frequency response function" cannot be established the rolling behaviour in irregular longitudinal waves must be assessed by a different method. In recent years, several proposals were made on how to estimate the danger of capsizing in this special case. Before dealing with this question it will be shown that a view of the degree of excitation can already be taken by a closer consideration of those parameters being responsible for the behaviour in a longitudinal seaway. In particular, the conditions

in which excitation will be severe, shall be pointed out. Thus the designer can find out whether dangerous rolling is to be expected or not.

One of the points indicating that serious rolling may occur is the narrowness of the sea spectrum. Under unfavourable conditions the spectrum, referred to a coordinate system moving with the ship, becomes so narrow that there is only one dominating frequency. In such a case the effect of an irregular sea is nearly the same as the effect of a regular sea and the problem can be treated as shown in Section 4. The transformation of the sea spectrum from a fixed to a moving system of coordinates is necessary because the ship responds to the seaway with the frequency of encounter ω_E . In the top graph in Figure 6 the corresponding frequencies are plotted:-

- a) Following sea ($c \geq V$) :-

$$\frac{V}{g}\omega = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{V}{g}\omega_E}$$

- b) Following sea ($c \leq V$) :-

$$\frac{V}{g}\omega = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{V}{g}\omega_E}$$

- c) Head sea (dotted line) :-

$$\frac{V}{g}\omega = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{V}{g}\omega_E}$$

The formula for the conversion of the spectra can be deduced by taking into account that corresponding parts of the spectra contribute equal amounts to the energy of the seaway:-

$$S_{\phi}(\omega) \cdot \delta\omega = S_{\phi}(\omega_E) \cdot \delta\omega_E$$

or

$$S_{\phi}(\omega_E) = \frac{d\omega}{d\omega_E} \cdot S_{\phi}(\omega)$$

The results of the differentiations are:-

- a) Following sea ($c \geq V$) :-

$$\frac{d\omega}{d\omega_E} = \frac{1}{\sqrt{1 - 4 \frac{V}{g} \omega_E}}$$

- b) Following sea ($c \leq V$)
c) Head sea

$$\frac{d\omega}{d\omega_E} = \frac{1}{\sqrt{1 + 4 \frac{V}{g} \omega_E}}$$

A plot of the factor of conversion against frequency of encounter is given in the lower graph in Figure 6. From the curves for following seas (solid lines) and head seas (dotted line), it follows that the shape of the transformed spectra will differ considerably. If the ship moves in the direction of the waves we obtain at frequencies of up to $\omega_E = 0.25 \text{ g/V}$ growing spectral ordinates because of the rising factor of conversion and the normally dominating wave velocities of $c > V$. If the ship moves head on into the waves, however, the spectral ordinates will be reduced because the conversion factor is smaller than unity. Since the energy of the seaway does not change by the transformation the areas of the spectra are the same. The consequence is that in the first-mentioned case the transformed spectrum is a rather narrow one (an example is shown in Figure 7). Therefore, the conclusion can be drawn that in irregular following seas rolling may become nearly as severe as in regular longitudinal waves. It is to be assumed that this is the main reason why more ships capsized in following seas than in head or beam seas.

The narrowest spectrum imaginable will be attained when the factor $d\omega/d\omega_E = \infty$ must just be applied to the maximum ordinate of the original spectrum. In fully developed seas the highest spectral density is to be expected in the region of

$$\omega_1 = \frac{c}{\sqrt{\bar{H}_{1/3}}}$$

where $c = 1.258 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}$ and $\bar{H}_{1/3} =$ average of the 1/3 highest waves (significant wave height). This is exactly

true if the actual seaway complies with the formula of Pierson and Moskowitz, Reference (5):-

$$S_{\zeta}(\omega) = \frac{a}{\omega^5} \cdot e^{-\frac{b}{\bar{H}_{1/3}^2 \cdot \omega^4}}$$

where $a = 0.78 \text{ m}^2 \text{ s}^{-4}$ and $b = 3.136 \text{ m}^2 \text{ s}^{-4}$.

The corresponding frequency to the conversion factor of $d\omega/d\omega_E = \infty$ is $\omega_2 = 0.5 \text{ g/V}$ (from Figure 6).

Hence, in order to avoid a spectrum of extreme narrowness, it must be seen that there is an inequality between ω_1 and ω_2 :-

$$\frac{c}{\sqrt{\bar{H}_{1/3}}} \neq \frac{0.5g}{V}$$

or

$$V^2 \neq 57.4 \text{ km}^2 \text{ m}^{-1} \cdot \bar{H}_{1/3}$$

A plot of unfavourable combinations of $\bar{H}_{1/3}$ and V is given in Figure 8.

It is of some practical interest that risk of capsizing in irregular following seas may be as great as in regular longitudinal waves even if the significant wave height is small. As can be seen from Figure 8, the corresponding speed of the ship then must also be low. Ships operating under such conditions (e.g. coasters) are therefore endangered to a greater extent than ships of higher speed. A further reduction of safety of these generally small ships is caused by the fact that there are more short than long waves. Hence, they will meet wave lengths which are equal to ship length relatively often. Since, as a rule, the differences between the righting-arm curves will be rather large if $\lambda \approx L$, there is a greater probability of these ships capsizing.

As a result of these considerations, the conditions for the occurrence of unfavourable situations can be formulated as follows:-

- a) Provided that there are differences between the righting moments in the crest and trough condition, dangerous

rolling is to be expected if the transformed spectrum is narrow and if the corresponding wave length of the predominating frequency $\omega = 0.5$ g/V is more or less equal to the ship's length. In deep water:

$$\lambda = 2\pi g/\omega^2$$

If $\omega = 0.5$ g/V we obtain:

$$\lambda = 8\pi V^2/g$$

As $\lambda \approx L$ we obtain as a criterion for an unfavourable situation

$$\sqrt{8\pi} \cdot V \approx \sqrt{gL}$$

or Froude number

$$F_n \approx 0.2$$

- b) An essential condition for an excitation by a following sea is the existence of a natural circular frequency ω_ϕ reaching a value in the vicinity of integral multiples of $\omega_E/2$ ($\omega_E = 0.25$ g/V = predominating frequency of encounter). If $F_n \approx 0.2$ (or $V \approx 0.2 \sqrt{gL}$) following natural periods of roll should be avoided:

From

$$\omega_\phi = \frac{1}{2}\omega_E = \frac{0.25g}{2V} = \frac{0.25g}{0.4 \sqrt{gL}}$$

$$= 0.625 \sqrt{\frac{g}{L}}$$

follows:

$$T_\phi \approx 10 \cdot \sqrt{\frac{L}{g}}$$

From

$$\omega_\phi = \omega_E$$

follows:

$$T_\phi \approx 5 \cdot \sqrt{\frac{L}{g}}$$

From

$$\omega_\phi = \frac{3}{2}\omega_E$$

follows:

$$T_\phi \approx 3.35 \sqrt{\frac{L}{g}}$$

- c) As mentioned previously, in fully developed seas the transformed spectrum will be extremely narrow if $V/\bar{H}_{1/3}$ is about $57.4 \text{ km}^2 \text{ m}^{-1}$ or 15.2 m s^{-2} . The spectral density is then concentrated at

$$\begin{aligned} \omega_E &= \frac{0.25g}{V} = \frac{0.25g}{\sqrt{15.2 \text{ ms}^{-2} \bar{H}_{1/3}}} \\ &= \frac{0.63 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}}{\sqrt{\bar{H}_{1/3}}} \end{aligned}$$

Unless the differences between the righting moments are small, rolling motion may become unstable if:

$$\omega_\phi = \frac{1}{2}\omega_E = \frac{0.315 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}}{\sqrt{\bar{H}_{1/3}}} \quad \text{OR}$$

$$T_\phi = \frac{2\pi}{\omega_\phi} = 20 \text{ m}^{-\frac{1}{2}} \text{ s} \cdot \sqrt{\bar{H}_{1/3}}$$

$$\omega_\phi = \omega_E = \frac{0.63 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}}{\sqrt{\bar{H}_{1/3}}} \quad \text{OR}$$

$$T_\phi = \frac{2\pi}{\omega_\phi} = 10 \text{ m}^{-\frac{1}{2}} \text{ s} \cdot \sqrt{\bar{H}_{1/3}}$$

$$\omega_\phi = \frac{3}{2}\omega_E = \frac{0.944 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}}{\sqrt{\bar{H}_{1/3}}} \quad \text{OR}$$

$$T_\phi = \frac{2\pi}{\omega_\phi} = 6.66 \text{ m}^{-\frac{1}{2}} \text{ s} \cdot \sqrt{\bar{H}_{1/3}}$$

One of these results was already achieved formerly by Grim. In Reference (6) he points out that in following seas dangerous rolling motions may be built up if

the Froude number is equal to $F_n = 0.2$ and the natural period of roll equal to $T_\phi = 5 \cdot \sqrt{L/g}$.

The mode of calculation, however, was quite different and more complicated.

6. EVALUATION OF SAFETY IN FOLLOWING SEAS

Since capsizing as a result of rolling chiefly occurs in following or quartering seas, it would also be most desirable to get statistical information about the roll amplitudes for these special sea conditions. But unfortunately, an appropriate method does not exist. Therefore it is all the more important to observe the advice and relationships demonstrated in Sections 4 and 5. In particular, the simple formulae of Section 5 often clearly indicate whether changes or modifications of the design are advisable or not. Hence, for all practical purposes a closer consideration of rolling is only necessary if a definite judgement cannot be given because an application of the aforementioned formulas leads to different results.

In recent years several approaches were made to gain a better insight into the problem of evaluating the probability of capsizing in following seas (Reference (7), (8) and (9)). A final solution, however, could not be found though some of the experimental and theoretical results represent remarkable progress. Among other things, it was possible to develop new criteria which are also applicable to those ships whose degree of safety cannot be estimated by qualitative considerations. One of these criteria will be discussed in the following (Reference (9)).

As pointed out by Grim a long-crested irregular sea can always be approximated for a short time $\delta t = T$ and within a given range of length $\delta x = L$ by a harmonic standing wave whose amplitude η_∞ is random, Reference (6). This approximation is obtained by two steps:

1) Replacement of the irregular wave pattern by a sine wave within the extent of the ship's length (top diagram of Figure 9). Its amplitude η is determined by the condition that the sum of the squared deviations shall be as small as possible. The result is an expression for η being similar to that for the elevation of the sea surface ξ . From this the conclusion can be drawn that the statistical properties of η and ξ are the same. Thus, the dis-

tribution of the amplitudes η must be normal and for the maximum amplitudes η_{max} a Rayleigh distribution can be assumed.

2) Replacement of the record of successive values η , which would be obtained by determining η at different times at the moving location of the ship's midlength, by a sine wave (Figure 9, lower diagram). This means that the ship can be imagined for a limited time T to be in a wave with a harmonically varying amplitude ("standing" wave). It can be shown that by an analogous procedure an expression for the amplitude η_∞ of the standing wave is obtained which is similar to that of η and ξ and that the normal distribution must also be valid for η_∞ .

A rough idea of the degree of excitation can be obtained by calculating the rolling motions in standing waves of different heights. As the righting-arm curves generally do not follow a linear function, the equation of motion must be integrated by one of the known graphical or numerical approximation methods. If damping is neglected the equation is reduced to

$$T' \frac{d^2\phi}{dt^2} + \Delta \cdot h(\phi, t) = 0$$

or

$$\frac{d^2\phi}{dt^2} = - \frac{g}{k_{1/2}} \cdot h(\phi, t)$$

A graphical representation of a typical function $h(\phi, t)$ is given in Figure 10.

By systematic calculations, the critical initial angles of inclination $\phi_{0, crit}$ can be determined which should not be exceeded in a dead centre of the rolling motion because otherwise the motion may be built up to amplitudes from which the ship cannot return to an upright position since water is running into openings, cargo is shifting, or because similar reasons are causing a new state of static equilibrium. The critical initial angle of inclination $\phi_{0, crit}$ depends on the amplitude η_∞ of the standing wave. As shown in Figure 11 it generally decreases rapidly with growing wave heights.

Due to the single-valued association of $\phi_{0 \text{ crit}}$ to η_{∞} it is possible to extend Grim's approximation in such a way that at any time not only the existence of a fictive amplitude η_{∞} but also the existence of a fictive angle $\phi_{0 \text{ crit}}$ can be assumed. According to the definition of $\phi_{0 \text{ crit}}$ the conclusion can be drawn that capsizing is to be expected as soon as a roll amplitude ϕ_M is reached which exceeds the fictive angle $\phi_{0 \text{ crit}}$ being present at this very moment. It depends to a high degree on the shape of the curve $\phi_{0 \text{ crit}}(\eta_{\infty})$ whether the condition $|\phi_M| > |\phi_{0 \text{ crit}}|$ is fulfilled or not. If in Figure 11 the $\phi_{0 \text{ crit}}$ axis is replaced by a ϕ_M axis we can see that the critical angle $\phi_{0 \text{ crit}}$ will more likely be exceeded in great wave heights than in small ones. A general calculation of the probability that $\phi_{0 \text{ crit}}$ will be exceeded is also possible provided that the two-dimensional distribution density of the standing wave amplitudes η_{∞} and the roll amplitudes ϕ_M can be set up.

Though the statement is not quite correct, it shall be assumed that the random variables η_{∞} and ϕ_M are independent. The distribution density $f(\eta_{\infty}, \phi_M)$ can then be obtained by multiplying the one-dimensional densities $f(\eta_{\infty})$ and $f(\phi_M)$:

$$f(\eta_{\infty}, \phi_M) = f(\eta_{\infty}) \cdot f(\phi_M)$$

As pointed out above, the wave amplitudes η_{∞} are distributed according to the Gaussian law:

$$f(\eta_{\infty}) = \frac{1}{\sqrt{2\pi m_0 \eta_{\infty}}} \cdot e^{-\frac{\eta_{\infty}^2}{2m_0 \eta_{\infty}}}$$

The distribution of the roll amplitudes ϕ_M , however, is uncertain and it can only be supposed that they will follow a Rayleigh distribution:

$$f(\phi_M) = \frac{\phi_M}{m_0 \phi} \cdot e^{-\frac{\phi_M^2}{2m_0 \phi}}$$

The resulting two-dimensional density is:

$$f(\eta_{\infty}, \phi_M)$$

$$= \frac{\phi_M}{m_0 \phi \sqrt{2\pi m_0 \eta_{\infty}}} \cdot e^{-\left(\frac{\phi_M^2}{2m_0 \phi} + \frac{\eta_{\infty}^2}{2m_0 \eta_{\infty}}\right)}$$

where

$m_0 \phi$ = variance of the amplitudes ϕ_M
(in design stage $m_0 \phi$ can be obtained by assessing the significant roll amplitude on the basis of the known values of similar ships:

$$m_0 \phi = \frac{1}{4} \bar{\phi}_M^2$$

$m_0 \eta_{\infty}$ = variance of the amplitudes η_{∞}
($m_0 \eta_{\infty}$ can be obtained by converting the sea spectrum S_{ω} into the corresponding spectrum $S_{\eta_{\infty}}$ and by determining the area under its graph:

$$m_0 \eta_{\infty} = \int_0^{\infty} S_{\eta_{\infty}}(\omega) d\omega$$

The imaginable combinations of values η_{∞} and ϕ_M as a whole can be divided into two groups, viz. in "safe" and "unsafe" combinations. The curve $\phi_{0 \text{ crit}}(\eta_{\infty})$ forms the boundary between these two groups. In a η_{∞} - ϕ_M -coordinate system any point lying under the curve $\phi_M = \phi_{0 \text{ crit}}$ represents a "safe" situation, whereas a point above this curve indicates that capsizing is to be expected. The knowledge of the density function $f(\eta_{\infty}, \phi_M)$ enables us to calculate the probability of the occurrence of a "safe" or "unsafe" couple of amplitudes η_{∞} and ϕ_M . We only have to draw the graph of this two-dimensional function and determine the volume of the space above the "safe" or "unsafe" region, Figure 12. The sum of these volumes must be unity since, according to supposition, one of these events occurs. Therefore it will be sufficient to determine only the integral taken over the "unsafe" region. The calculation of the corresponding probability P can be carried out with great accuracy because the volume is rather small ($P \ll 1$).

Though there are still some unsolved problems the probability P is an appropriate criterion for evaluating the danger of capsizing in a following sea. It takes into account the irregularity of the seaway as well as the parametric excitation. Experiences in practical application, however, could only be obtained to a small

extent. A rough idea of the range of the probability values P of actual ships can be obtained by two examples: a calculation of the capsizing index P for two small cargo vessels which capsized in a following sea, resulted in $P = 10^{-5}$ and $P = 10^{-6}$. For the moment, it may be recommended that the proposed index P shall not be greater than about 10^{-7} .

7. CONCLUSIONS

Though capsizing by rolling does not occur frequently, it would be a serious fault if in a new stability standard the dynamic behaviour of the ship in a seaway is disregarded or taken into account in an incorrect manner. Of course, for reasons of practicability, simplifications are necessary. One feasible way would be to prescribe in a special paragraph of the stability regulations that in all cases where the righting moments in the crest and trough condition differ widely and also the Froude number and period of roll (e.g. $F_n \approx 0.2$ and $T_\phi \approx 5\sqrt{L/g}$) indicate dangerous rolling, a probability criterion like the proposed capsizing index P must be applied. On the basis of this value it can then be seen whether the risk of capsizing by rolling can be considered as small enough or not. The main advantage of such a rule would be that, in order to be exempted from the application of the probability criterion, more attention would be paid to the factors being responsible for the rolling behaviour in a following sea. For instance, the influences of the shape of the hull and location of the superstructures, as demonstrated in Figure 5, would no longer be disregarded.

It was pointed out that rolling in beam seas is generally less dangerous than in following seas. Therefore this kind of excitation may be neglected also in future stability provisions. This does not mean that the effect of transverse waves can be ignored in the design stage. For vessels which often embark passengers or take over cargo when lying in the roads, it is particularly important to avoid troublesome rolling in beam seas. If the maximum wave length and the maximum wave steepness are known, which must be expected in a given sea territory, it is useful to draw a diagram as shown in Figure 2. From it the values GM can be obtained which must be observed if a given amplitude of roll is not to be exceeded.

At the standard of knowledge which has been reached today, it cannot be proposed to change the existing stability

criteria radically. Since until now the distribution functions of the heeling moments are unknown and also the method of including them in a comprehensive probability criterion, and the method of comparing the righting and heeling moments in the still water, crest and trough condition should be maintained. A supplementation by adding a rule which will consider the danger of capsizing by rolling in waves, however, is desirable and also possible.

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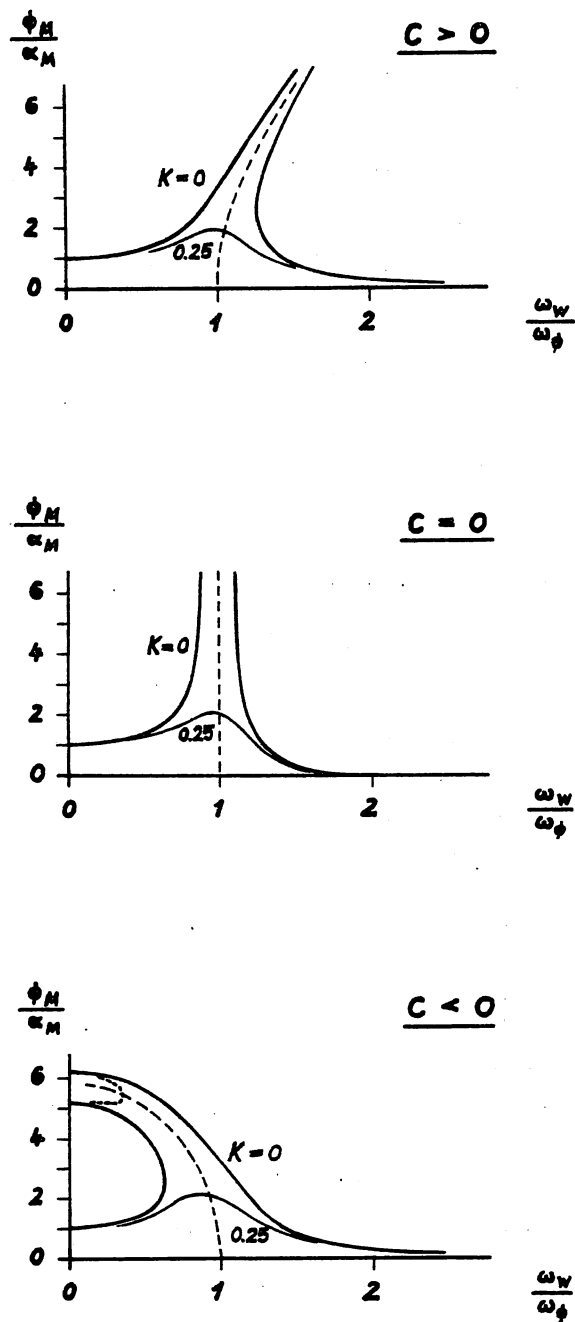


Figure 1

Limitation of rolling in beam seas
by avoiding unfavourable values \overline{GM}

Example: Unresisted rolling ($K=0$) of a ship with a linear restoring moment ($C=0$) and a breadth of $B=10\text{ m}$ ($k'=0.4\text{ B}=4\text{ m}$).

If at a given wave steepness $(H/\lambda)_0$ or maximum wave slope $\alpha_{H_0} = \pi \cdot (H/\lambda)_0$, a certain angle of roll ϕ_{H_0} shall not be exceeded the magnification factor must be smaller than

$$\frac{1}{|1 - (\frac{\omega_w}{\omega_\phi})^2|} < \frac{\phi_{H_0}}{\alpha_{H_0}}$$

Since $\omega_w = \sqrt{\frac{2\pi g}{\lambda}}$ and $\omega_\phi = \sqrt{\frac{g \cdot \overline{GM}}{k'^2}}$ we obtain

$$|1 - \frac{2\pi k'^2}{\lambda \cdot \overline{GM}}| > \frac{\alpha_{H_0}}{\phi_{H_0}}$$

There are two solutions: $\frac{2\pi k'^2}{\lambda \cdot \overline{GM}} < 1 - \frac{\alpha_{H_0}}{\phi_{H_0}}$ and $\frac{2\pi k'^2}{\lambda \cdot \overline{GM}} > 1 + \frac{\alpha_{H_0}}{\phi_{H_0}}$

$$\text{or} \quad \frac{2\pi k'^2}{\lambda (1 - \frac{\alpha_{H_0}}{\phi_{H_0}})} < \overline{GM} < \frac{2\pi k'^2}{\lambda (1 + \frac{\alpha_{H_0}}{\phi_{H_0}})}$$

In the diagram the areas indicate the range of unfavourable values \overline{GM} as a function of λ and H/λ . Within these ranges amplitudes of roll $\phi_H > 6^\circ$ must be expected.

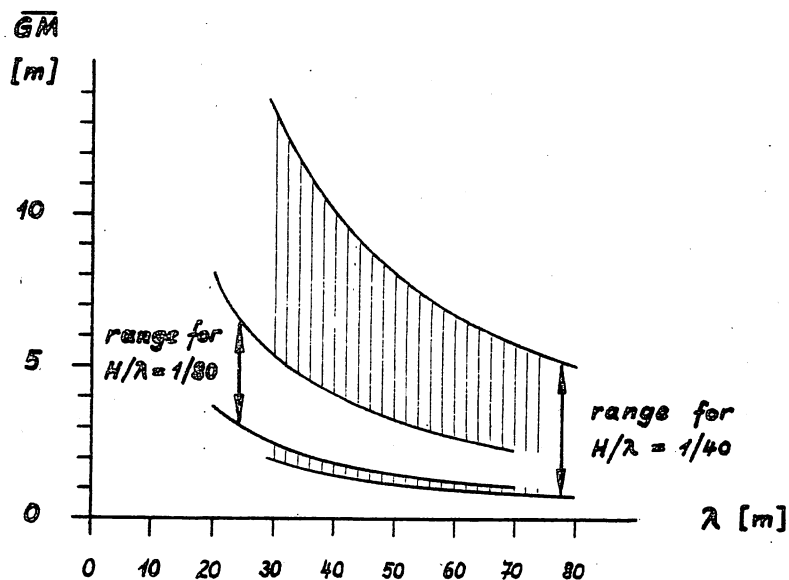


Fig. 2

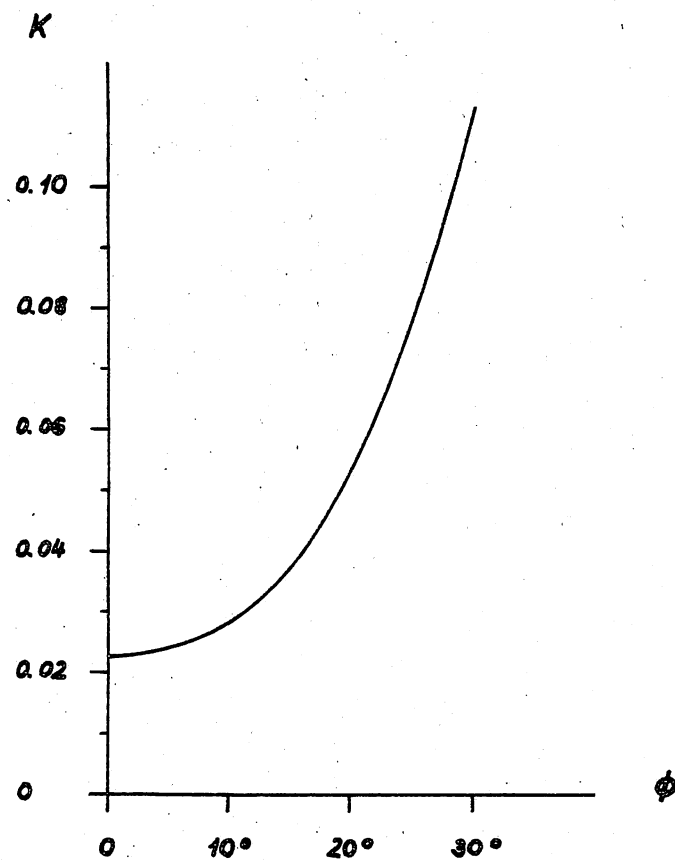


Fig. 3

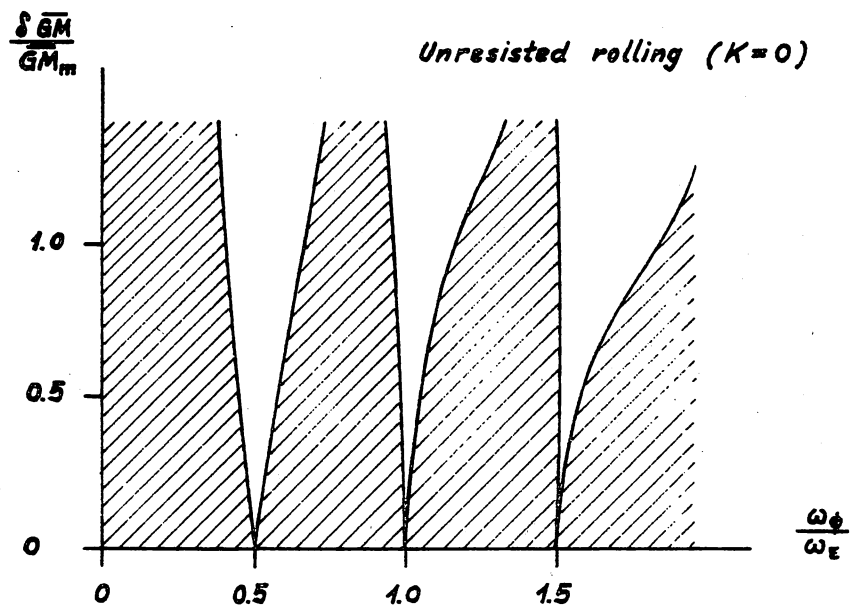
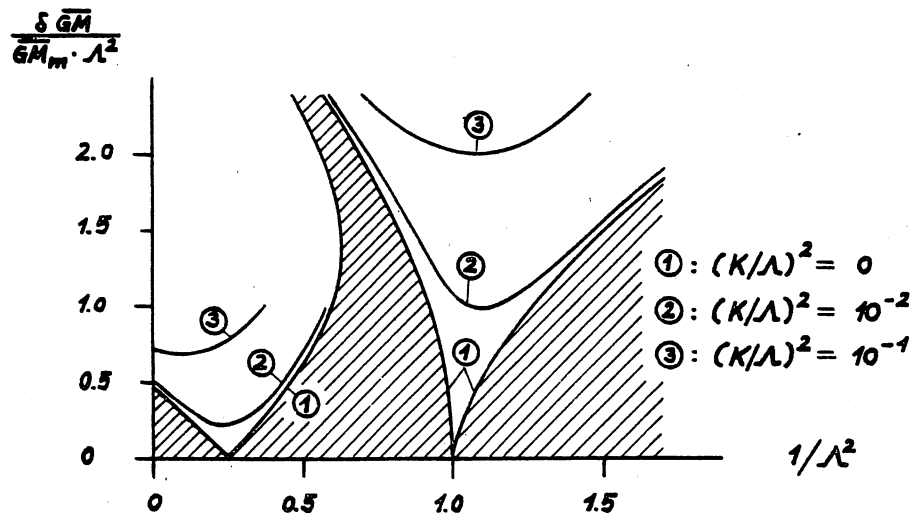
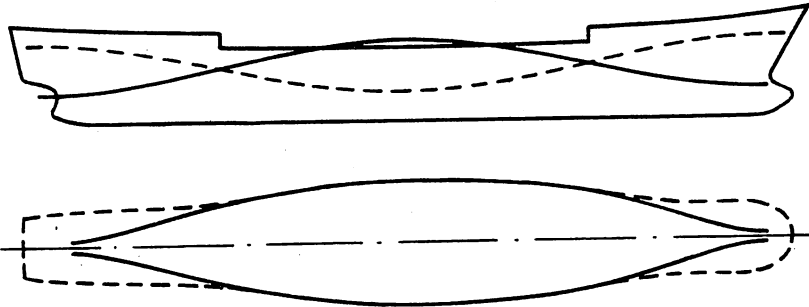


Fig. 4

1. Ship with considerably fluctuating stability curves :



2. Ship with only small fluctuation of stability :

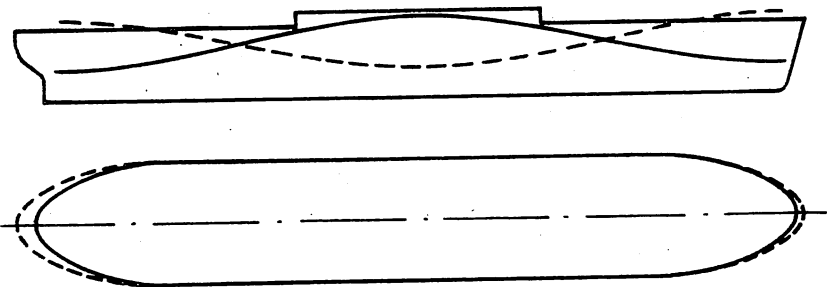


Fig. 5

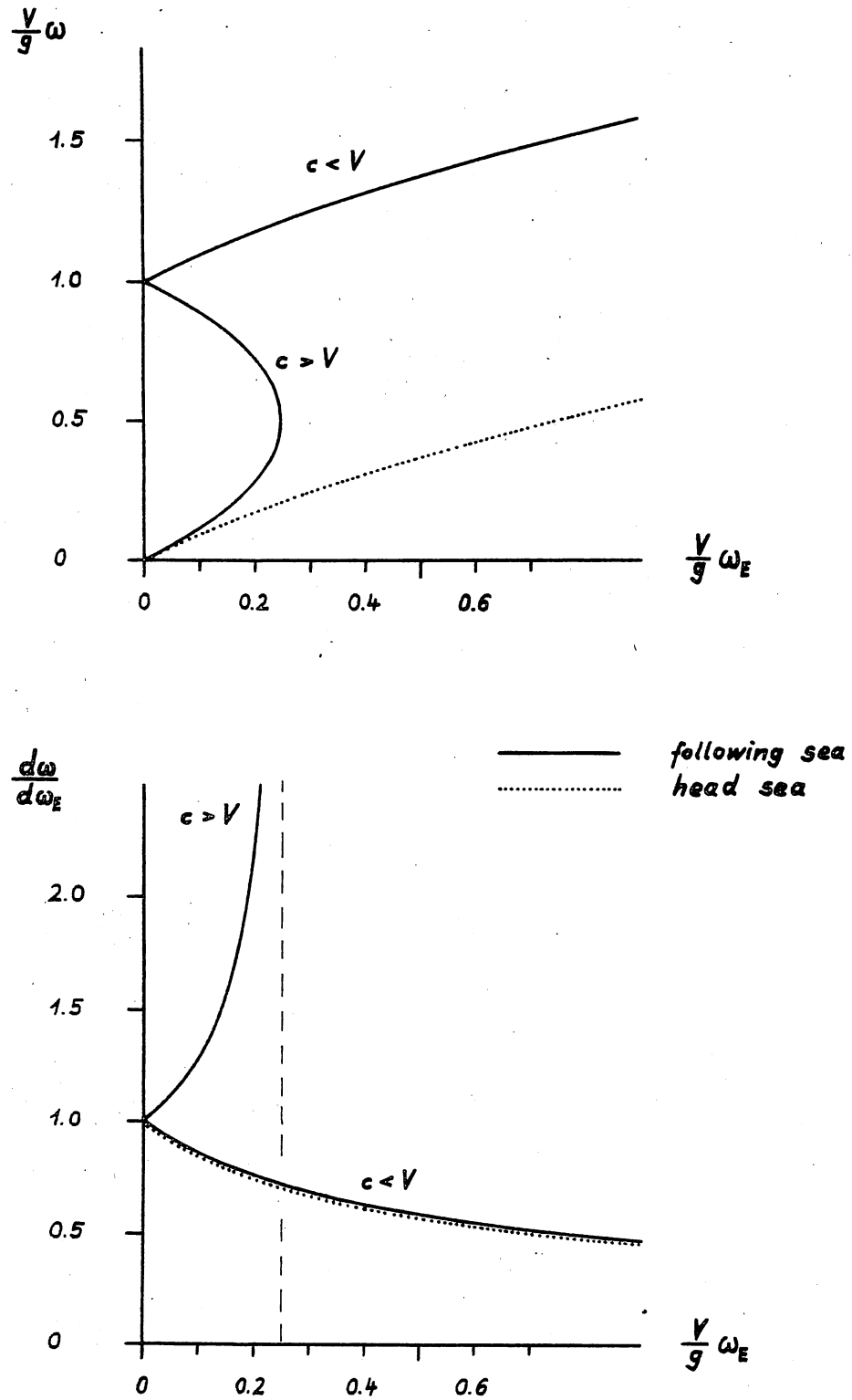


Fig. 6

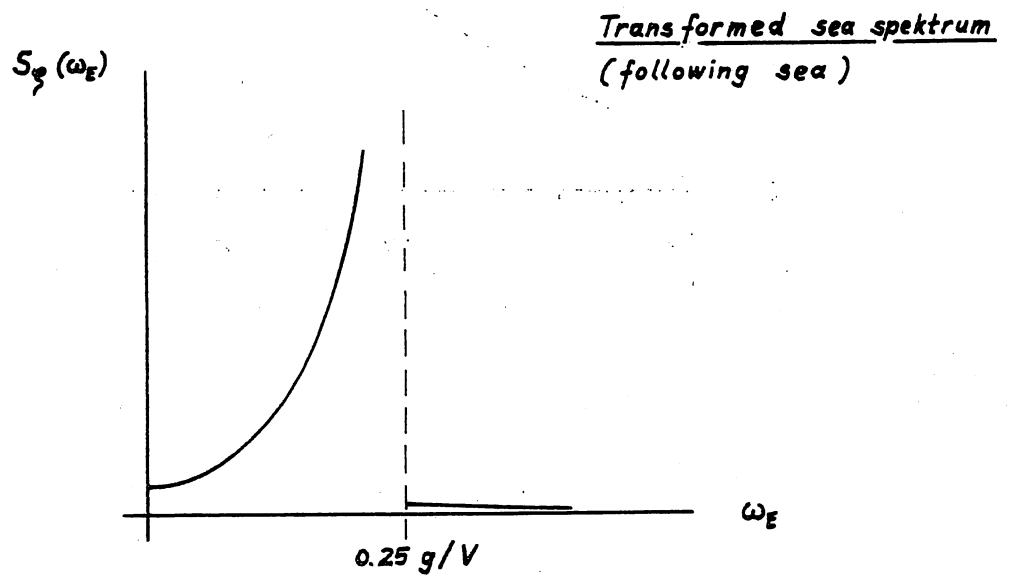
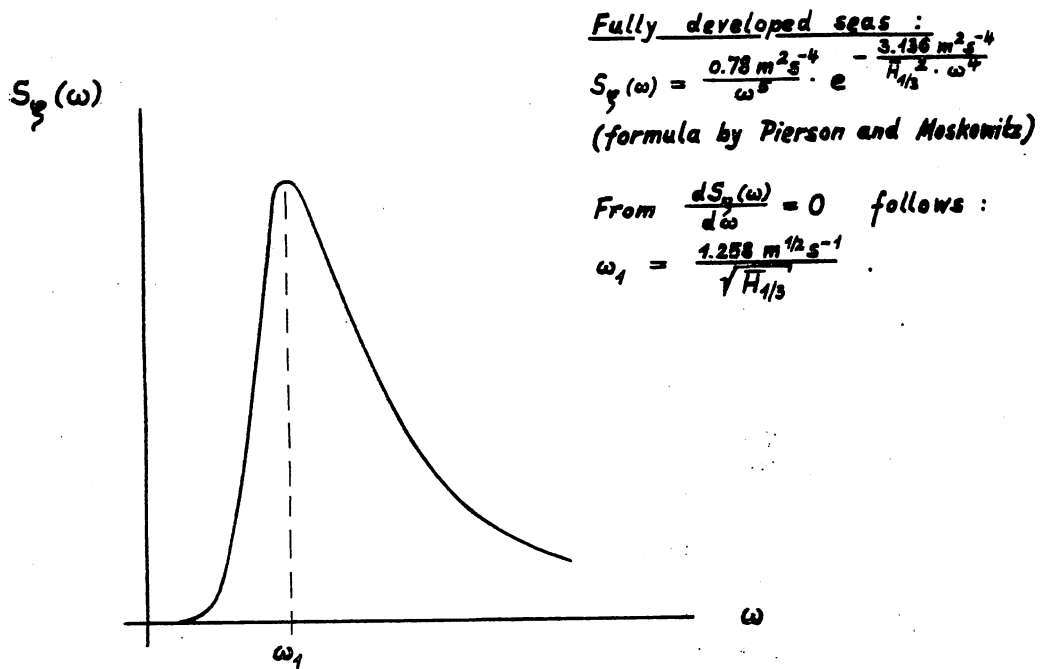


Fig. 7

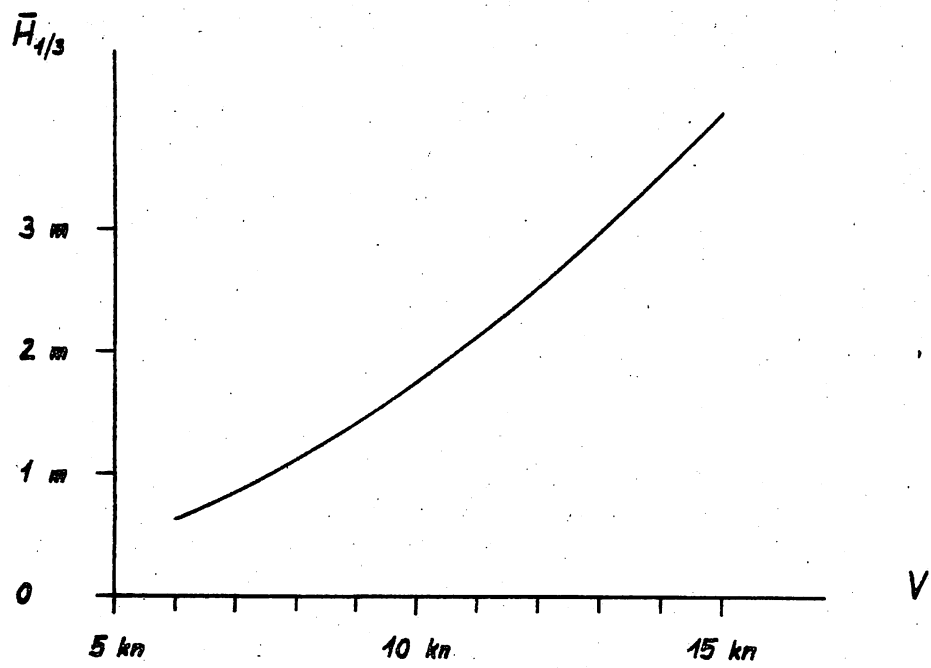


Fig. 8

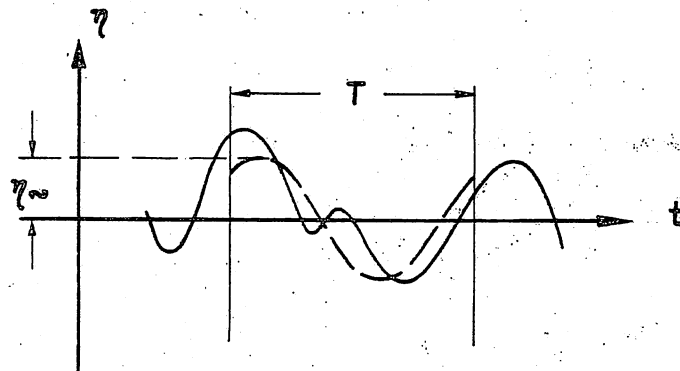
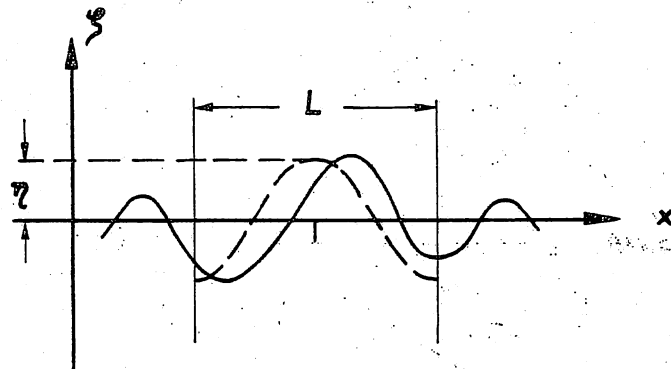


Fig. 9

Curves of equal righting arms :

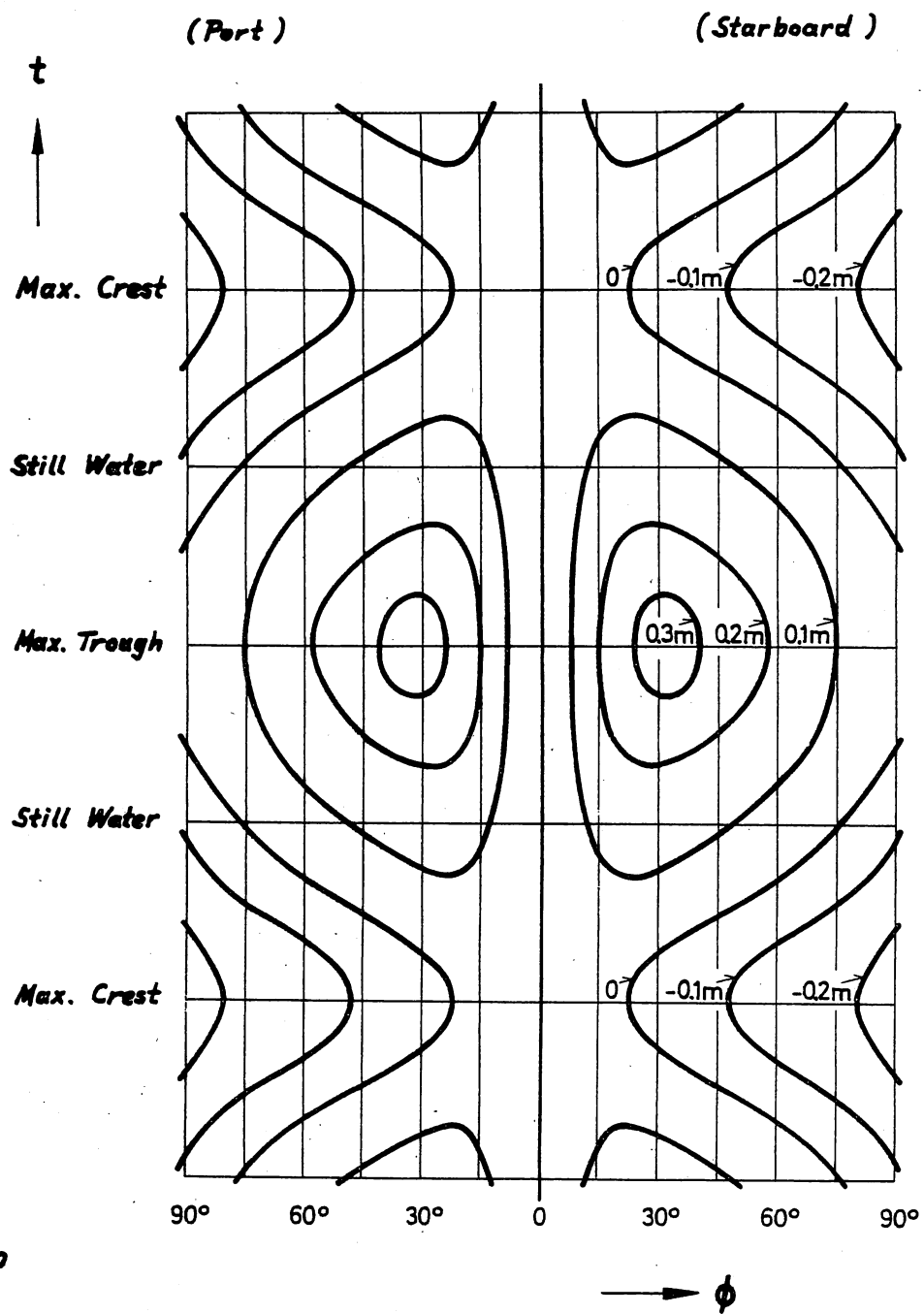


Fig. 10

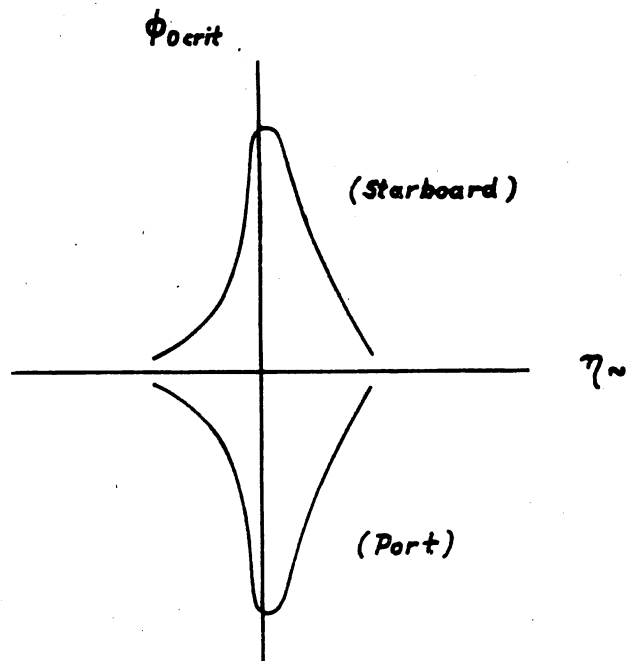


Fig. 11

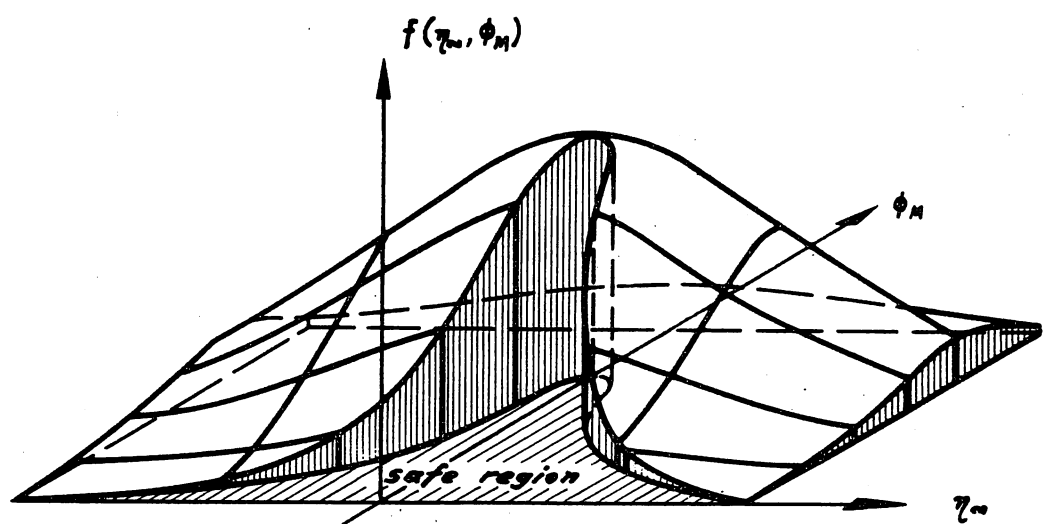


Fig. 12

ON THE STATISTICAL PRECISION OF DETERMINING THE PROBABILITY
OF CAPSIZING IN RANDOM SEAS

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1. INTRODUCTION

In studying the capsizing of a ship in a random seaway we are faced with an event which appears rarely and unpredictably but which must be accounted for in order to survive at sea. Since 1961, model tests have been carried out on the study of extreme roll and capsizing under severe irregular sea conditions, lateron supported by time domain computer simulation. For experiments, relatively large testing areas even at model scale are required, in particular for quartering seas, in order to gather a sufficient sample of rare extreme motion records. This led to open water model tests.

Determining the capsizing probability in random seas requires an enormous amount of running distance and testing time, in order to ensure sufficient statistical precision. This was already pointed out by Krappinger 1962 at the Symposium on Ship Theory at the Institut für Schiffbau in Hamburg. He therefore recommended evaluating capsizing tests in more detail, i.e. which parameters affect the capsizing rather than trying to measure the capsizing rate.

However, it is worthwhile to be aware of the statistical relations involved. In this paper, we want to evaluate the classical hypothesis testing in statistics applied to the random capsizing problem. It may help for the appropriate design of stochastic experiments of rare events, and of stochastic calculations as well. Furthermore, the hypothesis testing will show the difficulties in setting up safety standards based on only a few extreme events.

2. PROBABILITY BACKGROUND FOR
RANDOM EXPERIMENTS

Performance tests with a free running ship model in an irregular seaway in open waters, as well as corresponding time domain computations, carry all signs of a random experiment which can be described by methods of probability theory and can be evaluated by means of mathematical statistics. Given a ship model with certain determined parameters (i.e. displacement, freeboard, centre of gravity, speed and wave heading) because of the randomness of the seaway and of the random initial phase conditions at the beginning of a model run, we do not know the outcome of any model run in advance. Even though we are close to the point where a capsizing of the model is very likely to occur, since we are particularly concerned about capsizing and will have ballasted the model appropriately, we will not be able to predict the outcome of a particular experiment; that is, whether on this run the model will or will not capsize and if so at what point during the run. On the other hand, what does a "very likely" occurrence of an event mean? Mathematical statistics provide the tools to solve this problem and to bring about a quantitative description of the possible outcome of this kind of experiment - see the Appendix for some definitions.

3. PRECISION AND TESTING

From our random experiments we want to derive some numerical characteristics of the samples. By definition,

we consider a set of k samples of a random vector \mathbf{x} with the size N , as being a random choice out of all possible events in the sample space Ω . The calculation of functions $g(\mathbf{x})$ is a well-known statistical procedure, generally called estimation, providing the best statistical result from a given sample. In evaluating the results of this procedure it is necessary to consider the following questions: is this sample representative of the population we want to look at? How precise is the estimation relative to the true population? We can control the precision of the estimate, i.e. how close our estimate is to the true parameter in the population, partly through the design of the experiment.

The basic question here is, how many trials, i.e. model runs and of what duration, have to be performed, even if model similarity and accuracy of the measurements are guaranteed through appropriate design of the physical model and the instrumentation. Therefore we refer to the design of an experiment if we mean the appropriate choice of the model environment and the way we are sampling, i.e. how we choose the samples, here in particular, for what length of time T we run the model, and how many runs we perform.

Some questions, undoubtedly, will arise in planning and performing our random experiments:

- i) If we run the model N times and observe capsizing in n of these runs, what can we generally say about safety from capsizing under the given conditions?
- ii) How many model runs will we need to conduct in order to allow the conclusion with a high degree of certainty that the model or ship under the given conditions will not be very likely to capsize? In other words, under which conditions will we consider the ship safe from capsizing?
- iii) Say we observe a series of capsizings in the course of the experiments which let us intuitively think of an unsafe ship. Thus it is clear that we would not recommend the corresponding conditions for a ship, as this would obviously result in a large danger from capsizing. Now we improve the conditions of the model somewhat by lowering the centre of gravity of the ship model, thus increasing the righting moment, meanwhile leav-

ing all other parameters unchanged. Repeating the same test series now we might still observe some capsizings, but not as often as in the series before. Obviously the ship model is safer now, but how much safer? How can we tell when the ship will be safe enough? We will now go into more detail to answer these relevant questions.

4. TESTING OF CAPSIZING HYPOTHESIS FROM RANDOM EXPERIMENTS

Let us consider only two possible outcomes of one model run of length T , capsizing or noncapsizing. For capsizing experiments this is the simplest information we can get from the model runs, without looking into other statistical events with more detailed information. Then our experiment can be seen as a binomial trial, where we shall denote the capsizing by C and the noncapsizing by NC (failure of capsizing). If we repeat the model runs N -times under comparable conditions independently, we have performed N binomial trials with an unknown capsizing probability, say θ . Thus we have a sample of size N from the population of all possible outcomes.

Note : As we assume independence of all samples, drawn from a population with finite sample space Ω , we select the individual sample points $\omega_i \in \Omega$ successively at random with replacement. Without replacement the population would clearly be altered by any drawn sample, thus making them dependent. Another case would result, for example, if we consider the probability of capsizing of a fleet of say N ships each running under the same extreme seaway conditions for T hours in their lifetime, but after loss of n ships from capsizing they will not be replaced, thus leaving the fleet with size $(N-n)$.

For our capsizing probability model let us consider a binomial trial with replacement. It is convenient to use the following definition:

If A is any event, the random variable I_A is defined by

$$I_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where I_A is called the indicator of the event A.

Using the indicator function for N binomial trials, we let x_i be the indicator of the capsizing event, that is, capsizing on the i th trial. Then the $x_i, i = 1, \dots, N$ are a sample from the binomial distribution, which assigns

probability θ to x_i equal 1:

$$P[x_i = 1] = \theta$$

and probability $(1 - \theta)$ to x_i equal 0:

$$P[x_i = 0] = 1 - \theta$$

Then clearly the number of capsizings occurring in repetitions of the trial are expressible as the sum

$$n = \sum_{i=1}^N x_i \quad (2)$$

and the relative frequency of capsizings which occurred is

$$\bar{x} = n/N = 1/N \sum_{i=1}^N x_i \quad (3)$$

This \bar{x} is a sufficient statistic. That means we might as well keep this statistic only, thereby reducing the data with no loss of information.

It can be shown that \bar{x} is also the maximum likelihood estimate (MLE) as well as the uniformly minimum variance unbiased estimate (UMVU) of the unknown parameter θ , the true probability of capsizing. (We generally denote the unknown parameter by θ , so in our special case now θ is the unknown capsizing probability. The statistic \bar{x} in equation (3) is our estimate of θ).

Then the probability of noncapsizing is $(1 - \theta)$. If C_n is the event, that in our sample of n model runs $\omega_i \in \Omega$ exactly n capsizings have occurred, the probability of C_n is given by the binomial density function

$$P(C_n) = \binom{N}{n} \theta^n (1 - \theta)^{N-n}, \quad n = 0, 1, \dots, N \quad (4)$$

with $0 \leq \theta \leq 1$

where $\binom{N}{n} = N! / n! (N-n)!$

It is important to understand that we imagine there is some true θ involved in the process we are looking at determined by the physical nature of the problem but we are faced with the uncertainties involved in the practical estimation of θ .

We want to test the significance of experimental results for the parameter θ . Generally this can be done by means of a testing hypothesis at a certain confidence level.

The hypothesis H_0 in a mathematical form is

H_0 : Given P_θ on R for $\theta \in \Theta$, and given $\Theta_0 \subset \Theta$, then the true θ is in Θ_0

Which hypothesis we are going to test depends on the problem and our intention. In the case of capsizing we want to make sure that the ship is safe. In other words, we want to be confident that the true probability of capsizing, θ , is at a low level, say in the interval $[0, 0.05]$. Therefore it is appropriate to choose the following hypothesis for the capsizing probability

H_0 : the ship is safe, $0 \leq \theta \leq \theta_0$

versus

H_1 : the ship is not safe, $\theta_0 < \theta \leq 1$

We will test this hypothesis with the experimental statistic \bar{x} . We will attempt to avoid what are known conventionally at Type I and Type II errors.

5. TYPE I ERROR

We consider the number of capsizings unacceptably high if c_α capsizings or more occur out of a total of N trials. In other words, with c_α

capsizings we will accept the hypothesis that the ship is not safe. So we test how bad things can be. Where indeed the hypothesis that the ship is safe is wrongly rejected we are committing an error of type I.

The test procedure is (introduced by Neyman and Pearson) : reject the hypothesis H_0 if

$$P_\theta(\text{reject}) \leq \alpha \text{ for all } \theta \in \theta_0 \quad (5)$$

In other words, we use a test in which the probability of rejection is $\leq \alpha$, where α is the level of significance, also called confidence level.

The smallest number c_α yielding a level α test is called the level α critical value, which leads to the test of the form

$$\text{Reject } H_0 \text{ if } \bar{x} \geq c_\alpha \quad (6)$$

We know from equation (3) that \bar{x} is the natural estimate of θ , where θ is the true proportion of potential capsizings out of N runs.

We can find c_α from tables for the binomial cumulative distribution function:

$$c_\alpha = B_\alpha(N, \theta_0) \\ = \sum_{n=0}^N \binom{N}{n} \theta^n (1-\theta)^{N-n} \quad (7)$$

See a graph of B for some parameters in Figure(2).

We may restrict our test to level $\alpha = .05$. Also commonly used is $\alpha = .01$ which asks for more precision in the estimation and therefore will have a wider confidence range

$$c_{\alpha=0.01} > c_{\alpha=0.05} \quad (8)$$

Figure 3 shows a graph of c_α versus N .

Resulting c_α of this hypothesis testing according to Figure 3 (values read from tables, see Figure 2) yield θ_0 and N with λ_c the next largest

natural number to $c_\alpha * N$

$$\lambda_c' > c_\alpha * N \quad (9)$$

For larger N the binomial distribution can be approximated by the normal distribution, which yields [1] :

$$c_\alpha(N, \theta_0) \\ = N\theta_0 + \frac{1}{2} + \Phi^{-1}(1-\alpha)\sqrt{N\theta_0(1-\theta_0)} \quad (10)$$

$$\text{for min } \{N\theta_0, N(1-\theta_0) \geq 5\}$$

where Φ^{-1} inverse normal cumulative distribution function.

The resulting λ_c' , see Figure 3, might be interpreted according to equation (6) as follows. If the sum of capsizings out of N trials

$n = \sum_{i=1}^N x_i$ is equal to or more than the critical value $\lambda_c' = c * N$, we reject the hypothesis $H_0 : \theta \leq \theta_0$.

For $\theta_0 = .05$ we might consider the hypothesis H_0 equivalent to the statement: the ship is safe. This means we allow only a small amount, say 5%, of capsizings in order to consider the ship as being safe from capsizing. Thus, for example, if two capsizings occur in only five runs, we consider the ship unsafe at the confidence level $\alpha = .05$. But if one capsizing in five occurs, we have no evidence for rejecting H_0 : the ship is safe.

At size 0.01 we require three or more capsizings out of five in order to consider the ship unsafe, but we allow two capsizings out of five runs and still consider the ship as being safe. From that we can see already that with only five runs the estimation of the unknown parameter θ , the true capsizing probability, is not very precise. This result leaves us dissatisfied if we learn that the there is no evidence to reject θ being only 0.05, i.e. a very small probability for capsizing, if we have a critical capsizing number of two respectively three out of five total runs, where we would estimate a θ of .4 and .6 respectively.

Therefore this test shows clearly that the estimation of θ using only five runs is not very precise, since from two capsizings we calculate $\bar{x} = 0.4$, but the test shows that even with this estimate the true θ might be as small as 0.05.

The only possible way to improve the precision in estimating the capsizing probability is to increase the number of model runs N . From Figure 3 we see, for example, that for $N = 20$ the critical number of capsizings goes down to four for $\theta_0 = 0.05$ at level $\alpha = 0.05$.

Compared with the five runs above, now we might estimate $n = 3$, that is $\bar{x} = 3/20 = 0.15$ and there would be no evidence to reject the hypothetical $\theta_0 = .05$. So $\bar{x} = .15$ for $N = 20$ is much closer to $\theta_0 = 0.05$ than $\bar{x} = .40$ for $N = 5$.

On the other hand, the result of this test does not mean that we can accept the hypothesis the ship is safe, if we cannot reject it. This is obvious as two or three capsizings out of five appears to be rather a high capsizing rate. The test only says that e.g. with $N = 5$ trials, i.e. model runs, we do not have more precision for estimating the true θ . If we test for a higher θ , say $\theta_0 = .4$, then we would expect two capsizings in five runs on the average. But there is no evidence to reject $\theta_0 = .4$ if four capsizings in five runs occur which would let us estimate $\bar{x} = 4/5 = .8$ for θ .

If we cannot reject our hypothesis, though, from lack of more experimental results we might accept the hypothesis H_0 provisionally. In practice this is often done, but we should be very careful about it. Experience shows that H_0 is generally accepted when N is small and rejected when N is large. Applied to our capsizing problem, this says that we are more apt to conclude a ship being safe from capsizing at a small number of model runs even if this is not true. This stresses again the need for a fairly high number of identically distributed model runs in order to estimate the capsizing probability precisely.

6. TYPE II ERROR

So far we have been dealing with the probability of a type I error only. That is the error we commit if we reject the hypothesis which we should have accepted. We have limited the maximum probability of committing a

type I error by means of the size to a small amount.

Now let us consider the error type II, that is, the error we commit when we accept the hypothesis if we should have rejected.

To avoid a type II error, we accept the hypothesis H_0 if

$$P_{\theta}(\text{accept } H_0) \leq \beta \text{ for all } \theta > \theta_0 \quad (11)$$

We have denoted the critical function of the test by c_{α} . From c_{α} we can easily calculate the function β_{δ} of a test for type II error, called the power function:

$$\beta_{\delta}(\theta) = P_{\theta}(\text{accept } H_0) \text{ for all } \theta > \theta_0 \quad (12)$$

We get then for the one sided test for the probability of capsizing in N binomial trials

$$\begin{aligned} \beta_{\delta}(\theta) &= P_{\theta}(\bar{x} \geq c_{\alpha}) \\ &= P_{\theta} \left[\sum_{i=1}^N x_i \geq x^1 c_{\alpha} \right] \\ &= \sum_{j=c_{\alpha}}^N \binom{N}{j} \theta^j (1-\theta)^{N-j} \\ &= B_{\alpha}[N, \theta] \end{aligned} \quad (13)$$

For each θ , $\beta_{\delta}(\theta)$ is just the sum of the binomial distribution from the critical number of capsizings, $x^1 c_{\alpha}$, divided by the number of trials N .

The power function β_{δ} gives the probability of rejecting H_0 , if θ is indeed larger than θ_0 . With increasing number of trials N the test has more power (see Figure 4 through 6 for three different θ_0).

Practically, we want again to have as small a probability of committing error type II as possible, or a high probability of not committing an error type II, in other words, high power β . We choose again $1-\beta = .05$, $\beta = .95$.

The complement of the power function is called the operation characteristic

$$O.C. = 1 - \beta_S(\theta) \quad (14)$$

It gives the probability of accepting θ_0 if the true θ is some θ_1 according to equation (11). For $\theta_1 = \theta_0$ the power is smallest or the complementary O.C. is largest as it obviously should be.

Let us discuss the power function in one example, see Figure 4 for $\theta_0 = .05$. The power, i.e. the probability of rejecting is smallest at $\theta = \theta_0 = .05$. We recall that θ_0 was set by us earlier as the limit, that is we should reject if the true $\theta > \theta_0$.

For $\theta > \theta_0$, by definition we must have higher probability of rejection, i.e. higher power. Thus β_S is an increasing function in θ and approaches the ideal value 1. As the power increases, the probability of type III error decreases. The most desirable case is when β_S approaches 1 for θ close to θ_0 so we are sure that accepting θ_0 is a better estimate than any other $\theta_1 > \theta_0$. This is the case for large N or in other words the test has more power for larger N .

In other words, the power is a measure of how likely it is to mistakenly accept H_0 . Thus again, if we want higher precision for our estimate, we need to carry out more experiments.

7. POISSON AND EXPONENTIAL DISTRIBUTION

The statistical study of the capsizing phenomenon by applying the discrete binomial distribution alone does not leave us fully satisfied. We have treated the capsizing probability so far in model runs of duration T as the probability of only two outcomes, capsizing or ;not capsizing. There is no specific reference made to the chosen length of time, T , for the model run, so the results seem to be independent of T . However, the length of time we run a model obviously has some influence on whether the model will capsize or not. For example, if a model run is only one minute instead of seven minutes, and no capsizing occurs in this one minute, this study does not tell us anything about

a possible outcome in the next six minutes. Therefore we would like to know more of the capsizing probability with respect to time.

Let us think of a model run of length T as of N independent repetitions of independent identically distributed binomial trials in every Δt seconds:

$$\Delta t = T/N \quad (15)$$

Since we consider extreme large roll amplitudes which may include capsizing as "rare events", the probability p of capsizing in Δt is very small. The probability of the number of successes n in N binomial trials $\{X_N\}$ is

$$P_X \left\{ \sum_{i=1}^N x_i = n/N \right\} = b(n, p) \\ = \binom{N}{n} p^n (1-p)^{N-n} \quad (16)$$

For p small, $N \rightarrow \infty$, $0 < \lambda < \infty$, we get the Poisson distribution

$$P_X^{(n)} = (e^{-\lambda} \lambda^n) / (n!), \quad \lambda = Np \quad (17)$$

If we set the p proportional to the time interval Δt

$$p(\Delta t) = \mu \Delta t = \mu \frac{T}{N} \quad (18)$$

with $\Delta t \rightarrow 0$ we get from equation (17)

$$P \left\{ \sum_{i=1}^N x_i = n \mid \Delta t \right\} = p_T(n) \\ = \frac{e^{-\mu T} (\mu T)^n}{n!}, \quad \mu T = \lambda \quad (19)$$

$p_T(n)$ is the probability of n events occurring during the time interval $[0, T]$. Now the Poisson distribution allows us to count rare events on a time scale.

Mean and variance of the Poisson distribution are

$$E(\bar{x}) = \text{Var}(x) = \mu T = \lambda \quad (20)$$

We can derive the time elapsed until capsizing from the Poisson distribution. The probability that no capsizing has occurred in the time interval $[0, t]$ is from equation (19)

$$\begin{aligned} \text{Prob \{no capsizing in } [0, t]\} \\ = \text{Prob \{ } t_c > t \} = P_t(0) = e^{-\mu t}, \quad t \geq 0 \end{aligned} \quad (21)$$

The probability of capsizing in the time interval $[0, t]$ is just the complement

$$\begin{aligned} \text{Prob \{capsizing in } [0, t]\} = F_c(t) \\ = 1 - e^{-\mu t}, \quad t \geq 0 \end{aligned} \quad (22)$$

This is the cumulative distribution function for the time until capsizing.

We find the probability density function by differentiating $F_c(t)$

$$f_c(t) = \mu e^{-\mu t}, \quad t \geq 0 \quad (23)$$

This is called the exponential distribution.

The mean and variance are

$$E(c) = 1/\mu_c \quad (\text{MLE, but not UMVU})$$

$$\text{Var}(c) = 1/\mu_c^2 \quad (24)$$

Therefore the standard deviation is

$$s_c = \text{Var}^{-\frac{1}{2}}(c) = 1/\mu_c = E(c) \quad (25)$$

The negative exponential distribution is like the binomial and Poisson distribution a one parameter distribution, for only one parameter must be determined from the sample. It has one important feature, its "lack of memory" or "forgetfulness". This is obvious if we consider the derivation from both the binomial and Poisson distribution from which we have seen the time until the first failure as a summation of independent discrete time intervals. In other words, the probability distribution of the remaining time until capsizing does not depend on how long

the model has been running.

Even when there first seemed to be some restriction because of the independence of every time interval, this distribution function is very appropriate for long statistical model runs, where, for instance, the wave which passed the model some minutes earlier does not at all affect the roll of the model caused by another new wave or a wave group, but is only statistically related to it as a member of an independent identically distributed sample.

8. PROBABILITY OF TIME UNTIL CAPSIZING

From the derivation of the probability distribution for the time a model is running until capsizing, we conclude that there is no absolute safety from capsizing for the whole time a ship is operating at sea under severe conditions. We can merely say that a capsizing in any given operation time T of a ship may not be very likely to occur. To express this more precisely, we would like to say: make the capsizing probability in the whole lifetime of the ship as small as possible; try to develop design principles that allow this goal to be achieved by means of ensuring sufficient righting arms, taking into account speed and heading of the ship in the most severe seaway she will encounter on her route, and any other parameter that turns out to be of influence. Though we are primarily concerned with the development of some basic safety criteria for some classes of endangered ships, like fast container ships or fishing vessels, we do not want to set up a special rule for every single ship. On the other hand, for lack of specific data of average routes and operational times, we want to guess an average safe operation time, SOT , in severe weather conditions, in order to show the basic idea in the relations involved. Generally, every ship should be able to withstand severe seas for some hours at least, so we choose the following example.

Say for the ship $SOT_S = 10$ hours = 600 minutes. This is at model scale in our experiments

$$\begin{aligned} SOT_M &= \frac{SOT_S}{(I_S/I_M)^{\frac{1}{2}}} = \frac{600 \text{ mins}}{5.5} \\ &= 109 \text{ mins} \end{aligned} \quad (26)$$

Within the safe operation time (SOT) we now allow a small probability of $\alpha = 0.05$ for capsizing, that is, only 5% of all potential capsizings may occur within this time. Then we will consider the ship as safe from capsizing.

We see that we need to make basic assumptions about SOT and α , which depend on operational conditions of ships as well as on a compromise for safety and economic reasons. One might consider $\alpha = 0.05$ as still being too large for the sake of safety, so we might decrease α and make it equal to 0.01. However, we do not consider the choice of α , or SOT as too important. Rather we like to stress the equal treatment of the safety of several ships on rational grounds. Here we want to give just one example under reasonable assumptions.

From SOT, α , and the probability distribution in equation (22) we calculate its parameter.

$$\bar{t}_{NC} = \frac{1}{3} SOT (1 - \alpha) = .95 = \frac{1}{3} SOT \quad (27)$$

Thus we know the capsizing rate we are willing to allow for the operating ship

$$\mu_{NC} = 1/\bar{t}_{NC} = 3/SOT \quad (28)$$

With $SOT_M = 109$ minutes, $\alpha = 0.05$, we get an average operation time until capsizing at model scale of $\bar{t}_{NC} = 36.3$ minutes. Just one run of this time length takes about five runs of seven minutes each, which was the length of a single run in the 1971 testing season in San Francisco Bay. This, of course, does not yet yield a statistic of capsizings, so we have to multiply by the number of capsizings we want to measure. Say with only ten runs, which still leaves us with a low precision of our estimate, we would have to conduct 10 times 36.3 mins = 363 mins = 6 hrs, which is more than fifty model runs of seven minutes each and this is only for one model condition and a poor statistical precision.

However, obviously we will not run an almost safe ship model over and over again in order to measure a capsizing distribution. More important, this example shows that with one run of seven minutes with no capsizing we can only say there is apparently a rather small capsizing probability as

we cannot estimate its value from a single run of seven minutes, and we cannot say the ship at these model conditions will be safe from capsizing.

We might better conduct experiments in conditions where a high capsizing rate is likely to occur. From that we may try to extrapolate for the safe condition.

It should be mentioned that this holds already for actual severe sea conditions and the ship in her lifetime will meet these extreme conditions only occasionally. Thus the probability of capsizing during the lifetime is even smaller.

In this section we have dealt with only a statistic for determining the capsizing probability. However, even though it now appears a tremendous task to conduct a large enough number of experiments, we are still able to look more specifically into the model behaviour during the runs, i.e. how the roll motion builds up and which parameters are involved. Eventually, using a theoretical model of the motions, a large statistic may be obtained better by computation rather than by experiment. Model tests are still necessary to find out the basic pattern of ship behaviour in severe seaway at the nonlinear range of the roll motion, as well as for proof of theory. But our remarks about a statistical evaluation of the capsizings might help prevent us from drawing too general a conclusion from single random experiments concerning safety from capsizing.

We cannot use the estimation of the distribution parameter T_C from the mean of all times elapsed until capsizing, as not all runs ended up with capsizing, if we stopped all runs intentionally, say after seven minutes. Therefore all runs which did not end up with capsizing must be taken into account too. Thus we may use the relation between the binomial and exponential distribution as described earlier. The probability of capsizing according to the binomial distribution in equation (3) must be equal to the probability of capsizing measured in the time T , therefore

$$P_T = 1 - \exp\left(-\frac{T}{T_C}\right) \quad (29)$$

and

$$T_C = -T/\ln(1-P_T) \quad (30)$$

$T = 7$ mins duration of model testing in single run

P_T = capsizing probability in time T

We can estimate this parameter, T_C , on the basis of 64 runs in 1971. There were 21 capsizings, therefore an estimate of P_T is

$$P_T = n/N = 21/64 = 0.33 = \hat{\theta}$$

Using this estimate in equation (3) yields

$$T_C = -7/\ln(1-0.33) = -7/-0.40 = 17.5 \text{ mins}$$

Furthermore, with equation (1) we may find the precision of the estimate and the confidence region for the time until capsizing in testing with the binomial distribution

$$H_0: \theta_0 = P_T \text{ against } H_1: \theta_1 = P_1 > P_T$$

In Figure 7 the corresponding probability distributions for the time until capsizing are plotted. We see that on the average we measure right in the middle between the safe and unsafe ship, obviously for practical reasons.

9. DESIGN OF RANDOM EXPERIMENTS

As we have seen in the preceding chapter, random experiments require a larger number of long time model runs in order to get sufficient information about rare events, such as extreme roll motion amplitudes in severe seas, and particularly capsizing. On the other hand, for each change in the model parameter, the performing of another random experiment might be necessary. Considering only the most important parameters we already know of, which are freeboard, speed, heading, and centre of gravity, it seems impossible to carry out with reasonable effort all combination experiments or calculations in full detail.

It is therefore obvious that this type of experiment must be especially carefully designed with respect to

the parameters involved and the model conditions actually to be tested.

Let us roughly estimate the amount of parameter combinations we might need for testing the safety from capsizing. Experiments with a large number of parameters whose influence is to be measured are called factorial. Generally, we get the combinations in Table I.

In this table there are nine parameters involved, each having between two and four levels. Table 2-I gives about the minimum number of parameters involved. This leads to a number of combinations of

$$M_9 = \prod_{i=1}^9 l_i = 2^3 \cdot 3^5 \cdot 4 = 5184$$

If there is some nonlinear component to be estimated, more than three levels might be advisable. This is the case for the ship speed where only three levels might not be sufficient, and for the GM where the small GM-level could be again subdivided into three more levels. A sufficient number of levels for speed and GM are advisable because of the search for roll resonance and because we would like to measure some GM-levels in the close vicinity of a high capsizing probability. However, we have to restrict ourselves to a considerably smaller amount of combinations for testing.

In our planning strategy for the model experiments we can take advantage of two facts:

- i) On the basis of our knowledge to date, from theory or similar earlier experiments, we can choose only the parameter combinations of special interest.
- ii) As the actual test runs are made with one specific ship model in a time sequence, we can use the results of the first runs for the planning of the remaining combinations to be tested.

Thus, it is possible to reduce the required number of test combinations considerably.

We know from the probability considerations that, e.g. five runs give a poor precision for estimating the capsizing probability. On the other hand, we have to limit the total

number of runs because of the effort connected with conducting the model experiments. In choosing five runs we make sure to get at least a significant sample of the motion pattern involved as this certainly will not come out sufficiently from just one single run because of the statistical fluctuations of many parameters, e.g. initial phases, waves, heading, etc.

10. CONCLUDING REMARKS

The theoretical probability background of random time domain capsizing model tests or computations is discussed in this paper, together with an appropriate design of test parameters. Picking out relatively short time histories of the ship motion in question, containing some extreme roll amplitudes, enables us to evaluate the motion pattern. But the question arises how significant this sample would be for safety from capsizing. Therefore a method of statistical hypothesis testing is being applied.

In this paper some probability distributions are used to develop criteria on the statistical significance of a time domain sample. The binomial distribution is applied to the question: capsizing or not capsizing? The negative exponential distribution is applied to the operation time of a ship until capsizing in a certain stationary seaway. Both distributions have one important feature, the lack of memory or forgetfulness for the previous history.

The concept of confidence levels is evaluated. But particular emphasis is placed on the statistical precision of the determination of the capsizing probability, which is mostly overlooked in using statistics. Both confidence level and precision require a large number of samples.

The practical results of this analysis might be summarized as follows:

- i) It can serve as a tool for planning the amount of experiments of calculations of rare events like extreme roll and capsizing in order to assure accuracy in a statistical sense. The parameter combinations must be carefully selected in order to avoid an intolerable amount of work.
- ii) It shows the danger of misjudging and misinterpreting of single rare events as being dominant for the conditions in question.

- iii) Caution is recommended in considering safety standards for ships with respect to capsizing from scarce statistical information. It is not advisable to determine safety standards solely on a statistical basis as the size limited samples of rare events will barely include all the information on safety needed.

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LIST OF SYMBOLS

ϵ	Belonging to ($\omega \in A$: ω is belonging to A)
\subset	Inclusion ($A \subset B$ means: A is a subset of B, or A is included in B)
$X Y$	X given Y
θ	Unknown statistical parameter
$\hat{\theta}$	Estimated statistical parameter

Only symbols used in probability theory which are not well-known are shown here. All other symbols are always explained in the text or can be understood from the context in which they are used.

TABLE IFactorial Table for Capsizing Experiments

i	parameter	Rough grouping of the parameter				1_i
1	freeboard	low	medium	high		3
2	ship speed	low	medium	high		3
3	ship heading towards waves	following	quartering	(head or beam some-times)		
4	GM in still water as a measure of the height of centre of gravity in ship KG	negative	small	moderate	high	4
5	initial list of model in still water	zero	10°			2
6	waves	small	medium	high		4
7	rudder control (autopilot)	low setting		high setting		2
8	roll damping	low	medium	high		3
9	transverse radius of gyration	low	medium	high		3

APPENDIX I

First we have to define a probability model for random experiments. We will generally use the Kolmogorov probability model. In the notation we will follow Bickel. We will introduce here only some of the most important features for better understanding of how to handle our problems, without proof or mathematical completeness.

Definition:

A random experiment is an experiment where any outcome would be possible. In other words, we do not require to have the same result for every repetition of the experiment, but we do not exclude the possibility of some experiments having the same results. The experiments must be repeatable. We consider then a

probability model (Ω, σ, P)

where Ω sample space - is the set of all outcomes of random experiments.

An event is any subset of Ω , denoted by A, B, etc. One sample point or an elementary event is any member of Ω , denoted by ω .

σ sigma field - is the class of events to which we are able to assign probabilities.

We assume that all our sets (sample space Ω) are measurable in the sigma field, generally called borel sets B.

P probability - a function which assigns to each event in σ a non-negative number such that $0 \leq P \leq 1$

with the properties

- i) $P(\Omega) = 1$
- ii) $P \sum_{i=1}^{\infty} A_i = \sum_{i=1}^{\infty} P(A_i)$

where

$\sum_{i=1}^{\infty} A_i$ = union of the disjoint events A_i , i.e. of all events that do not occur simultaneously.

If the outcomes of the experiments are real numbers, the sample space Ω

of the corresponding probability model can be thought of as a Euclidean space, denoted by R^k , with k the dimension of the space, and

$$\underline{x} = (x_1, \dots, x_k) \quad \text{members of the space } R^k$$

The interesting quantities in a probability model are called random variables, denoted by x , and random vectors* (denoted by \underline{x}). Thus for $k = 1$, random vectors \underline{x} are just random variables x .

Although the sample space Ω for our model experiments is very diverse, we are interested primarily in only some numerical characteristics of an elementary event that has occurred. For example, for safety from capsizing we might be interested mainly in the roll motion, or even only whether capsizing has occurred during this run.

The probability measure of random vectors in the Euclidean space is called a probability distribution:

$$P(A) = \int_A \dots \int p(t_1, \dots, t_k) dt_1 \dots dt_k$$

with p the probability density function.

It is appropriate to describe random variables purely in terms of their probability distributions. The study of functions g of a random vector \underline{x} is central to the theory of probability and statistics. Such well-known functions g are, for example, the sample average or the standard deviations of the samples. Generally, the probability distribution of $g(\underline{x})$ is completely determined by that of \underline{x} through

$$P[g(\underline{x}) \in A] = P[\underline{x} \in g^{-1}(A)]$$

Figure A1 gives a graphical explanation of the relations within the defined probability model. Each space or field is shown as an area, and the derived vectors, probabilities or distribution parameters are shown on the connecting lines.

*Strictly speaking, a random vector is a function which assigns a vector of real numbers to an event.

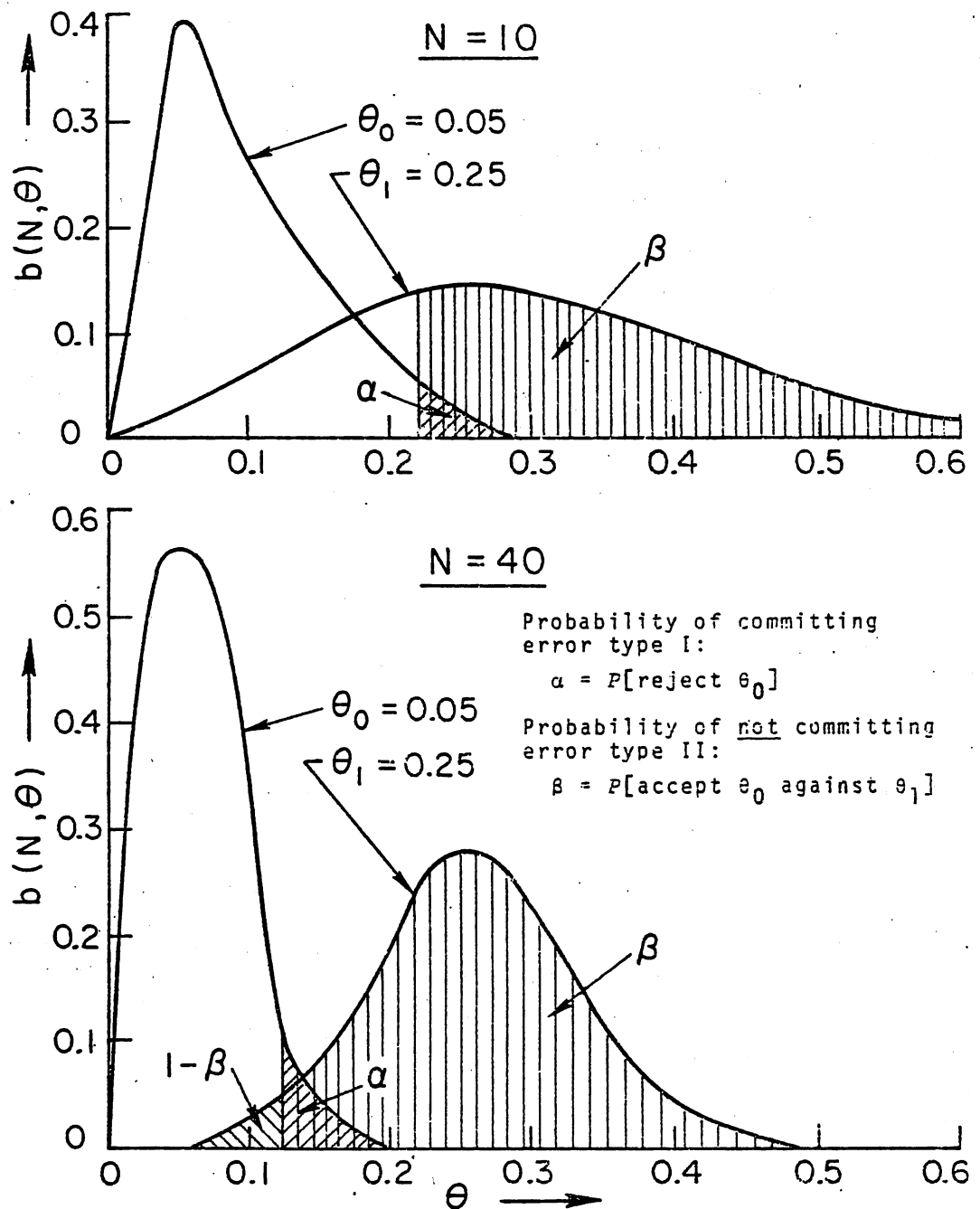


Figure 1 Definition of Confidence Level $(1-\beta)$ and Power α for One Sided Test in Binomial Trials

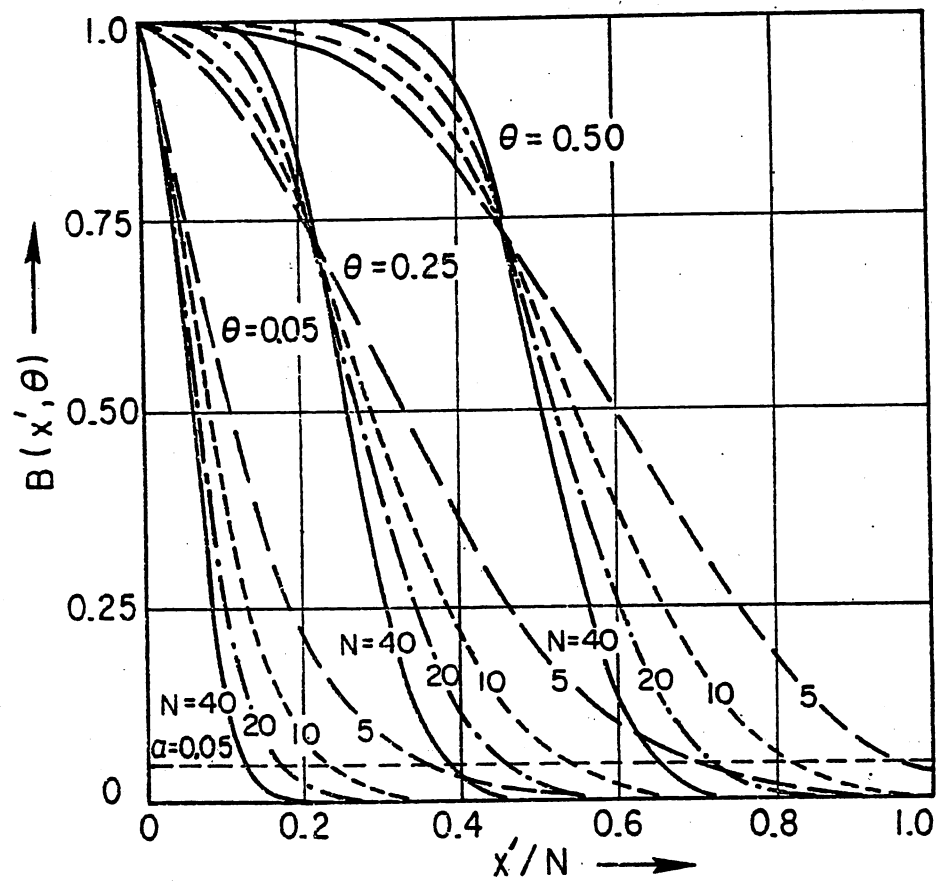


Figure 2 Binomial Cumulative Probability Distribution (Values from Tables)

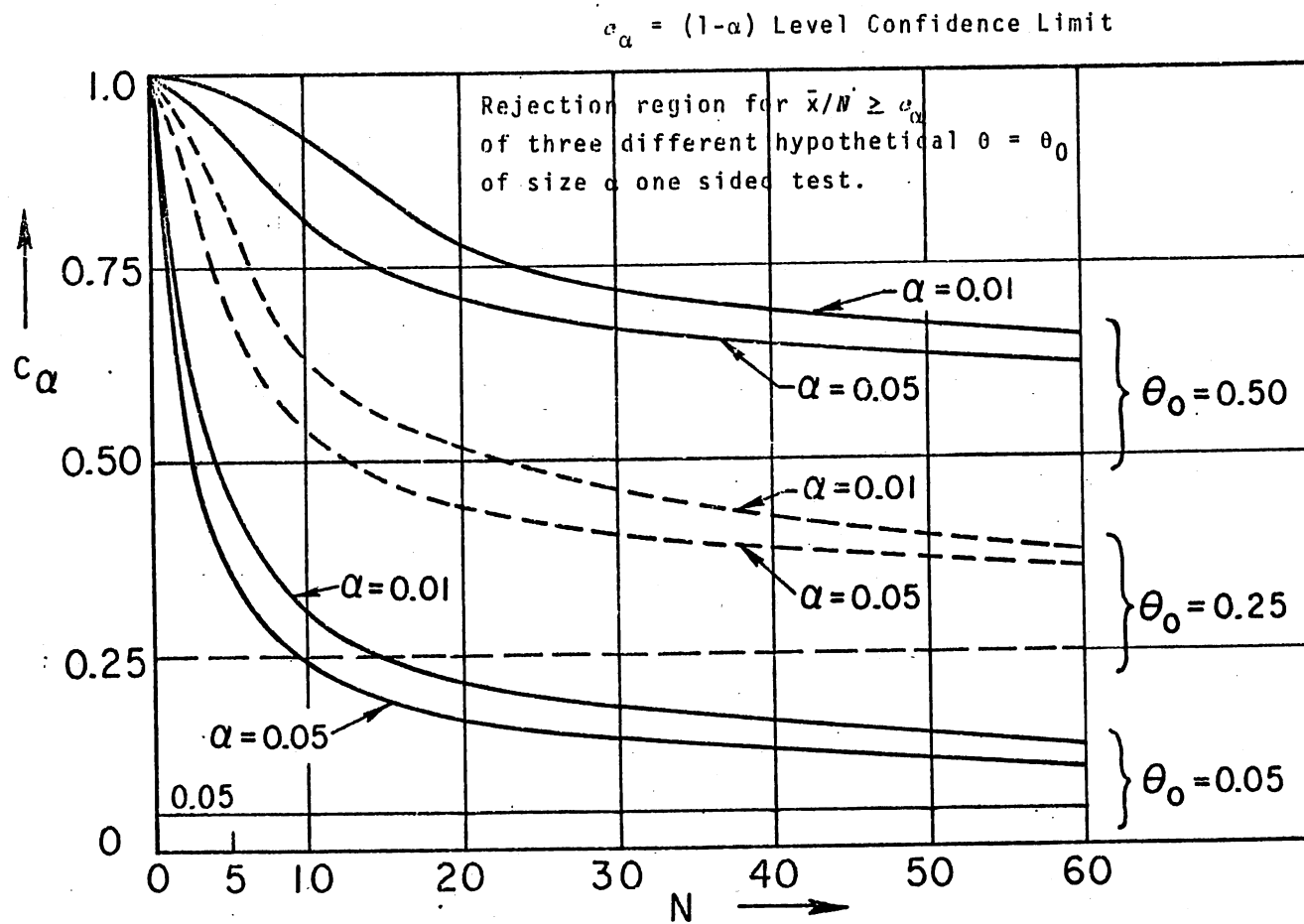


Figure 3 Critical Function C_α for Rejection (Type I Error)

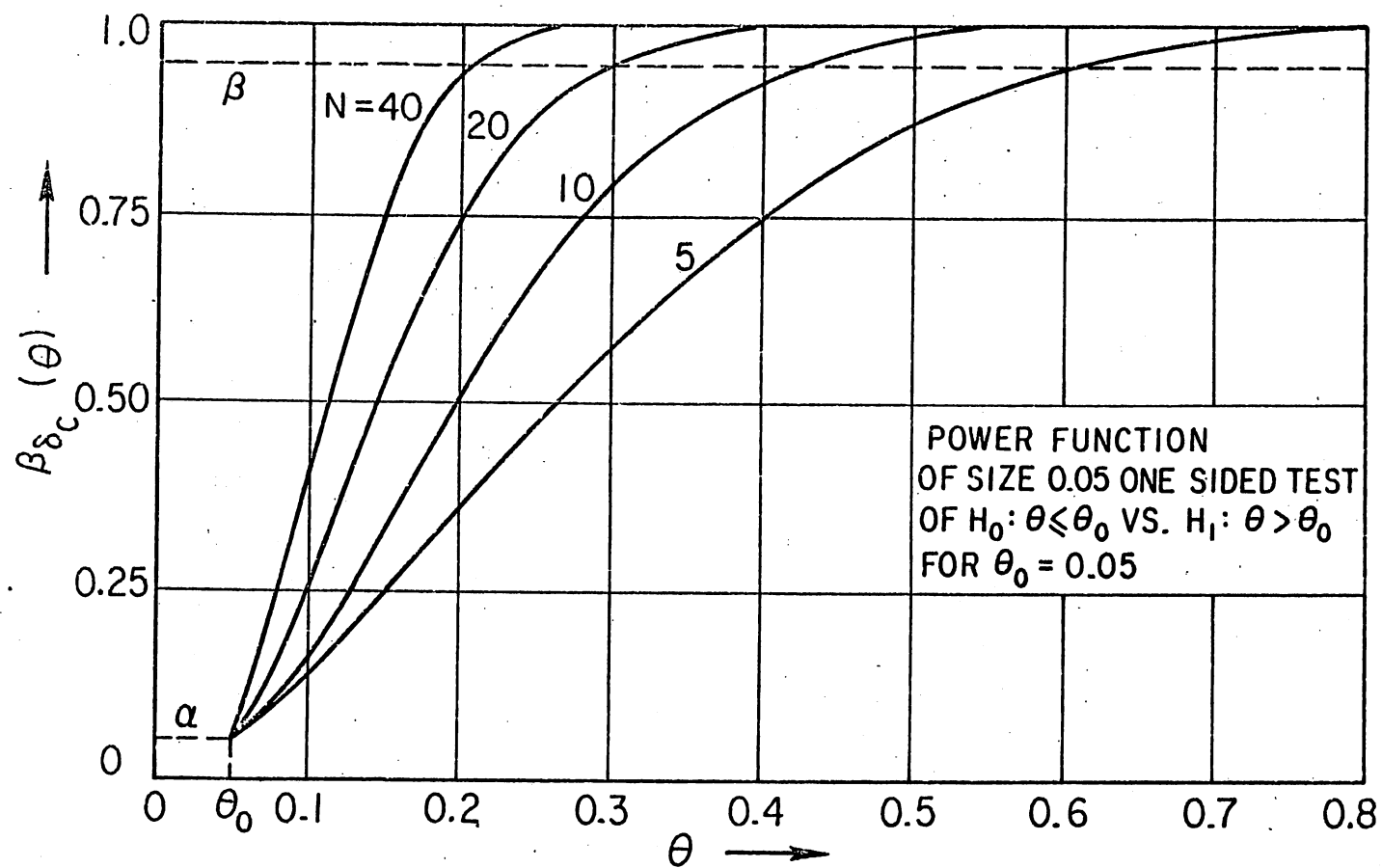


Figure 4 Power Function for Binomial Trials, $\theta_0 = 0.05$ (Type II Error)

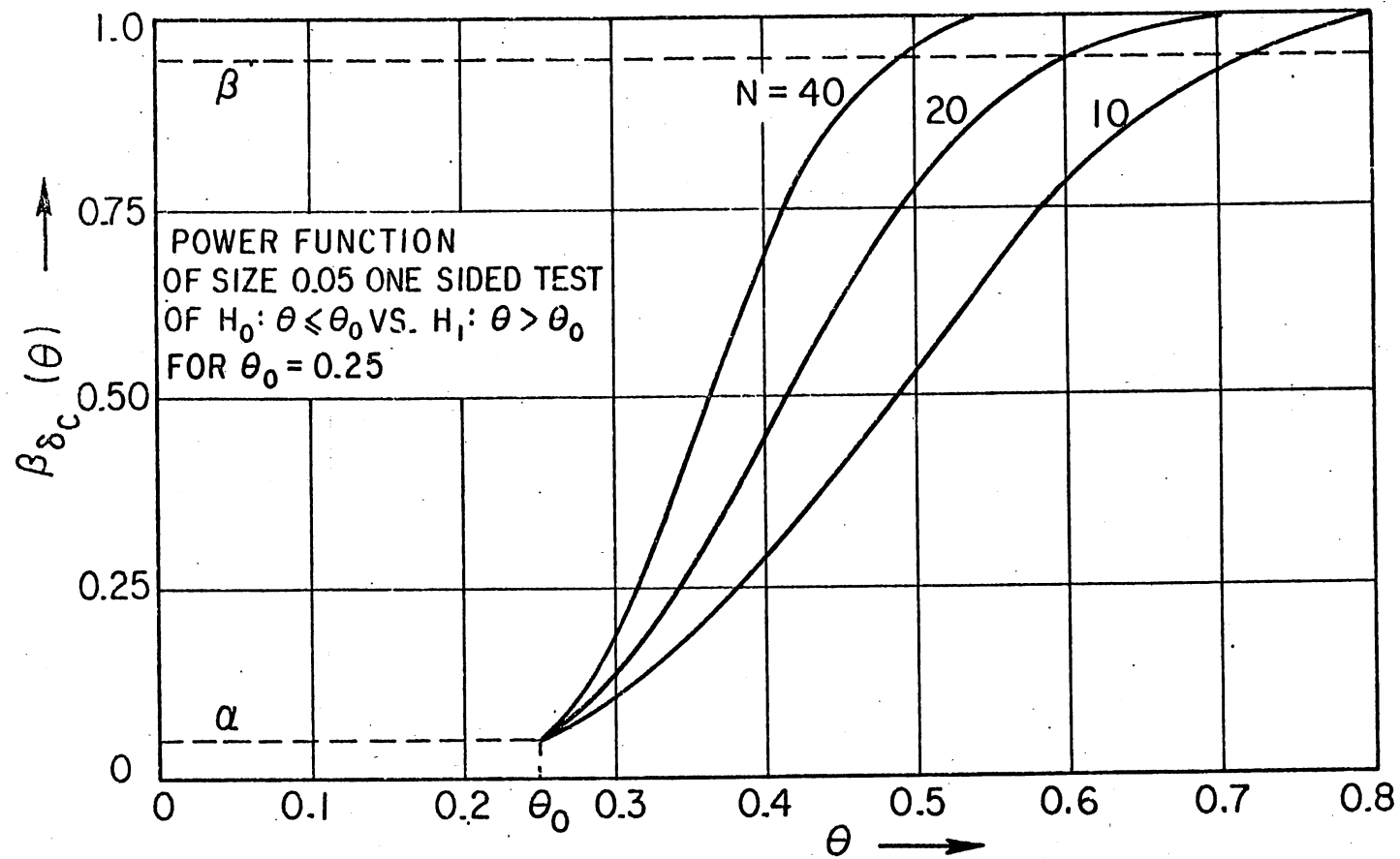


Figure 5 Power Function for Binomial Trials, $\theta_0 = 0.25$ (Type II Error)

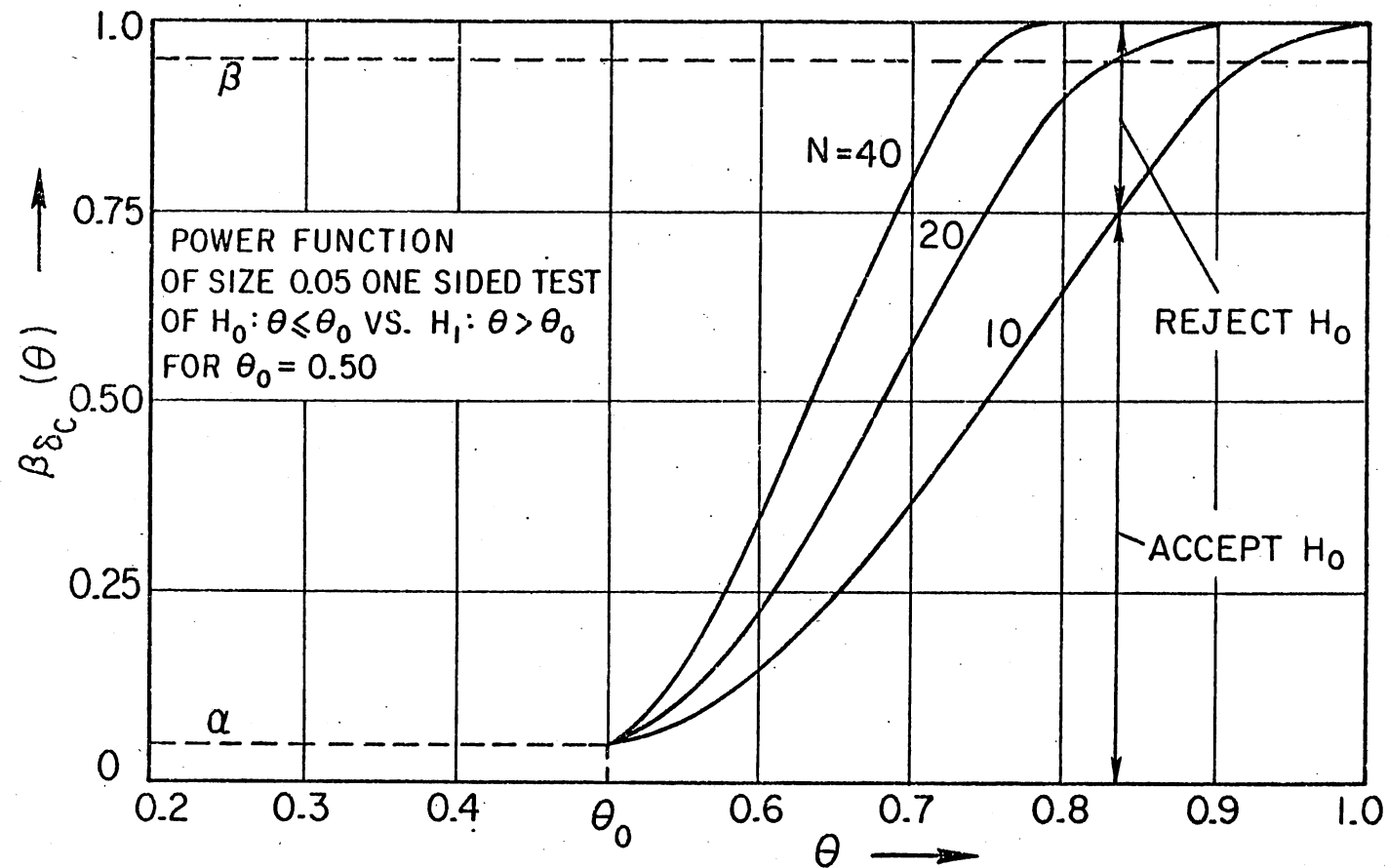


Figure 6 Power Function for Binomial Trials, $\theta_0 = 0.50$ (Type II Error)

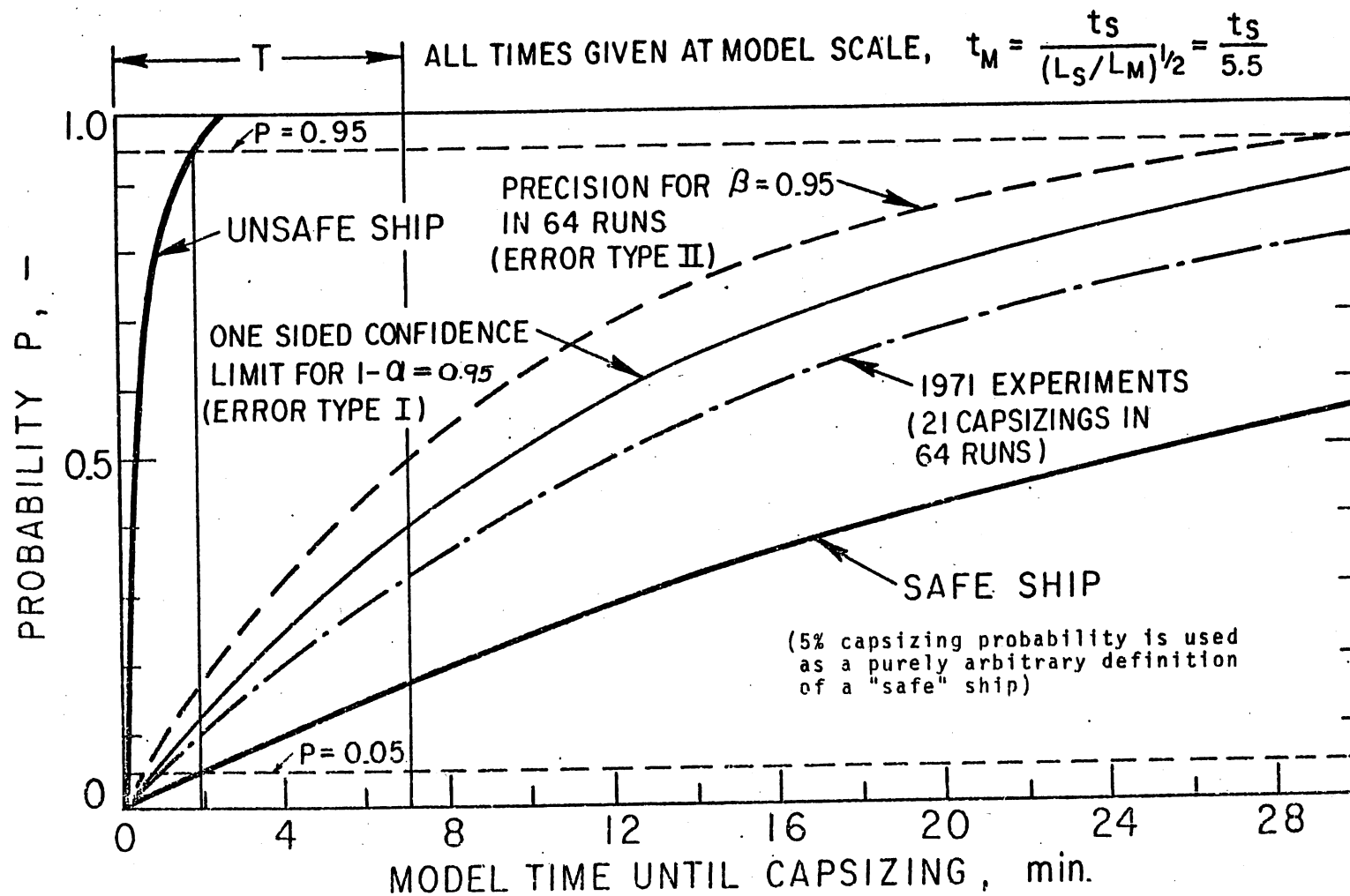
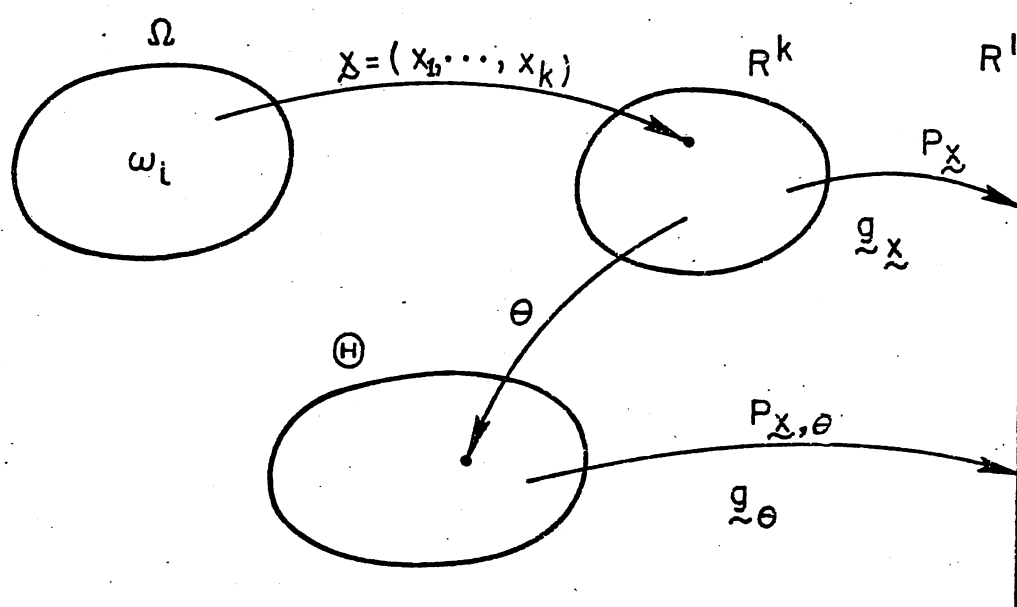


Figure 7 Probability of Time Until Capsizing (Cumulative Exponential Distribution)



Ω sample space

R^k observation space
(Euclidean space)

Θ parameter space

R real line

P probability

\tilde{x} random vector

x_i random variable

k dimension of Euclidean space
(number of random variables)

N sample size

θ unknown parameter

g_θ any function of θ

g_x statistic, any function of

Figure A1 Probability Model for Random Experiments

METHOD FOR ESTIMATING THE SHIP'S STABILITY IN IRREGULAR SEAS

by

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1. INTRODUCTION

It has become usual to mention the reduction of transverse stability in the following waves as one of the factors responsible for a decrease in the safety of a ship in stormy conditions. However, not all the problems related to stability of ships in waves have been solved so far, while the assumptions in respect of the conditions under which stability estimations are carried out are leading, as a rule, to an oversimplification of the phenomenon. For example, the assumption that the speed of a ship is coincident with that of a regular wave and that the length of the latter is equal to the length of the ship.

This paper deals with the results of studies of the transverse stability of ships in waves, the restoring moment variations being considered as a random process with inherent statistical characteristics. It is from this standpoint that consideration is given to the problem of a suddenly applied wind load acting on the vessel and the problem is of paramount importance for the estimation of the ship's safety in waves. In this respect the investigations described are very close to the problem of the probability of capsizing of a randomly oscillating vessel with an invariable statistical stability curve. This problem was solved by G.A. Firsoff in 1959 (2).

2. EQUATION OF MOTION

Let us assume that at a moment $t = 0$

a transverse wind load $P_x(t)$ is applied to the vessel moving in waves on an arbitrary course. Subsequently, in analysing the behaviour of the vessel exposed to the wind we shall follow the methods described in References (1), (8).

Using d'Alembert's principle, we shall write the equation of the ship's heeling about the longitudinal axis GX as

$$-(J_{xx} + \mu_{44})\ddot{\theta} + M_{a1} + M_{a2} + M_c + M_p - M_g = 0 \quad (1)$$

where J_{xx} - mass moment of inertia about axis Gx;
 μ_{44} - added mass moment of inertia;

M_{a1} - noninertial hydrodynamic moment due to drifting;

M_{a2} - inertial moment due to drifting;

M_c - moment of resistance to side inclinations of the vessel;

M_p - moment of aerodynamic force $P_x(t)$;

M_g - restoring moment to be determined with allowance for the effect the ship speed has on stability.

Since moment M_g includes the wave-induced component (4), thus in the absence of wind load $P_x(t)$ Eq.(1) describes the ship's rolling motion in waves taking into account the effect of the waves on stability.

The value of moment M_c being small in comparison with other components of (1), this moment need not be taken into account in the practical estimation of the dynamic angle of heel as an error tends towards the side of safety.

For a restoring moment the following presentation given by (3) holds true:

$$M_g = M_g^0 + m_v + m, \quad (2)$$

where M_g^0 - still water restoring moment;
 M_v - component due to ship motion in still water;
 M - additional restoring moment in waves.

If Z is the resultant of aerodynamic forces along Z-axis, then we have

$$M_p = - P_\alpha Z_n$$

In order to define the structure of moments M_{a1} and M_{a2} it is necessary to make an analysis of the ship's drifting. As is usually accepted, the effect of velocities and accelerations of the ship's rolling oscillatory motions on her transverse movement will not be taken into consideration, see (7), (8).

The equation of drifting will be written as

$$\frac{D}{g} \dot{v}_{\eta g} = P_\alpha(t) + H(t) \quad (3)$$

where D = displacement
 $v_{\eta g}$ = projection of the CG transverse velocity onto axis for a vessel moving in waves;

$H(t)$ = lateral hydrodynamic force.

For the case when the ship speed v is sufficiently high when compared to the speed of drifting, we represent force $H(t)$ as the sum

$$H(t) = -\mu_{22} \dot{v}_{\eta g} - \lambda_{22} v_{\eta g} + H_g \quad (4)$$

where H_g is the lateral force due to waves, while the hydrodynamic characteristics μ_{22} and λ_{22} as well as the added mass moment of inertia in (1) are dependent on the angle of heel θ . Owing to linearity of Eq. (3) and taking account of (4) the velocity $v_{\eta g}$ can be written as the sum of the velocity of transverse motions $\dot{\eta}_g$ in waves without regard for the force P_α and the speed of drifting v_η in still water under the action of aerodynamic load:

$$v_{\eta g} = v_\eta + \dot{\eta}_g \quad (5)$$

Component $\dot{\eta}_g$ is determined by the methods of the theory of ship motion in waves; the velocity v_η is obtained from equation

$$\dot{v}_\eta + v_\eta v_\eta = P_\alpha(t) \quad (6)$$

where $\dot{v}_\eta = \frac{\lambda_{22}}{\frac{D}{g} + \mu_{22}} \quad (7)$

$$P_\alpha = \frac{P_\alpha}{\frac{D}{g} + \mu_{22}} \quad (8)$$

With $t=0$; $v_\eta=0$, this solution takes the form

$$v_\eta = e^{-v_\eta t} \int_0^t P_\alpha(\xi) e^{v_\eta \xi} d\xi \quad (9)$$

Relation (9), given the wind load variation, makes it possible to calculate the speed of drifting. The resistance coefficient included in (7) can be represented as

$$\lambda_{22} = C_1 v \frac{\rho S_0}{2} \quad (10)$$

where S_0 - area of underbody projected on the central longitudinal plan;

C_1 - empirical coefficient

In a more general case, when the ship speed is not too high, equation (6) can be substituted for analysis purposes by the equation

$$\dot{v}_\eta + v_\eta v_\eta + v_\eta v_\eta^2 = P_\alpha(t) \quad (11)$$

The following designations are used here:

$$v_\eta^* = \frac{\lambda_{22}^*}{\frac{D}{g} + \mu_{22}} = \alpha_1 v \frac{\rho S_0}{2(\frac{D}{g} + \mu_{22})} \quad (12)$$

$$v_\eta^{**} = \frac{\lambda_{22}^{**}}{\frac{D}{g} + \mu_{22}} = \alpha_2 \frac{\rho S_0}{2(\frac{D}{g} + \mu_{22})} \quad (13)$$

where α_1, α_2 are the coefficients obtained by model testing in towing tanks. Some recommendations with respect to numerical values of these coefficients can be found in Ref. (8).

eq. (11) is a particular case of the more general Riccati equation, on solving one can obtain in principle the speed of drifting v_η .

When passing from eq. (3) to Eq. (11) it is assumed that equation (5) is equally valid for the case where due to non-linearity of the equation of drifting there is no ground for breaking its velocity into component v_η and the component determined by wave-induced swaying. However, taking into account the semi-empirical nature of estimated velocity v_η and the possibility of neglecting the value of $\dot{\eta}_g^2$ as one of the second order smallness, let us consider expression (5) to be acceptable.

Thus, with the speed of drifting in calm water known, the noninertial lateral force in the general case can be represented by the relation

$$R_\eta = -(\alpha_1 v + \alpha_2 v_\eta) \frac{\rho S_0}{2} v_\eta \quad (14)$$

and its moment by

$$M_{a1} = -R_\eta z_R, \quad (15)$$

where z_R is the Z-coordinate of the point of application of R_η .

It should be noted that the hydrodynamic moment induced by the velocity η_g and acceleration $\ddot{\eta}_g$ of the ship's transverse motions in waves is included as a component in the restoring moment M_g (3). Hence, the moment M_{a2} will be expressed as

$$M_{a2} = -\mu_{24} \ddot{\eta}_\eta. \quad (16)$$

Substituting (2), (15), (16) into (1) we derive the equation of heeling:

$$(J_{xx} + \mu_{44}) \ddot{\theta} + M_g^0 + m = M_K, \quad (17)$$

where the value of M_K is

$$M_K = -\rho z_R - R_\eta z_R - \mu_{24} \ddot{\eta}_\eta - m v$$

and this should be considered as a heeling moment applied to the vessel.

3. APPLICATION OF THE MOTION EQUATION

Eq. (17) may serve as the basic one for solving the problem of the dynamic action of wind load on the

vessel in a seaway. Since no special assumptions were made regarding the characteristics of waves, it is felt that Eq. (17) can be extended to cover the irregular waves as well.

If we assume that the squall-excited aerodynamic force increases according to some law from zero value and then again decreases gradually to zero, the solution of the dynamic heeling problem may be reduced to the analysis of asymptotic stability of the ship's rolling motions in waves.

In the case of irregular seas this analysis will essentially consist of defining the time-dependent mean values of heeling angles and their dispersion. In ship theory, however, there is another accepted approach to the evaluation of the aftereffects of wind load action. As the duration of a wind gust is unlikely to exceed the time interval within which the vessel inclines to the lee side for the second time, the first inclination is considered to be the most dangerous. Though one cannot assert that the subsequent angular oscillations of the vessel will not exceed the first one in amplitude, such a situation is hardly probable. The evaluation of a ship's safety by analysing the first inclination can be made in a simple and most illustrative way on the basis of expression (17) by using the equation of actions.

In Eq. (17) the additional restoring moment m of a vessel moving in irregular waves can be described by the differential law of distribution assumed as recommended in (4), (6) in the form of the Gauss law. The problem of defining relevant statistical characteristics of moment viz. variance, mean value as well as correlation moments presents no special difficulties, see (4). In addition account can also be taken of the restoring moment components caused by ship's rolling and the diffraction of the oncoming waves, (3). For this purpose, however, it is necessary to know the damping coefficients and the added masses of the inclined ship's sections, which may sometimes prove to be difficult due to scant information about these hydrodynamic characteristics.

Figs. 1 and 2 show stability curves for two fishing vessel models underway in the following regular waves, where M_{gmax} , M_{gmin} are the maximum and the minimum values of the restoring moments. Apart from the

experimental points, plotted in the figures are the results of calculation based on the distribution law of hydrostatic pressures in wave, as well as those allowing for the Smith correction and inertial hydrodynamic component of moment M_g . The value of the added static moment of the model, which is required for carrying out the latter calculation, was determined by the electrohydrodynamical analogy method. In the evaluation of the results mentioned before it should be borne in mind that the effect of the hydrodynamic components will probably be more significant when the vessel is on an oblique course to the wave.

4. EFFECTS OF WIND LOAD

Let us analyse the effect of a suddenly applied wind heeling load on the vessel rolling in irregular waves. The random values of the heeling angle and the angular velocity peculiar to the ship at the moment of applying M_K will be denoted by θ_H and $\dot{\theta}_H$ respectively. It should be emphasized that the components of the heeling moment M_K included in (17), given the aerodynamic force, are quite definite nonrandom values.

We shall now multiply both sides of the equation (17) by $d\theta$ and integrate these between the limits $\theta = \theta_H$ and $\theta = \theta_0$ assuming that for $\theta = \theta_0$ the angular velocity $\dot{\theta} = \dot{\theta}_0$. Then in each particular case i.e. for a particular realization of random function $M_g(\theta, t)$ and particular values of θ_H and $\dot{\theta}_H$ the value of the dynamic angle of heel θ_0 can be derived from the equation of actions

$$A_g(\theta_H, \theta_0) - K(\dot{\theta}_H) \text{sign} \dot{\theta}_H = A_H(\theta_H, \theta_0) \quad (18)$$

where

$$A_g(\theta_H, \theta_0) = \int_{\theta_H}^{\theta_0} [M_g^0(\theta) + m(\theta) + m_{\dot{\theta}}(\theta)] d\theta$$

is the action of the restoring moment at angles of heel ranging from θ_H to θ_0 ;

$$A_K(\theta_H, \theta_0) = \int_{\theta_H}^{\theta_0} M_K(\theta) d\theta$$

is the action of the heeling moment;

$$K(\dot{\theta}_H) = J \frac{(\dot{\theta}_H)^2}{2}$$

is the kinetic energy of the rolling ship at the moment of the action of M_K ,

$$J = J_{xx} + K_{44}$$

The dynamic angle of heel θ_0 being a random variable, a particular value of that angle for any kind of realization of the process under study

will not characterize the dynamic angle completely. However, expression (18) enables the probability characteristics of angle θ_0 to be associated with the regularities of ship's rolling in waves. Let us formulate the condition which determines the probability of the dynamic angle of heel exceeding the arbitrary value θ_0^* with the moment M_K given. For each realization the above condition as based on Eq. (18), lies the fact that in the course of dynamic heeling the work of the heeling moment A_K will exceed the difference $A_g - K \text{sign} \dot{\theta}_H$ for all angles of heel ranging from θ_H to θ_0^* . Hence the required probability

$$P[\theta_0 > \theta_0^*] = Q(\theta_0^*) = P[A_g(\theta_H, \theta_0^*) - K(\dot{\theta}_H) \text{sign} \dot{\theta}_H < A_K(\theta_H, \theta_0^*)], \quad (19)$$

for all values of θ_H provided that $\theta_H < \theta_0^* \leq \theta_0^*$. In expression (19) symbol $P[\dots]$ is used to denote the probability of satisfying the condition enclosed in square brackets.

The combination of values θ_0^* and $Q(\theta_0^*)$ for a given heeling load P_K characterizes the effect of its action. By analogy with the terms commonly used in the theory of ship motions the value $Q(\theta_0^*)$ can be termed the probability of exceedance of the dynamic angle of heel.

The calculation of probability $Q(\theta_0^*)$ by Eq. (19) for any angle θ_0^* does not differ in the main from the calculation of the probability of ship capsizing, the latter to be found as the probability of the validity of inequality

$$P_0 = P[A_g(\theta_H, \theta) - K(\dot{\theta}_H) \text{sign} \dot{\theta}_H < A_K(\theta_H, \theta)], \quad (20)$$

for all $\theta > \theta_H$.

Further development of relations (19) and (20) and formulation of a method for estimating the values of P_0 and $P_{\theta_0^*}$ are described in Ref. (4). If $P_0(\theta_H, \dot{\theta}_H)$ is the conditional probability of capsizing obtained for fixed values of the initial angle of heel θ_H and velocity $\dot{\theta}_H$, the total probability will be determined by the result of integration

$$P_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_0(\theta_H, \dot{\theta}_H) f(\theta, \dot{\theta}) d\theta d\dot{\theta} \quad (21)$$

where $f(\theta, \dot{\theta})$ is the two-dimensional probability density of values θ and $\dot{\theta}$ assumed in terms of the Gauss law.

Symbol θ_{0H} in Eq. (21) designates the maximum amplitudes of rolling motion which may occur in irregular seas of a given intensity. Methods for the evaluation of maximum amplitudes of heeling and ways of correcting the corresponding laws

of distribution are given in Ref. (5).

For the probability of noncapsizing, P_H , the following relation is valid:

$$P_H = 1 - P_0 \quad (22)$$

If in the relations used for the estimation of P_0 the value of the additional restoring moment is assumed to be equal to zero, one can approach the problem of the probability of capsizing of a rolling vessel with a fixed curve of static stability and this problem has been analysed by G.A. Firsoff.

5. COMPARISON OF THEORY AND EXPERIMENT

The results of the computer calculation made for the curve of exceedance are given in Figs. (3) and (4) together with the data obtained in a model tank from testing a model of a seiner in two-dimensional irregular head waves for two conditions of loading. The external heeling moment applied to the model was effected by a sudden dropping of a small weight from an arm extended horizontally outboard for a significant distance. The calculations were performed with reference to this particular heeling moment.

The comparison, using the Kholmogorov criterion, between the curves of exceedance as obtained from experiment and those calculated showed a good agreement. This serves to confirm that the proposed method of determining the heeling load effect on the vessel can be accepted for practical evaluation of ships' dynamic stability in irregular seas.

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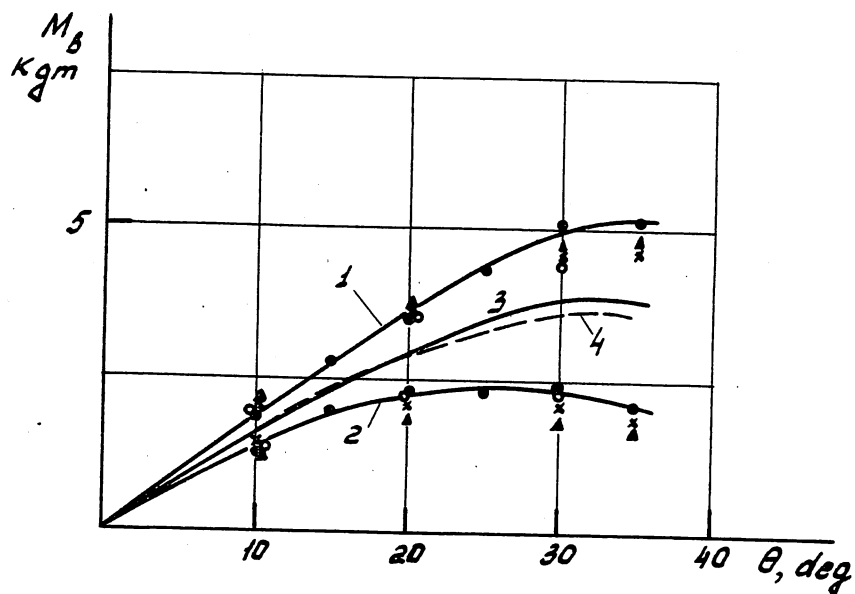


Fig. 1. Curve of stability for model No. 1 in the following waves

$$\frac{\lambda}{L} = 1.16; \quad Fr = 0.243$$

- 1 - M_{gmax} ; 2 - M_{gmin} ; 3 - M_g^0 ; 4 - $-\bar{m} + M_g^0$;
 ● - experiment;
 X - hydrostatic calculation;
 Δ - with allowance for Smith correction;
 ○ - with allowance for inertial component.

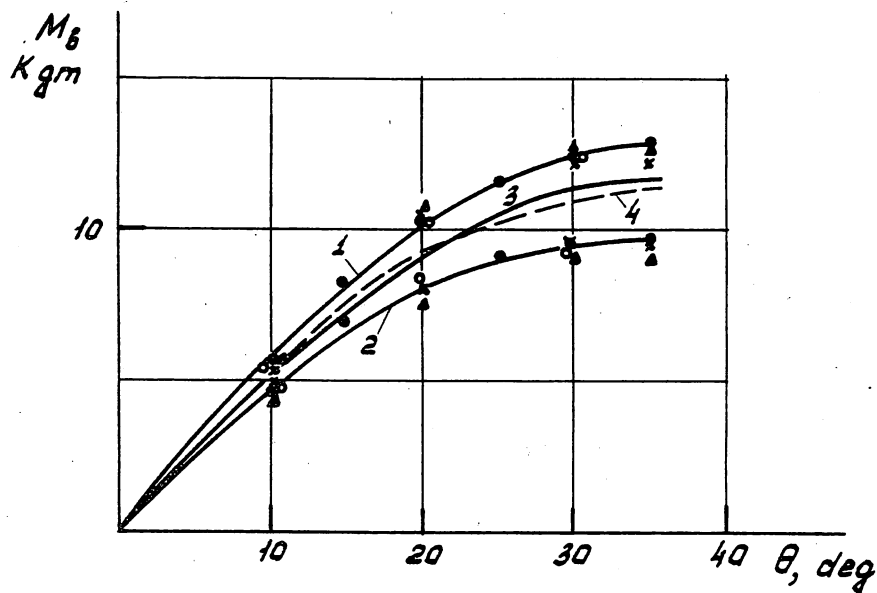


Fig. 2. Curve of stability for model No. 2 in the following waves

$$\frac{\lambda}{L} = 1.06; \quad Fr = 0.243$$

- 1 - M_{gmax} ; 2 - M_{gmin} ; 3 - M_g^0 ; 4 - $-\bar{m} + M_g^0$;
 ● - experiment;
 X - hydrostatic calculation;
 Δ - with allowance for Smith correction;
 ○ - with allowance for inertial component.

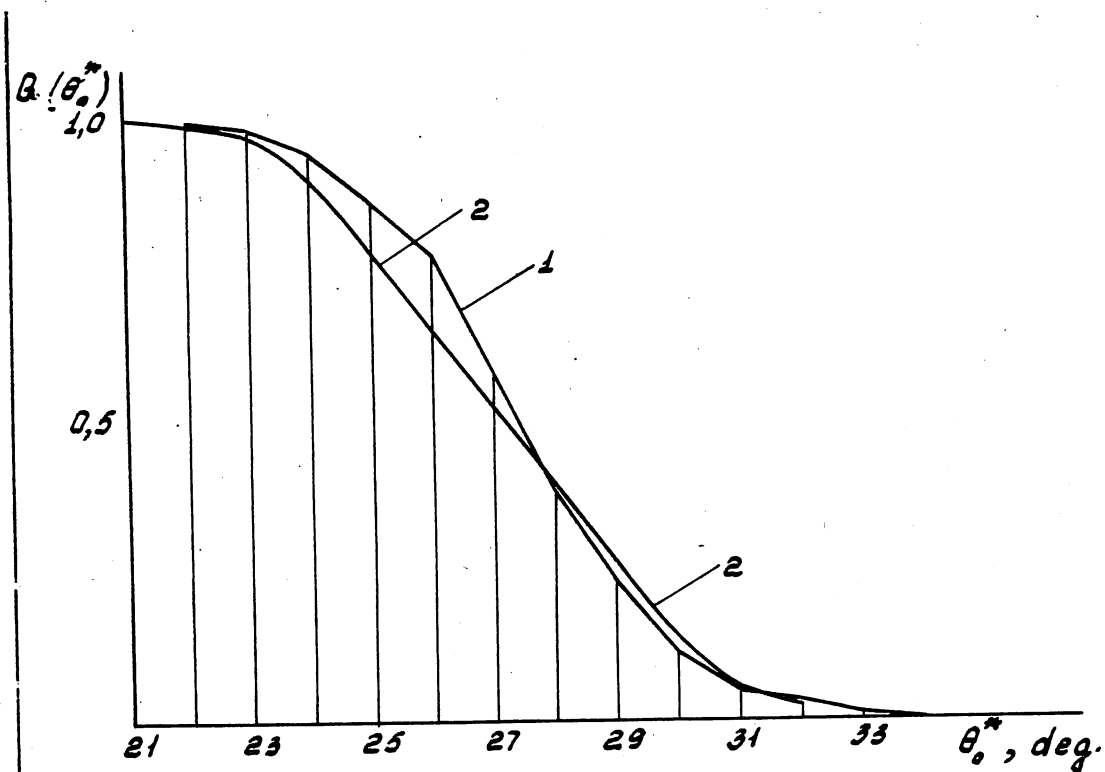


Fig. 3.
Probability curve of exceedance of dynamic angles of heel for a seiner (first condition of loading)

1 - experiment; 2 - calculation.

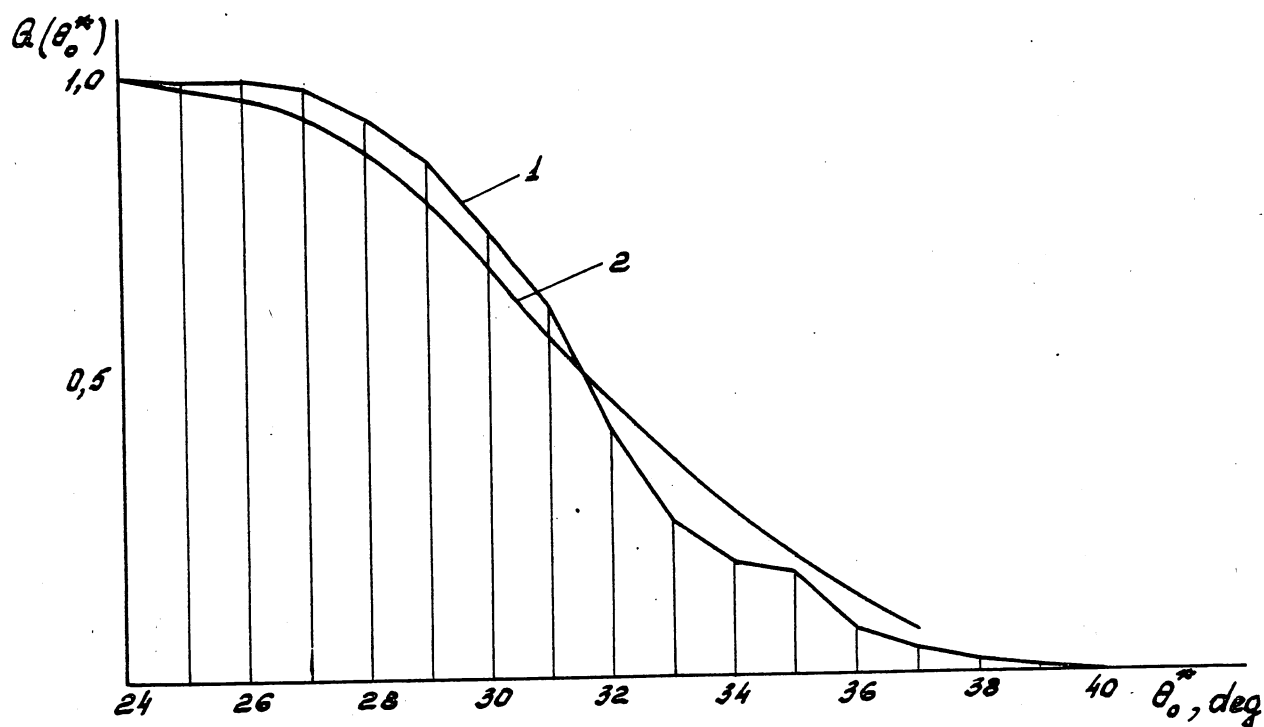


Fig. 4.
Probability curve of exceedance of dynamic angles of heel for a seiner (second condition of loading)

1 - experiment; 2 - calculation.

SESSION 4

CORRELATION OF THEORY AND EXPERIMENT

<u>Paper No.</u>	<u>Author</u>	<u>Title</u>
4.1	Kure, K. & Bang, C.J. (Denmark)	"The Ultimate Half Roll"
4.2	Tamiya, S. (Japan)	"Capsize Experiment of Box Shaped Vessels"
4.3	Paulling, J.R. Oakley, O.H. Wood, P.D. (U.S.A.)	"Ship Capsizing in Heavy Seas: the Correlation of Theory and Experiments"
4.4	Kobayashi, M. (Japan)	"Hydrodynamic Forces and Moments Acting on Two Dimensional Asymmetrical Bodies"
4.5	Morrall, A. (U.K.)	"Simulation of Capsizing in Beam Seas of a Side Trawler"

THE ULTIMATE HALF ROLL

by

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SUMMARY

Many years of research in the field of ship stability in waves, the results of which have partly appeared in the IMCO papers PFV X/8/2, PFV IX/7/3, formed the background for studies of the capsize of a coastal tanker in ballast condition, which complied with the IMCO recommendations for transverse stability.

Model tests were performed with a four metre wooden model in a 16 million litre model basin equipped with a digitally controlled pneumatic wave generator.

The reconstruction of the capsize in ballast condition was successful and was extended to higher GM values than that recommended by the IMCO regulations. Fully loaded it was not possible to capsize the model when it was even at much lower GM values than recommended. Wind (mean) and gust effects were simulated. Hard facts regarding the limit GM in the test environments were obtained.

A deeper analysis was subsequently attempted, with concentration on the observed ultimate half period of roll which progressed directly into the capsize.

An energy and moment balance analysis, using righting levers which varied during the wave passage, mean wind and gust moments and non-cancelling wave-forcing and motion-damping moments, did not explain the capsize.

The well-established effect of coupling moments into roll from heave and pitch and wave passage, described by the Mathieu type of equations, was not eff-

ective, because of the lack of necessary 2:1 tuning ratio for the wave encounter.

A Faltinsen, Salvesen, Tuck strip theory computer program using Potash close-fit hydrodynamic force values was applied to evaluate the coupling terms from sway and yaw. They were small for the loaded condition and large for the ballast condition in which latter case counteracted the roll motion and the large wave heeling-moment. The coupling moments were thus apparently not responsible for the observed rapid capsize.

However, a hypothesis could be built up to explain the capsize in terms of the above causes, by considering the lagging of the roll motion behind the other modes in the (then) ultimate half-roll cycle of this strongly non-linear system.

Conclusions and recommendations are given.

1. OBSERVATIONS

1.1 The Capsize

A small Danish tanker having GZ-curves complying with the IMCO recommendations capsized some years ago in the Baltic Sea near the Swedish island of Gotland. She was on a ballast journey and steamed south in a stern quartering sea. In between she rolled rather heavily and suddenly she capsized. The weather had a strength of Beaufort 6-7. Only two persons survived.

The main particulars of the ship are given in Table 1.

The loading condition was later analysed in detail and established to have been as specified in Table 2.

The characteristics of the GZ curve are given in Table 3.

The curve itself is shown in Figure 1.

The environments are not known very accurately. The wind force was stated to be Beaufort 6-7 with 4 or 5 m high waves. For a standard set of wave data in the Baltic Sea east of 14° East longitude, Reference [1] gives the following values:

<u>BF 6-7</u>	$H_{1/3} = 3.4\text{m}$
	$H_{\max} = 6.3\text{m}$
	$\lambda = 45\text{ m}$
<u>BF 7-8</u>	$H_{1/3} = 4.3\text{m}$
	$H_{\max} = 8.1\text{m}$
	$\lambda = 55\text{ m}$

It is usually assumed, when more precise information is lacking, that the visually observed wave height compares to the significant height $H_{1/3}$. The reported waves are then a little higher than the average standard value. The extreme heights of a seastate are a little less than double the $H_{1/3}$ value.

1.2 Model Tests

Laboratory attempts to reconstruct the capsizes as soon as possible after the disaster were made at the Danish Ship Model Basin. An existing fibre-glass model was adopted, having more or less the same main particulars as the present ship. This model had earlier been used for the reconstruction of the sea disaster of the ship and was an accurate model of that ship.

In the present reconstruction it capsized immediately and repeatedly in different weather states.

A one in sixteen scale model was then built of Obeisha wood in order to reconstruct the capsizes more precisely.

It appears from Reference [2] that the IMCO recommendations which in the present case were not sufficient to prevent capsizes, were almost entirely based on loaded ship in contrast to the present ballasted case. It was therefore decided that the model should also be tested in a loaded IMCO

condition in the same seaway as that of the reconstruction. See Figure 5 and Table 4 for details.

The main particulars of the model are given in Table 5. It was built with poop and forecastle, bulwark, bilge-keels and stylized deckhouses. The model was equipped with a propulsion motor and a model propeller, a steering gear and a rudder, a gyro-horizon and a wind moment and gust simulator. A king post with adjustable leaden weights to be fixed in a range of vertical positions were installed to be able to adjust the GM-value easily during the tests. See Figure 2.

The basic values of displacement and trim, metacentric height and transverse and longitudinal radii of gyration were obtained by adding and displacing leaden weights in the model during consecutive inclining, trimming and oscillation tests in still water and in air suspended on knife edges.

All instrumentation was remote controlled in the final set-up as shown in Figure 3.

The wind and gust simulator consisted of a mechanism to displace a weight suddenly from the centreline to the side of the model and to let it stay at side until it was released. The magnitude of the weight shift was selected in advance to comply with the wind moments apparent at the instant of capsizing. They were established from existing measurements on similar ship models in the laboratory's wind tunnels.

The model seastate was generated by the pneumatic wave-generator at one end of the 240 x 12 m model basin of 5.5 m depth of water. The wave-generator has been digitally controlled for more than 10 years and can generate regular, irregular and transient waves. Reference 3.

To facilitate analysis, a wave system was adopted comprising of a single representative component and one additional longer period component to shape the distribution of peaks.

A sample of the seaways used is shown in Figure 4 as measured by the laboratory's admittance type wave probe and recorded on a chart recorder.

The model tests proper were performed in the model basin in stern quartering seas in the set-up as shown in Figure 3. Colour film was taken of all tests and wave surface elevation

and roll angle were recorded continuously on a chart recorder.

For the ballast loading condition tests were made using the metacentric heights given as full-scale ship value in Table 6.

The observed number of capsizes are also given for each test condition. The ratio of number of capsizes to number of tests for each condition does not indicate the risk at the tested GM-values. Each test began close to the wave generator when the model seastate had been developed throughout the basin.

The character of the wave pattern is ever changing with time and location within the chosen statistical parameters. The model is sailed in piecewise stern quartering seas along the tank. It was therefore also a matter of luck when the model was proceeding suitably for a reconstruction sailing when the proper severe wave conditions developed locally.

However, it was a convincing experience to watch the model behaviour in the particular cases and judge its tendency to capsize or to be safe. It was safe in the model basin at a GM = 0.97m, but not at the smaller values. It was not necessary to include wind and gust heeling moments to 0.64m, 0.70 and 0.75m to obtain a capsize. At 0.97m GM it was not possible to obtain a capsize even by using the wind and gust simulator.

The fully loaded model was tested under the same circumstances in the loading conditions (Table 7). Wind and gust moments were used in all cases, but, nevertheless, the model was intuitively felt to be quite safe at 0.65m and 0.55m GM values. There was some slight doubt with respect to complete safety for the 0.45m case.

1.3 Hard Facts

The immediate results of the experimental investigation can be expressed in terms of the displacement of the vessel and the GM-value. A more detailed specification is given by the related GZ-curves and tabulations of the individual sub-criteria of the IMCO recommendations.

These data are presented in the diagrams Figures 5, 6 and 7 and in Table 8.

It should be remembered that the data describing the results of the tests were obtained in a model basin under

laboratory conditions, for the particular type of ship investigated, in a single seastate which was not extreme. The results represent single points only in a regression diagram for a statistical evaluation of required parameters for safety against capsize.

2. ANALYSIS

2.1 Introduction

The hard facts of the reconstruction of the capsize in model scale are direct results. Combined with a lot of corresponding results from model tests and - preferably - full-scale experience, a new statistical analysis as performed by IMCO, see Reference [2] for full load vessel capsizes, can be made. It might even be possible to introduce new significant parameters.

However, let us try to analyse the observed model scale capsize phenomena, which yields much more detail than sea-disasters do, to see if it is possible to dig deeper into the physics.

Such an exercise will, if we are lucky, provide us with information on possible new parameters in addition to the usual ones, the e_{30} , e_{40} etc. used in the regression analysis. It might also bring us a little step towards a rational solution of the safety against capsize problem.

There is not doubt that the capsize in the model basin was a dynamic phenomenon. Lack of static stability does not concern us in the following. We do know some facts from the real disaster, but the instrumented repeatable model phenomenon filmed at 50 frames per second will be our only reference to physics in the analysis.

The usual way of analysing dynamic stability phenomena is so far a rational one. It takes place in terms of ordinates and areas related to the GZ-curve using principles of energy and moment balance. The method studies in each case the work done by the heeling moments against the stability work done by the righting moments. Figure 8 indicates the method where the case shown is safe. The roll motion commences from the initial heel to port, passes through upright and the heeled equilibrium position where the kinetic energy of the roll motion is at its largest, and ends with the

utmost starboard heel angle where all the kinetic energy is absorbed by the stability work. The two shaded areas are equal. The righting lever at the final heel angle is positive and will accelerate the motion towards upright. Reference [4].

The half roll period studied was not the ultimate. The ship did not capsize.

2.2 Critical Examination

- 1) The external loads The analysis sketched above is physically sound. The question of capsize or not and the details of the heel motion are completely described by the diagram. At any instant from the port down position, to the starboard down position, the differential change of potential energy represented by the strip area Δ (GZ-heeling lever). $d\phi$ equals the change in kinetic energy $1/2 I (\phi + d\phi)^2 - \phi^2$ and thus by integration the instantaneous velocity of motion.

The success of the approach in predicting the effect presupposes, however, that all causes have been taken into account.

The wind and gust moments acting on the ship are not the only environmental effects. The waves also act on the ship. In the usual analysis it is assumed that the energy transferred from the waves to the rolling motion dissipates at the same rate into damping of the motion (wave generation damping, eddy and roll drag damping). This must be the case in the long run, when the ship does not capsize. This is not necessarily the case when a single half roll is considered, and especially not when a potential ultimate-half-roll is studied.

In a pure sinusoidal motion of roll in a pure sinusoidal wave exciting moment the motion is expressed as:

$$\phi = \phi_0 \sin \omega t$$

with the velocity

$$\dot{\phi} = \phi_0 \omega \cos \omega t$$

It is seen that the linear damping term $B \dot{\phi}$ is expressed by an

elliptical curve in the GZ-diagram because the above two equations form a parametrical representation of an ellipse. The same argument is valid for the out of phase component of the wave exciting moment. In the resonance case where the roll motion is largest it holds true for the complete exciting moment. See Figure 9. When the roll motion is stationary and sinusoidal as assumed above, the two contributions, exciting moment and damping moment, cancel each other identically at any instant. This is not so in factual cases which are neither stationary, linear nor sinusoidal. The elliptical form is probably more or less retained as a first approximation. Further research is needed.

The wind moment applied in the analysis needs further consideration. In many standard cases the heeling moment due to wind is assumed equal to the product of the wind force and the vertical distance between the geometric centroids of the lateral areas of the above-water ship and the under-water ship, thus assuming horizontal forces only. Any vertical component of wind force is compensated for by a very slight change of draft.

However, the pressure field over the above-water ship will transversely be equivalent to a non-horizontal force and a moment which also has contributions from local vertical forces. Measurements in wind tunnels of horizontal force and total moment will give a vertical position of centre of pressure distinct from the area centroid. The same holds true for the under-water hull. Dependent on above and below water shapes, the vertical wind moment lever will differ from the centroid lever, see Figure 10. In the case of gust contribution, the under-water pressure field and thus centre of pressure will be different from the stationary mean wind case. These effects and the ones caused by rolling motions and the other ship motions themselves have not been taken into account in the present investigation. Further research is needed.

- 2) The GZ-curve The righting lever curve in the diagram is of the utmost importance for the out-

come of the analysis of dynamic capsize, with equivalent consequences at sea. The curve is completely described for static heel of any ship in still water by the well-known expression:

$$GZ = GM \sin \phi + MS$$

in the usual notation provided the MS-values are computed for the ship free to change trim. The righting moment is the product of the lever and the weight of the ship equal to the total buoyancy force.

When the ship is labouring up and down in waves the instantaneous buoyancy force is not equal to the weight of the ship. The GM-value, the slope of the initial tangent of the GZ-curve, and the residuary lever MS differ from what they were in still water. Their origin, the pressure distribution around the underwater hull, differs from the hydrostatic valid in still water case.

The GZ-values depend on the heave motion, the pitch motion and the mere wave passage. It may be expressed precisely in mathematical terms as the derivatives:

$$\frac{\partial GZ}{\partial z} \neq 0 \text{ for heave } (z) \text{ dependency}$$

$$\frac{\partial GZ}{\partial \theta} \neq 0 \text{ for pitch } (\theta) \text{ dependency}$$

$$\frac{\partial GZ}{\partial \xi} \neq 0 \text{ for wave } (\xi) \text{ dependency}$$

They are all dependent on hull form, loading condition and wave parameters. The first two are conveniently determined for fictive motions in still water for which case the initial values $\partial GM / \partial z$ and $\partial GM / \partial \theta$ have been treated in great detail in Reference [5]. These initial derivatives have a great influence on the roll motion as they affect the motion stability by transferring roll energy from the heave and pitch motions respectively in the case that these motions occur twice as fast as the roll motion. Many ship models have capsized in the SL-model basin even in head

seas due to this effect. It is not thought to be acting in the present case where the heave and pitch motions were forced by the encountered waves having periods quite distinct from 1:2 of the roll period. The roll motion occurred approximately at resonant forced by the same waves.

Only the wave passage dependency need further consideration in the present case. The determination of the variation of GZ ordinates during the wave-passage is a rather complicated matter. As a first approximation, the GZ-curve in waves obtainable from existing computer programs was therefore chosen. They represent fictitious static situations of a ship being inclined in a wave at selected parameters and locations along the ship side. The computations take into account the apparent change of buoyancy in the wave, but not the disturbance due to the presence and the motions of the ship. The results of the calculations for ballasted and loaded conditions for a 100 m long wave of 9 m wave-height positioned with a crest respectively trough at midship section are shown in Figures 11 and 12.

In order to study the time history of a capsize it is necessary to know the time history of the GZ-curve. The best way of obtaining this on the adopted basis is to compute the GZ-curve for a set of locations of the wave crest and trough along the ship length and between the two shown in Figures 11 and 12. This has not been done. Instead a harmonic variation has been assumed as sketched in the figures. The average value is more or less equal to the still water values. This does not mean, however, that the effect of the variation in waves is nil in the long run, because the long run is made up from particular cases like Figure 13.

Commencing with the port down attitude in the wave crest, the righting lever curve is at its highest numerical value. As the roll develops the wave passes along the ship so that the GZ-curve is at its average value when the ship is upright and at its lowest when the need is largest, at the starboard down attitude in the right hand side of the diagram.

The potential energy bound in the port heeled condition due to the righting levers and the heeling levers, is exaggerated by the wave action which increases the righting lever. Later in the motion the righting levers, which must absorb the kinetic energy involved in the roll angular velocity, are diminished by the wave action. This causes the heeling angle to increase until the shaded areas become equal. In this example, the final righting lever is positive. The ship returns towards upright.

- 3) The initial angle of heel It is apparent that the result of the analysis depends strongly on the applied initial value of heel. For regulations and standard procedures it is necessary to specify this value directly or indirectly. In the present case we are in the lucky position of having recorded this angle during the model basin tests. It appears from Figure 14, which shows a typical time history of a capsize. A record of the time history of a set of the largest roll angles which occurred during the loaded condition tests is shown in Figure 15.

2.3 Analysis of the Capsize

A simple dynamic stability diagram like Figure 8 will not explain the capsize as it occurred. The hypothesis illustrated in Figure 13 tries to explain capsizes by the effect of variation of the GZ-curve ordinates during the wave-passage as discussed in a previous section.

The model experiments performed present an opportunity to test this hypothesis. We have quantified the causes, though only to a first approximation, and we have recorded a time history of the capsize. The main hypothesized cause, the GZ variation, is significant in the ballast case but much less so in the loaded case, where capsize was not encountered.

The GZ-variation for the ballast case appears from Figure 11. The wind moment was nil. The simulator in the model was not used in the analysed experiment. The moment from the waves is taken equal to the product of displacement, the metacentric height and the surface-wave slope corrected for angle of incidence, by the sine of this angle. The resulting wave slope was 8 degrees corresponding to a heeling lever of 8.9 cm. The damping moment of the roll motion is determined as $B \dot{\phi}$. The angular

velocity is estimated at the equilibrium position to correspond to the potential energy represented by the left hand side shaded area of Figure 17b. The velocity is about 10 deg/sec. The linearized damping coefficient B is found from the expression:

$$B = \frac{\Delta \cdot i}{F} \sqrt{\frac{GM}{g}} \quad (4)$$

derivable from any text book on linear oscillations. The amplification factor F is in the present case found from additional model experiments in beam waves and from roll decay tests. The value is on the average, about 3 for the roll angles occurring. See Figure 16. The resulting maximum damping lever is 6.2 cm. The time history of the capsize appears from Figure 17a.

Taking everything together the result appears as Figure 17b. The effective GZ-curve is drawn through the time mark plots on the instantaneous curves from Figure 11. The area between this curve to the left of the ordinate axis and the ellipse-like curve representing the difference between the wave forcing and the damping moments continued to the equilibrium heel of 2 degrees starboard, must be absorbed by the right hand side shaded area. This is easily accomplished at an angle of about 14 degrees. The capsize can hence be explained this way.

The left hand shaded area is almost of triangular shape corresponding to the case of a linear motion. The duration from the start, to the equilibrium heel, can therefore be found as one quarter of the natural period of roll associated with an effective GM value of 0.96 m. The duration then is about 2 seconds. The actual capsize heel was developed only a little further at 2 seconds from the start. See Figure 17. The actual heeling moments before then can therefore only have been a little larger than the ones used in the analysis. During the following part of the motion towards capsize, the moments acting in this direction must have been considerably larger than the values used in the analysis to provoke the capsize. The actual path of the wave minus the damping curve is rather indeterminate, but does seem seem to matter much. There is plenty of potential energy represented by the non-shaded area to the right at the final 14 degrees heel angle, and there is a considerable righting lever at that angle.

A heeling moment larger than this righting lever is needed to explain the capsize. The magnitude may be evaluated from the acceleration curve of Figure 17a. It has been determined from the capsize time history by graphical differentiation twice and is therefore rather uncertain. However, the moment lever required to accelerate the ship mass moment of inertia to the final value of 2 degrees per second squared is about 0.5 m or about twice the still water righting lever and three times the effective righting lever from the above analysis. A moment of this size does not seem to be the result of uncertainties.

The loaded condition did not capsize despite the much larger variation of GZ lever than in the case of the ballasted condition. The righting lever for the wave crest amidships attains negative values immediately from up-right. The righting lever for the wavetrough amidship has large positive values from zero heel to very large angles.

Consider Figure 18, which shows the outer end of the stability work area and the time increasing GZ-curve. A roll motion passing the apparent capsize point at a finite angular velocity, because all the kinetic energy of the roll motion has not been absorbed at that heel angle, is further accelerated outwards by the moment acting thereafter. The ship might still be saved if the rise of the GZ-curve is fast enough.

This possibility of rescue is not at hand for the loaded condition because of the all negative wave crest righting lever in the present analysis. No diagram is shown of this situation.

2.4 Further Consideration

At the present stage of the investigation of the dynamic capsize in stern quartering waves the hypothesis illustrated in Figure 13 is being studied with respect to its power to explain in this particular case, the GZ-curve variations as the cause or an indispensable part of the causes of the capsize. Related to the model experiments made with the loaded and the ballasted model, numerical values of the assumed cause have been obtained, though only as a first approximation.

Summarizing the circumstances which have all been made equal as far as possible for the loaded and the ballasted ship model, Table 9 may be set up [6].

This table taken together with the failed analysis of the ballasted case show that we are on the wrong track. The hypothesis has been falsified for the present particular case and therefore [7] also for the general case as a necessary and sufficient explanation of capsize in stern quartering waves.

One significant observation from the model tests might save the analysis and explain the capsize for the ballasted case contrary to the loaded case. The harmonic variation of the GZ-curve presupposes a wave passage along the ship at a constant velocity. The visual observations for the loaded ship model confirmed this pre-supposition. For the ballasted ship model, however, it was clearly seen that this was not the case. The ship model was carried along with the wave crests at their speed and retained during the passage of the troughs resulting in the maintained average speed. The instantaneous ship model speed relative to the constant wave profile celerity must have caused the GZ-curve variation to be strongly non-harmonic, more or less as shown in Figure 19.

Taking this into account in the capsize analysis of a half roll Figure 20 shows that it does not explain the capsize in spite of the much longer continuance of the effective GZ-curve on the wave crest amidship curve. This is due to the fact that the kinetic energy to be absorbed is also reduced, which is seen from the reduced size of the shaded area of the heel potential energy.

2.5 Strip Theory

An efficient tool which is now classical in seakeeping work is the strip theory computational method. It deals with regular motion in regular waves. The transfer functions which are the ship motion characteristics quantified by the above methods can perfectly be adopted for spectral analysis approaches to predict the behaviour in random seaways.

Consider the roll transfer functions for the ballasted and loaded conditions for the present ship. They have been computed using [8] which is a Danish version of the Salvesen, Tuck, Faltinsen strip theory [9] and the Potash potential flow calculations [10]. An important deviance from the original work [9] is that the non-linear damping of the roll motion, numerically treated by iteration, is associated, by the simultaneous

computation of the coupled motion modes. They are still treated as linear, but the contributions from the roll coupling terms are better balanced in this way, to make better transfer functions.

The computed transfer functions for the two loading cases appear from Figure 21. They have been computed from the linear equation of roll motion coupled with sway and yaw (equation IV of [8] for variable No. 4):

$$\begin{aligned} & (A_{44} + I_{44}) \ddot{\phi} + B_{44} \dot{\phi} + C_{44} \phi \\ & + (A_{42} - MZ_G) \ddot{y} + B_{42} \dot{y} \\ & + (A_{46} - I_{46}) \ddot{\psi} + B_{46} \dot{\psi} \\ & = F_{41} + iF_{42} \end{aligned} \quad (5)$$

The motion mode 4 is roll, 2 is sway and 6 is yaw. The exciting moment on the right hand side of the equation F_4 has a real component F_{41} and an imaginary component iF_{42} in the complex notation used.

The coefficients are given in Table 10 for a 100 m wave length as used in the model experiments. The table values are extracted from the complete computer output.

All values are stated per meter wave amplitude. The inertia product I_{46} is taken equal to 0 due to lack of information on the proper value. The I_{44} is taken to comply with an effective rate radius of gyration for the ship of about 0.4 ship breadth. The non-linear damping coefficient was given a value of 0.5 for both loaded and ballasted condition being an average of model test values from roll decays analysed by a numerical procedure [11].

The harmonic motion being dealt with has the motion amplitudes of Table 11 for the angular frequency of encounter ω_e .

Referring these amplitudes to the values of motion per metre wave height or radian of slope of Figure 21 and introducing them into the equations of Table 11, the individual terms will obtain the values given in Table 12.

The values for ballasted ship and loaded ship in Table 12 are more or less of the same magnitude except the sway mass coupling term and the wave excitation. They are both larger for ballast than for loaded by a factor of

2 respectively 5. In terms of righting levers the difference is more pronounced by a factor two between the ship weights of the two conditions. The wave excitations stated are the pythagorean sums of the components of Table 10.

2.6 Vector Diagrams

A convenient way of studying the individual contributions in their mutual relationship is the vector diagram (Gauss plane [12]) presented in Figure 23 for ballast condition and Figure 22 for loaded condition. They show to the same scale the amplitude and phase of each contribution in such a way that the length of each vector represents the magnitude and its direction represents the phase angle of each particular contribution.

Looking at the loaded condition diagram first it is seen that all vectors are less in size than the righting moment vector. All quantities are given per meter wave amplitude. The roll angle in the 100 m long waves encountered 30 degrees from aft is about ± 2 degrees and thus not comparable to any capsized situation, but does on the other hand compare to the rolling in the higher capsized waves.

The vectorial sum of all contributions is nil except for a small contribution from the second order roll damping which is not shown. The dynamic equilibrium diagram shown has had the excitation moment transferred to the left hand side of the equation which is the reason for the opposite sign in relation to the table values.

Turning now to the vector diagram for the ballast condition of Figure 23, it is seen that the exciting moment is more than twice the righting moment and that the active roll moment due to sway acceleration is almost twice the righting moment. The other contributions are relatively small or even negligible. The above two large contributions do each have the magnitude we were looking for in the analysis based on the Figure 13 hypothesis measured by the righting moment.

Vector diagrams describe harmonic motions and do not lend themselves to the description of a Figure 13 motion. However, for a more or less equivalent harmonic motion only four vectors would be present, the righting moment vector and the oppositely directed inertia vector. Perpendicular to this direction along the vector would occur the damping moment

vector. The size of this would be as many times smaller than the righting moment vector as the earlier used amplification factor F , i.e. a factor of about 3 in this case. The vectorial sum of the damping vector and the difference between the two opposite and, for this particular case, almost equal righting and inertia vectors would be equal and opposite to the wave exciting moment, see Figure 24.

The present diagram Figure 23 is much more complicated. The wave excitation is far from perpendicular to the righting moment vector due to the interaction of the coupling terms. Summarising the above exposition two large roll moments appear in the coupled harmonic motion. Each of them has the size required to capsize the ship in terms of the earlier analysis. One of them is the direct wave exciting moment. It is much larger by a factor of about 5 than the earlier assumed value based on surface wave slope. The other is the sway acceleration coupling moment. But they are mutually interlocked in opposite directions. Their difference has the magnitude of the earlier used exciting moment which means that they almost cancel each other.

2.7 The Non-Linear Case

The above assumptions quantified by the strip theory relate to harmonic motion which requires the righting moment to be proportional to the heel angle, i.e. a GZ-curve following the initial GM tangent. This required moment will accelerate the roll motion towards upright from the utmost heel angle to each side in the motion.

If now the heel in the roll motion has become relatively large due to the effect of direct excitation moments and the Figure 13 effect of GZ variation then the instantaneous righting lever, when the motion stops, is much less than the value of the corresponding linear case.

The roll motion can therefore not keep pace with the other motions sway and yaw and lags behind them disturbing the entire motion and thus the magnitude and phase of the individual moment components.

If especially the sway motion continues more or less unaffected by the delayed roll and the direct roll exciting moment is closely related to the roll motion itself, then the mutual interlocking between the two large moment components cause their total effect to

increase considerably. The other moment contributions are too small to affect the motion.

In the loaded case all moment contributions are small in relation to the righting moment and thus not able to cause trouble. The capsize in the ballasted case contrary to the loaded may then be explained by the following section.

2.8 Final Hypothesis

The ship was rolling moderately in stern quartering seas. The GZ-curve was varying as the waves overtook the ship in a non-harmonic way causing the particular motion from one side to the other which should end up as the ship's ultimate half roll to have much reduced righting levers. This caused a continuous lagging of the roll motion behind the sway and yaw motion. The lagging disturbed the balance between the large wave exciting moment and the large sway acceleration moment existing in the harmonic motion, where they almost completely cancelled each other out. The new resultant of these two outweighs the remaining righting levers and accelerates the roll motion into capsize.

3. CONCLUSIONS AND RECOMMENDATIONS

Reference [2] mentions two contemporary practices regarding ship transverse stability. The first balances heeling levers against righting levers and the second specifies coordinates of the GZ-curve. The first may be interpreted as a physical approach and the second as a statistical approach see also Reference [15]. The physical approach will be able to render new insight, whereas the statistical approach will be able to include all model and full-scale experience using regression methods, either formal mathematical ones or more intuitive practical ones as they appear from [2]. The regression parameters must anyway be determined from a physical theoretical insight in the phenomena.

There is not doubt that these can, in principle, be described and predicted entirely based on Newton's laws. But the circumstances are so complicated that special theories and hypotheses will be required to describe them. The analytical geometrical character of foundational work such as [16] and [17] is not equivalent to that of the present topic.

It is inherent in the present work that the parameters of [18] are not found completely representative. The hard facts have immediately been related to the displacement of the ship giving a few values for a coming regression analysis.

However, the main part of the investigation is concerned with the set of and the magnitude of individual contributions to capsize in stern quartering seas at moderate initial rolling. The investigation ends up with a hypothesis for this particular case that the sway coupling moment and the very large wave exciting moment which normally almost cancel each other, taken together with the effect of GZ variation expressed by an earlier hypothesis (Figure 13) and together with the effect of the strong non-linear character of the righting moment are responsible for the ballast capsize. No study was made of whether or not the magnitude of these heeling moment contributions are due to the light displacement as such, or if for example the high position of C.G. alone would be sufficient to cause the large moments, and that the strong surging moment was the only contribution from the light displacement as such. This would introduce as regression parameter also a quantity such as KG, BM or similar.

This investigation will be of wider interest only if the hard facts from the model experiments can be explained for this particular capsize by the analysis, and if it can lead to the explanation of the general case and thus contribute to a scientific theory of such capsizes. [14]

The attempted analysis, based on common theoretical aspects and on the existing hypothesis (Figure 13), concerns an aspect which is not usually treated and is believed to throw some light on these phenomena. The existing hypothesis was falsified. It did not explain a single cause for the capsize. Ad hoc hypotheses did not save it. Taking in further theoretical aspects from the theory of ship motions elucidated the problem when taken together with certain considerations peculiar to the non-linear case represented by the shape of ordinary GZ-curves and taken together with the above hypothesis on GZ variation and the surging motion peculiar to the ballasted ship model. These contributions together made up the final hypothesis which is believed to explain the present particular capsize.

Its corroboration by the present model

experiments is heuristic and a formalized treatment would give it strength. An analog computer analysis is being considered. It is anyway believed that the hypothesis will be able to explain and predict the general case.

Future work must include derivation of test implications and refinements of quantification procedures and probably introduce new concepts from the general theory of stability of dynamic systems [13]. Strong test implications are still the outcome: capsize, non-capsizes of experiments with ship models with and without the hypothesized cause, but having all other aspects, e.g. the GZ-curve more similar than in the present experiments. Deeply loaded models, with and without large coupling moments and large wave excitation, should be selected for tests if they exist.

The quasi static computations of the GZ variations should be tested in model experiments similarly to earlier reported work to IMCO. In the theoretical computations the harmonic motion presupposition should be dispensed with, leading probably to a numerical integration procedure, e.g. a finite element method.

The above discussion and the investigation is made within the paradigm of deterministic mechanics [19] [14]. It would be highly desirable due to the probabilistic character of the ocean environment to transfer the treatment to the paradigm of random mechanics using descriptions in terms of long term probabilities [20].

It is the hope of the authors that the paper will contribute to the establishment of rational criteria of ship transverse stability having sufficient details to promote sound design of safe ships.

4. ACKNOWLEDGEMENTS

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Table 1 Main Particulars		
Length	L_{pp}	= 58.60 m
Breadth	B	= 9.65 m
Depth	D	= 4.15 m
Tonnage		= 498 BRT
Engine Power		= 800 HP

Table 2 Capsize Particulars		
Displacement	645	m^3
Draft at	1.75	m
Trim aft	1.52	m
Metacentric Height	0.64	m

Table 3 GZ-curve Particulars						
	e_{30} m.rad	e_{40} m.rad	$e_{40}-e_{30}$ m.rad	GZ_{max} m	GM_{max} deg.	GM m
IMCO	0.055	0.090	0.030	0.20	25	0.15
Capsize Cond.	0.096	0.135	0.039	0.32	27	0.64

Table 4 GZ-curve Particulars - Loaded Condition						
	e_{30} m.rad	e_{40} m.rad	$e_{40}-e_{30}$ m.rad	GZ_{max}	GM_{max} deg.	GM m
IMCO	0.055	0.090	0.030	0.20	25	0.15
Loaded Cond.	0.055	0.098	0.043	0.29	46	0.65

Table 5 Model Particulars		
Length	L_{pp}	= 3.66 m
Breadth	B	= 0.603 m
Height	D	= 0.259 m

Table 6 Tests in Ballast Condition			
	GM	No. of Runs	No. of Capsizes
IMCO	0.64 m	5	2
	0.70 m	3	2
	0.75 m	3	1
	0.97 m	6	0

Table 7 Tests in Loaded Condition			
	GM	No. of Runs	No. of Capsizes
IMCO	0.65	4	0
	0.55	10	0
	0.45	3	0

Table 8 "Safe" Values from Model Tests						
	e_{30} m.rad	e_{40} m.rad	$e_{40}-e_{30}$ m.rad	GZ_{max} m	GM_{max} deg.	GM m
IMCO	.055	.090	.030	0.20	25	0.15
Ballast	0.141	0.216	0.075	0.47	28	0.97
Loaded	0.026	0.048	0.021	0.14	42	0.45

Table 9 Cause and Effect		
	Ballasted	Loaded
Assumed Cause	weak	strong
Effect	strong	nil

Table 10 Coefficients of Equation of Motion			
Coefficient	Ballasted	Loaded	Unit
Hydrodynamic Mass			
A44	405	339	tm.sec. ²
A42	- 69	75	t. sec. ²
A46	-262	511	tm.sec. ²
Hydrodynamic Damping			
B44	5	0.5	tm.sec.
B42	- 2	- 0.6	t. sec.
B46	356	-387	tm.sec.
Hydrostatic Restoring			
C44	413	1041	tm
Mass Dynamic			
I44	769	1910	tm.sec. ²
MZ _G	171	163	t. sec. ²
I46	0	0	
Wave Exciting Moment			
F41	10	- 6.9	tm
F42	54	- 6.0	tm

Table 11 Motion Amplitudes			
	Roll	Sway	Yaw
Motion	ϕ_c	y_c	γ_c
Velocity	$\omega_c \phi_c$	$\omega_c y_c$	$\omega_c \gamma_c$
Acceleration	$-\omega_c^2 \phi_c$	$-\omega_c^2 y_c$	$-\omega_c^2 \gamma_c$

Table 12 The Individual Terms of Equation (5)		
Per meter wave amplitude		
Term	Ballasted	Loaded
Unit	tm	tm
Pure Roll		
$(A_{44}+I_{44}) \ddot{\phi}$	17.4	20.6
$B_{44} \dot{\phi}$	0.2	0.1
$C_{44} \phi$	24.2	37
Sway Coupl.		
$(A_{42}-M Z_G) \ddot{y}$	-43.4	27.4
$B_{44} \dot{y}$	0.7	0.1
Yaw Coupl.		
$(A_{46}-I_{46}) \ddot{\psi}$	- 1.78	3.3
$B_{46} \dot{\psi}$	4.8	4.9
Wave Excitation		
F_4	55.4	9.1
Ship Weight	645 t.	1561 t.

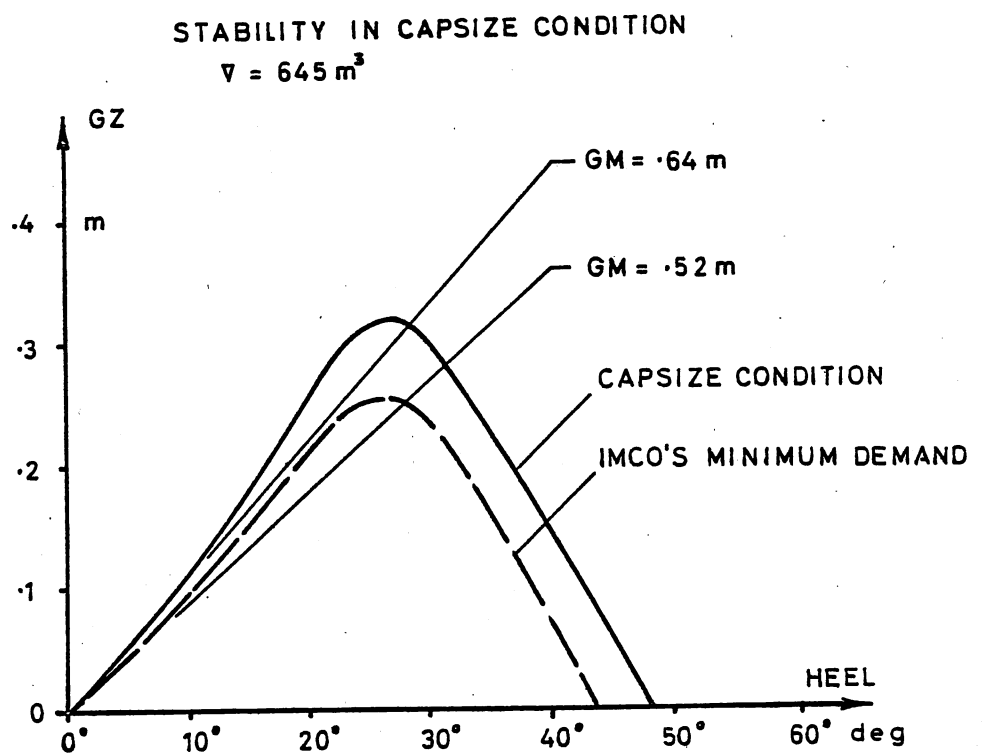


Figure 1 The Righting Lever Curve for the Capsize Condition and the IMCO Minimum Curve

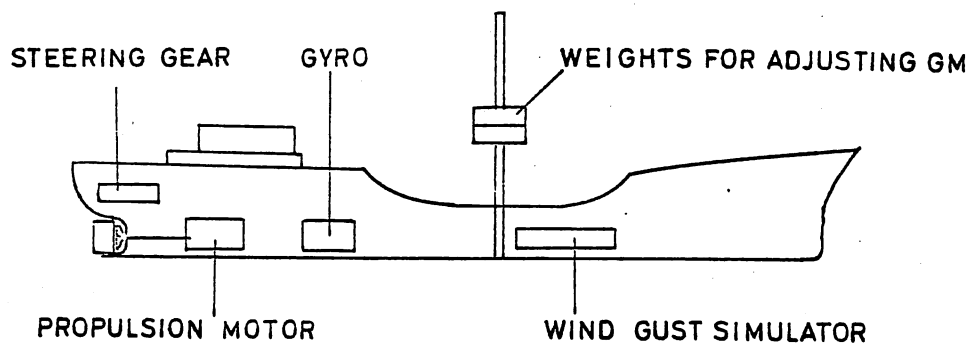


Figure 2 Instrumentation of the Model

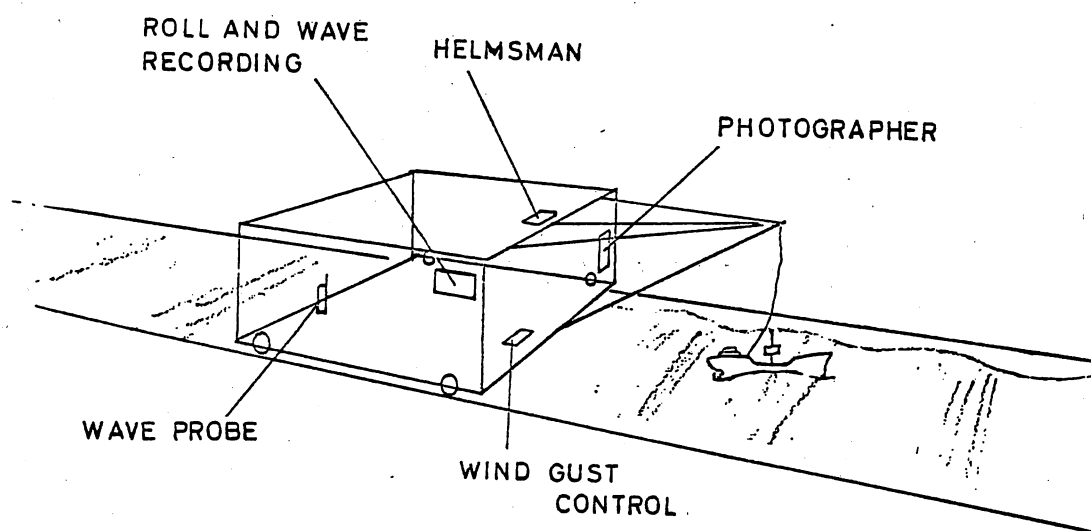


Figure 3 Experiments in the Model Basin

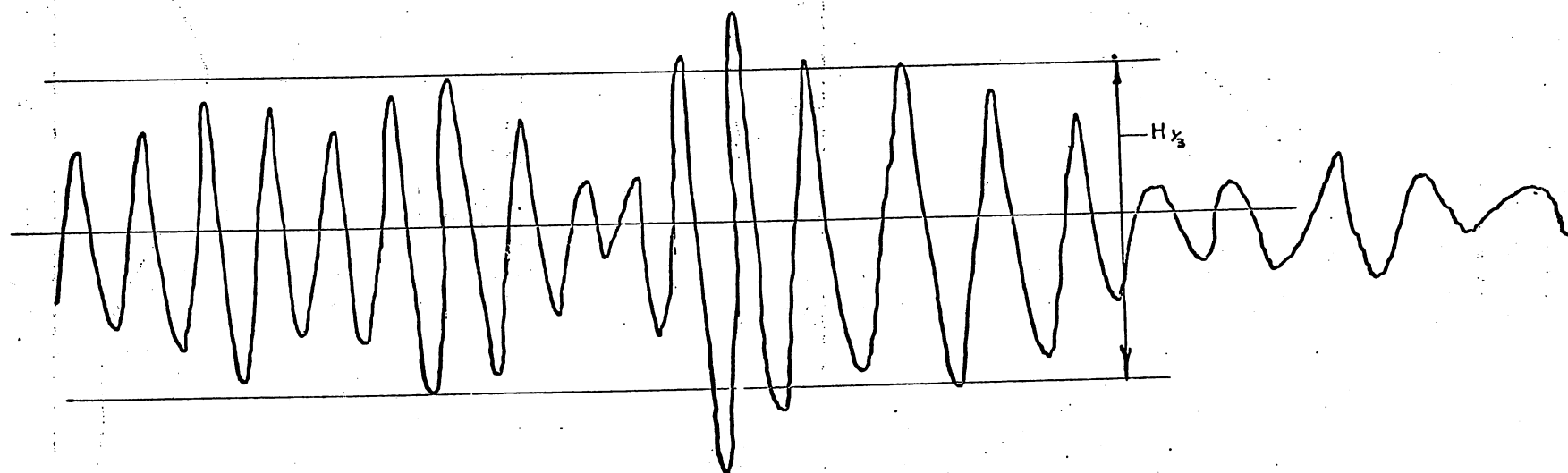


Figure 4 A Sample of the Model Waves

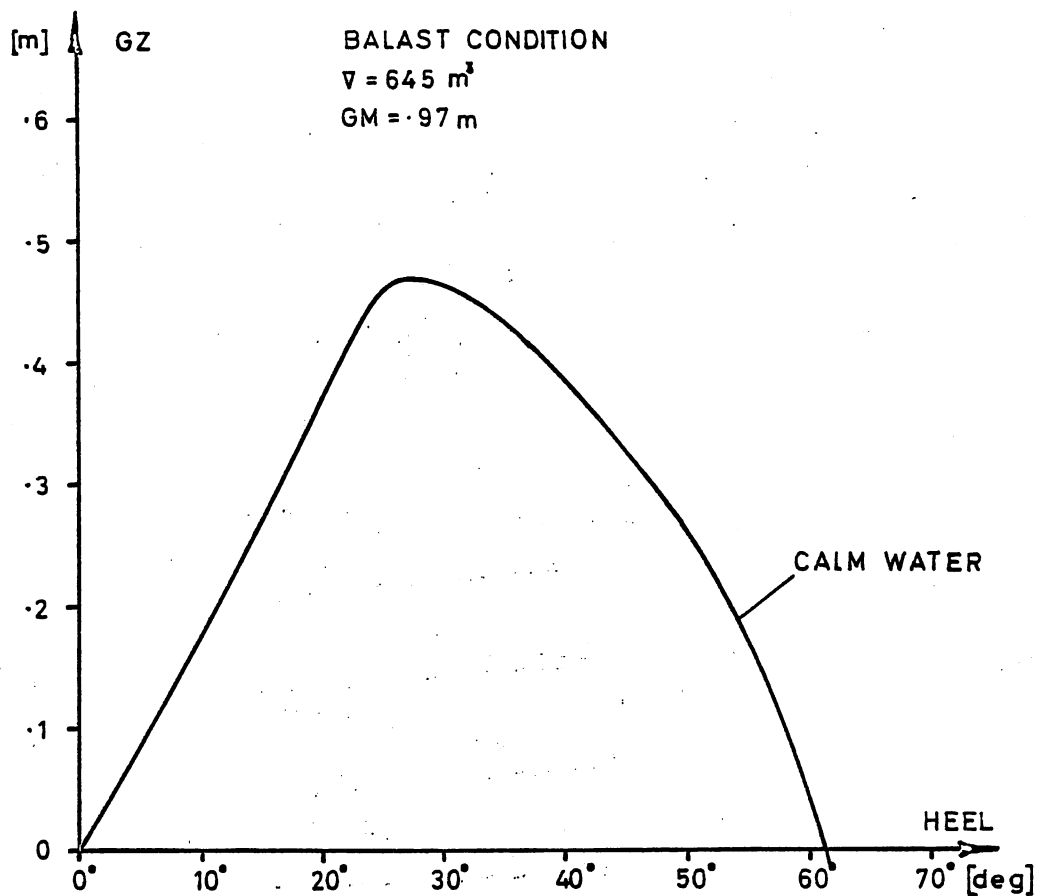


Figure 5 The Lowest Righting Lever Curve for the Ballast Condition which did not Capsize

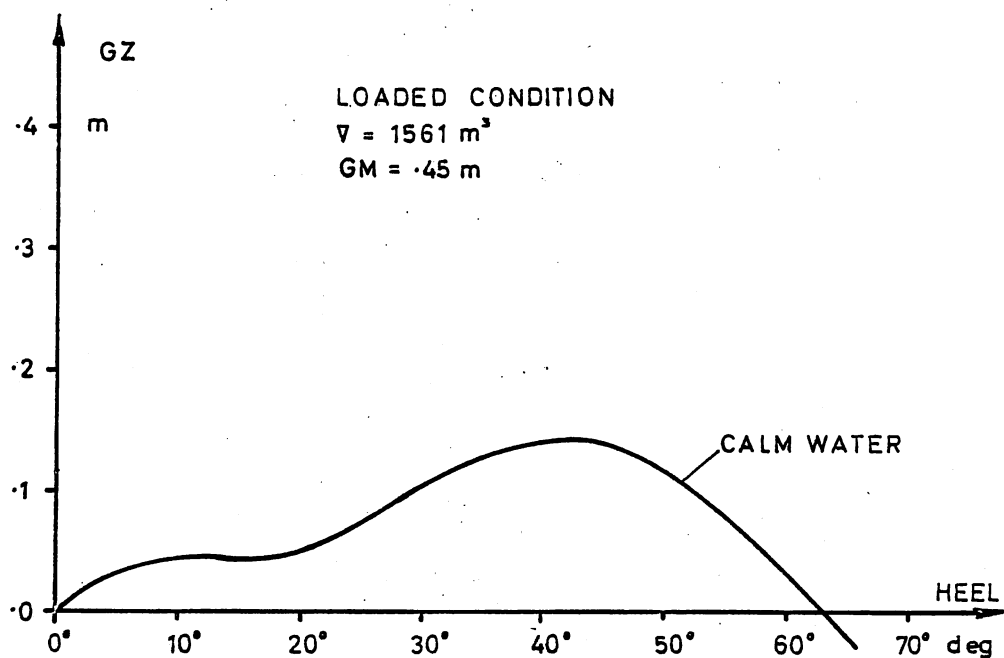


Figure 6 The Lowest Righting Lever Curve for the Loaded Condition which did not capsize

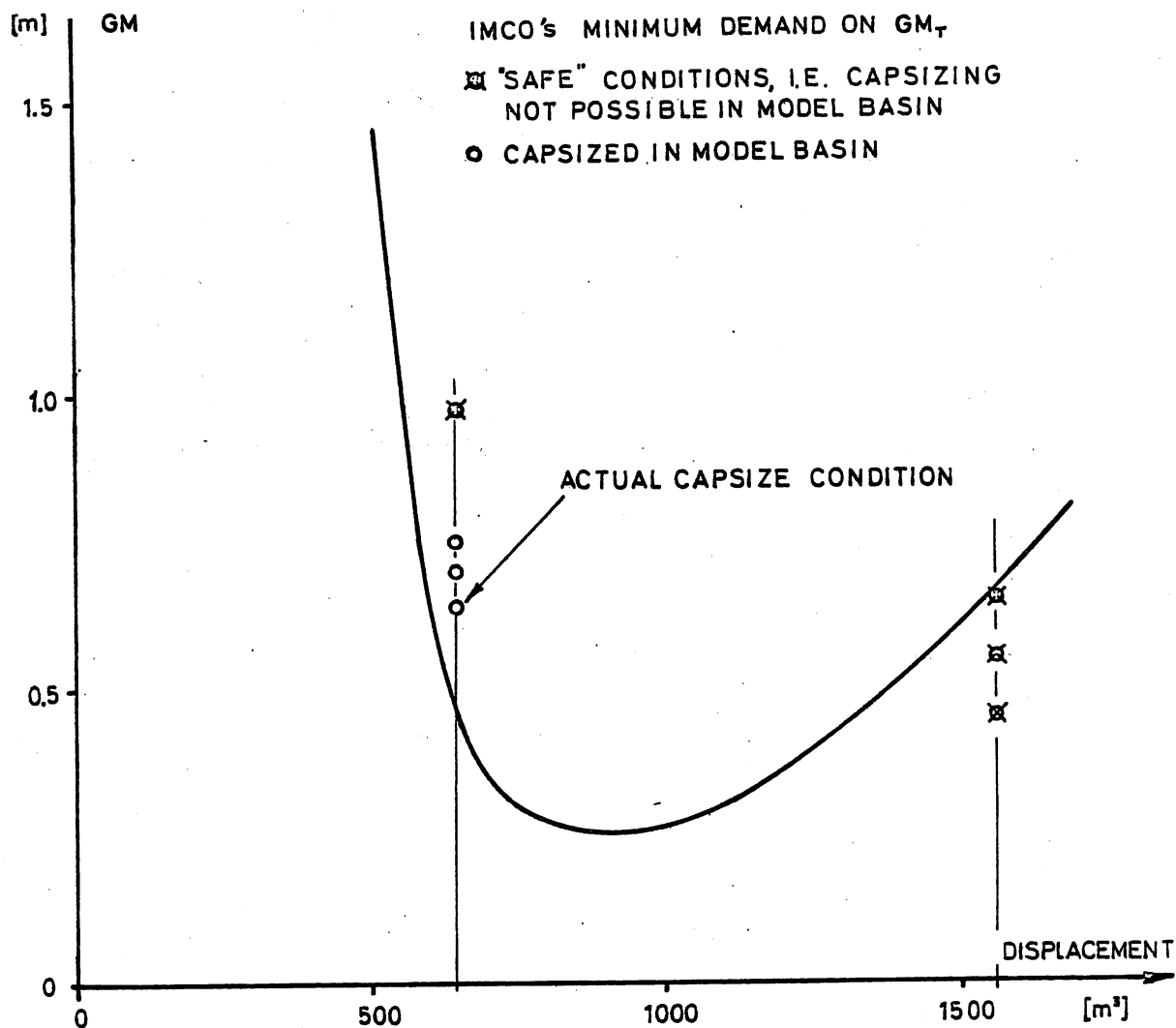


Figure 7 The Results of the Experiments

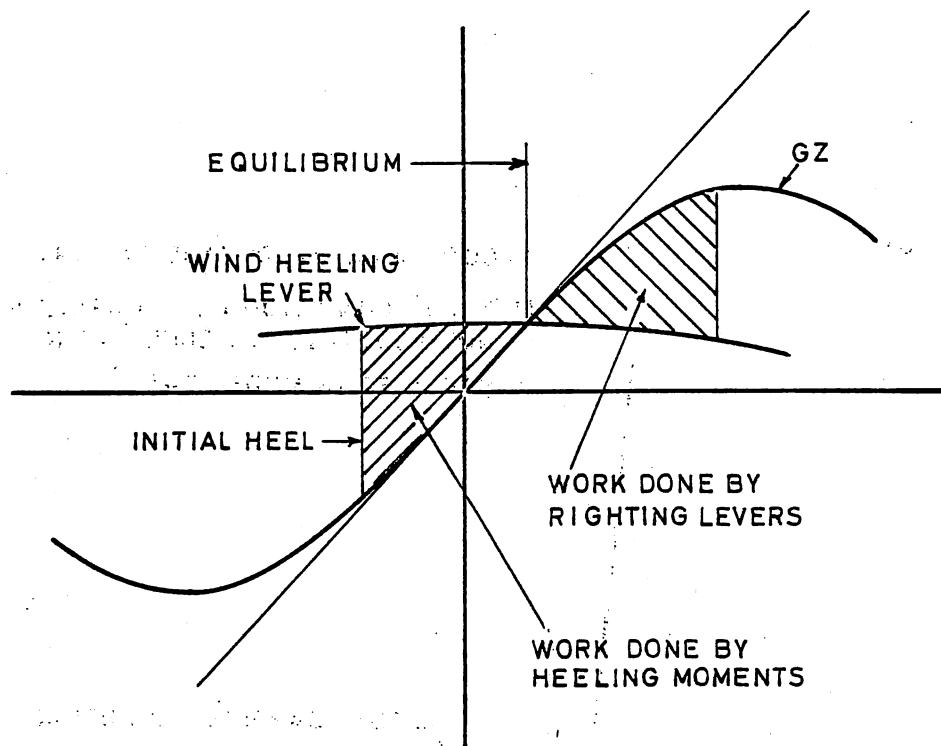


Figure 8 The Classical Dynamic Analysis

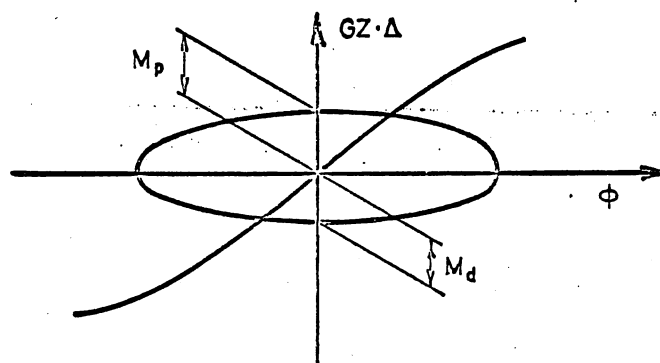


Figure 9 Wave Exciting Moment M_p and Damping Moment M_d in the GZ-Diagram

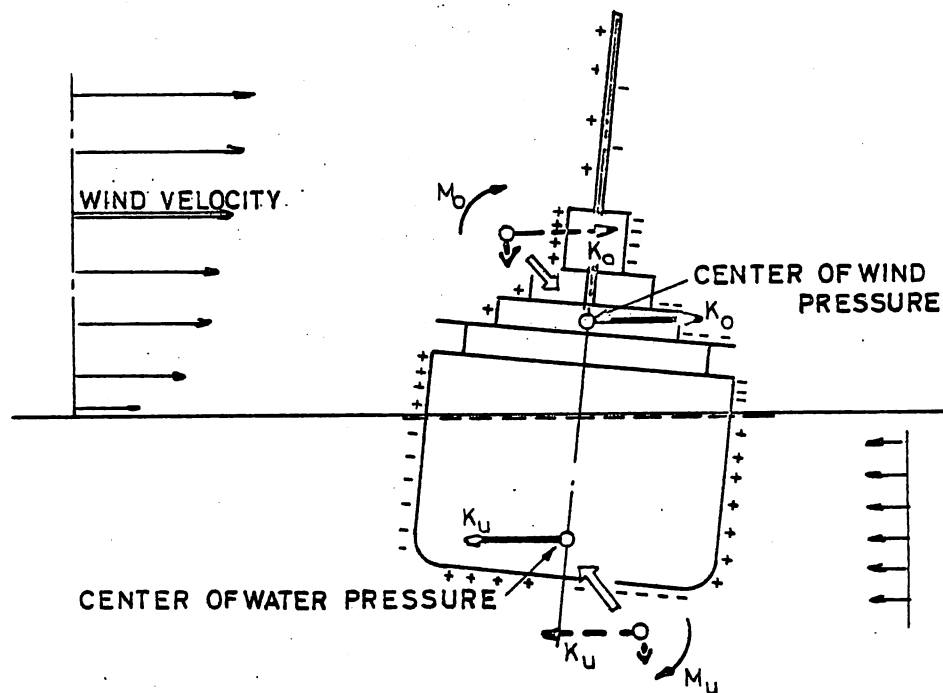


Figure 10 The Wind Pressures over the Hull causes a Heeling Moment and a Side Force

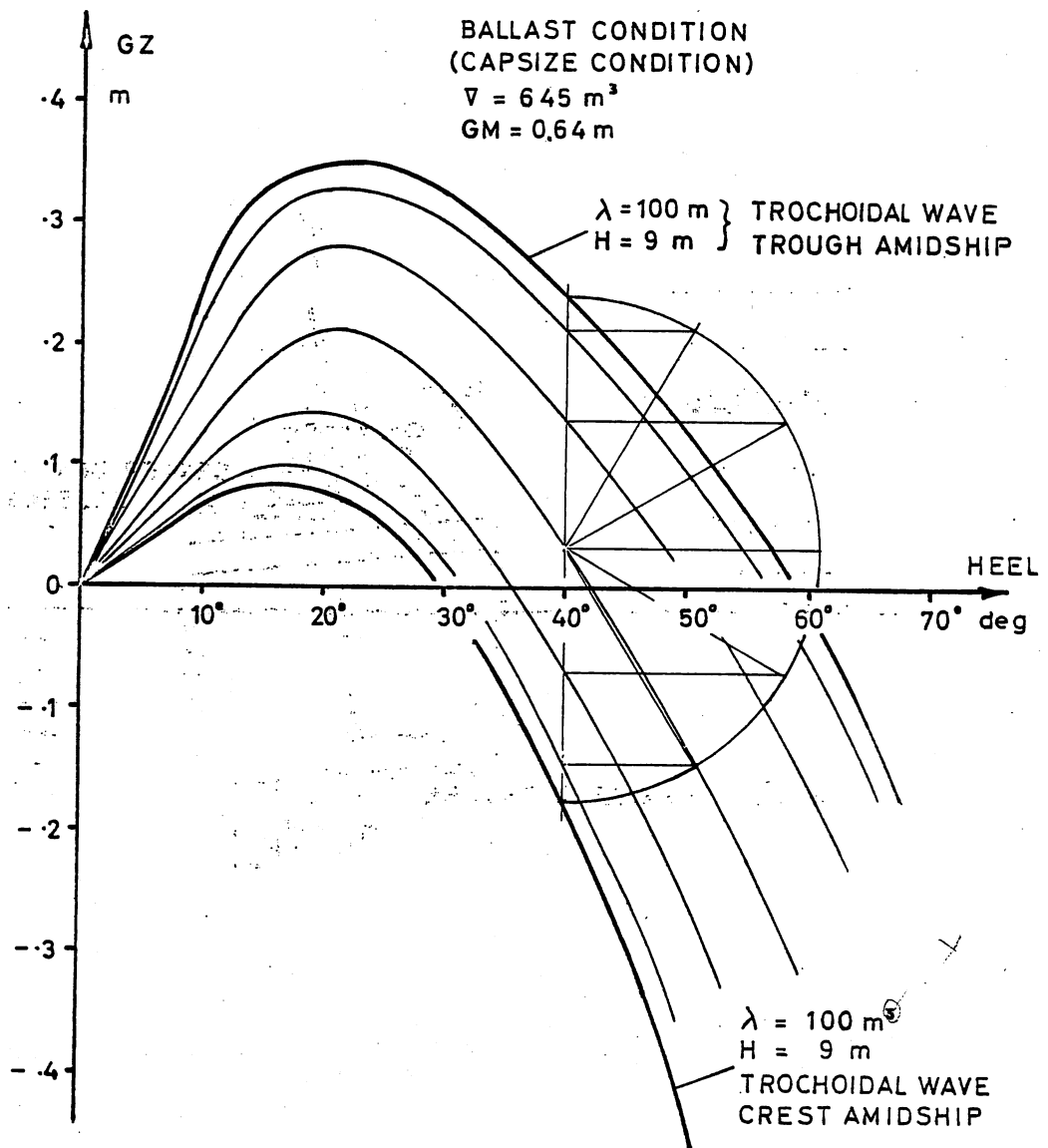


Figure 11 The Variation of the Righting Lever during Harmonic Wave Passage for Ballast Condition

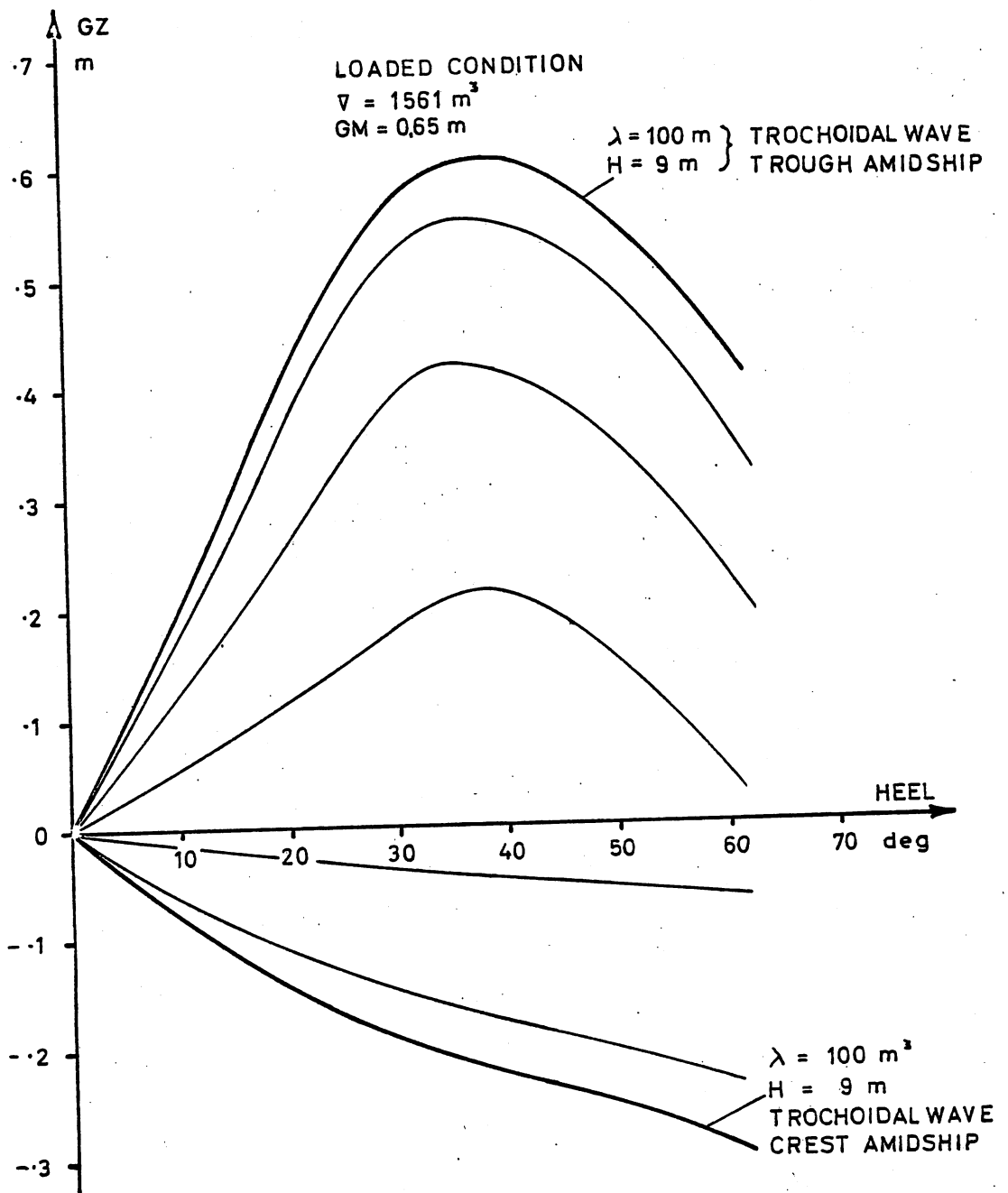


Figure 12

The Variation of the Righting Lever during Harmonic Wave Passage for Loaded Condition

FIG. 13

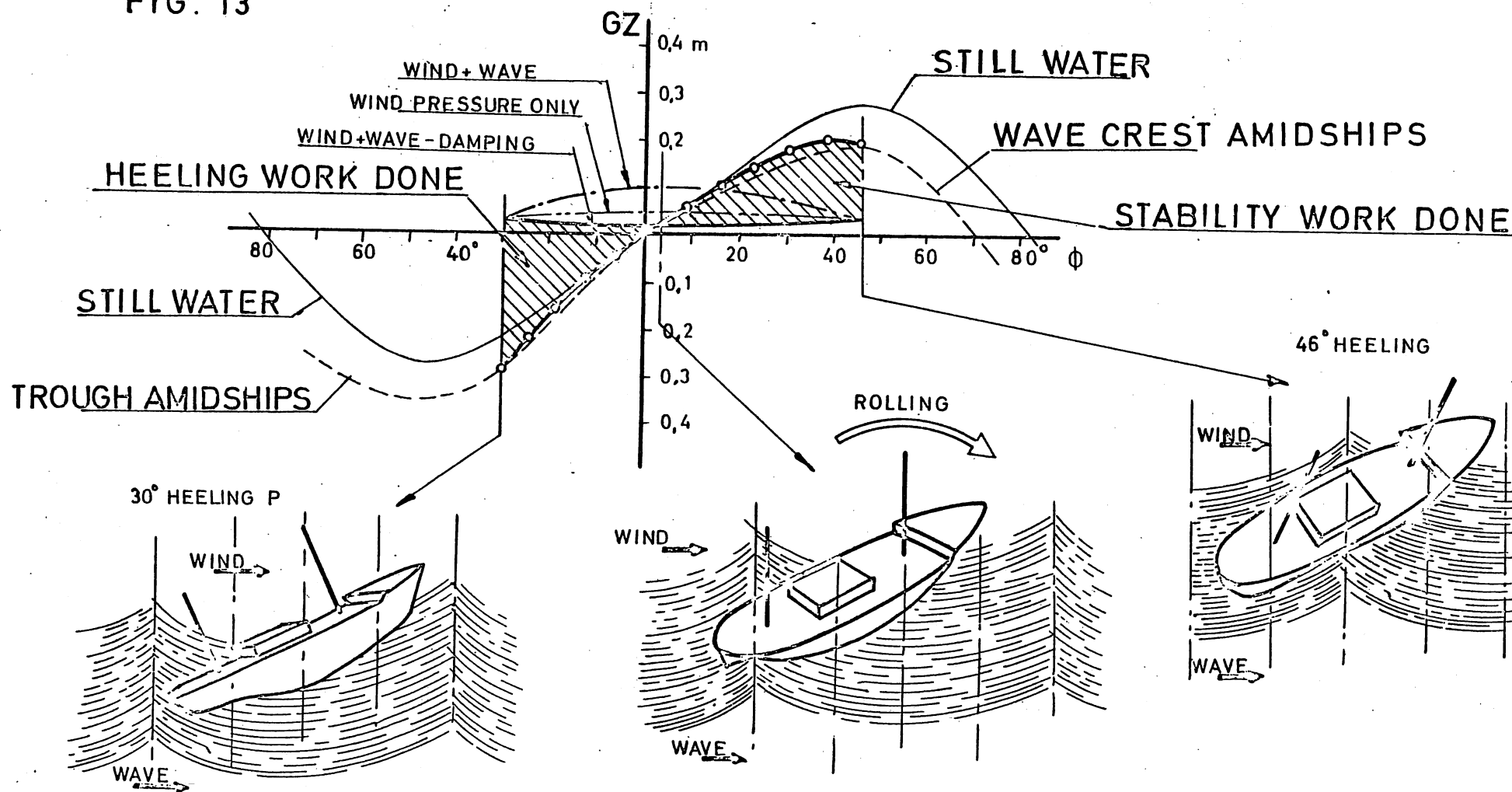


Figure 13 The Effect on Roll of GZ-Curve Variation in Stern Quartering Waves and Wind

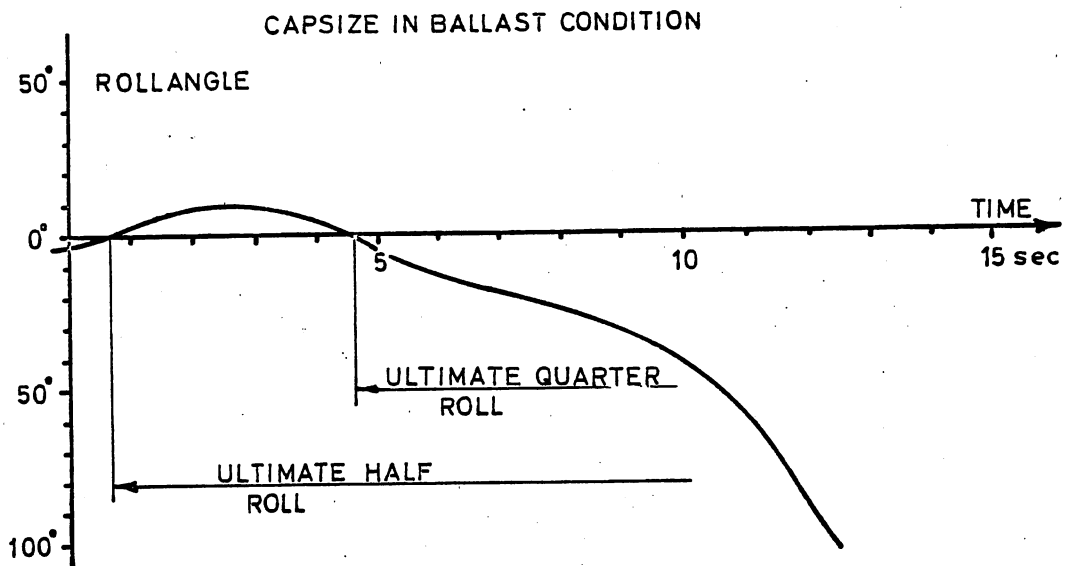


Figure 14 The Capsize Time History from the Model Experiments

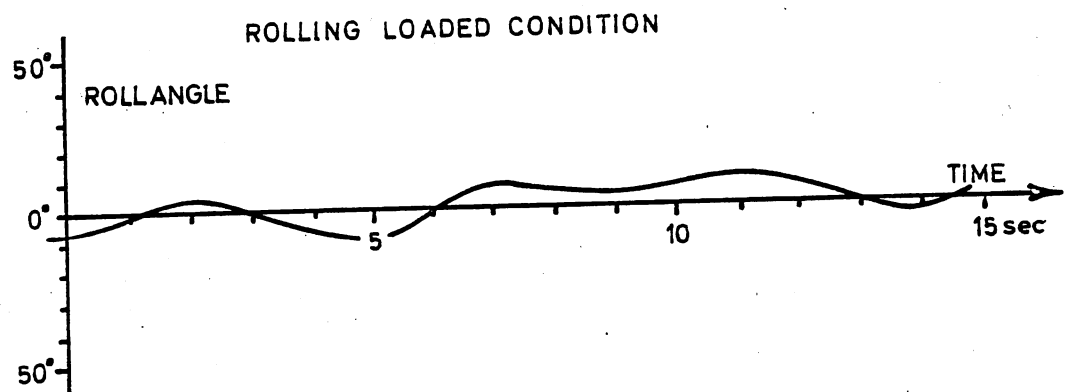


Figure 15 Representative Rolling for the Loaded Condition in the Model Experiments

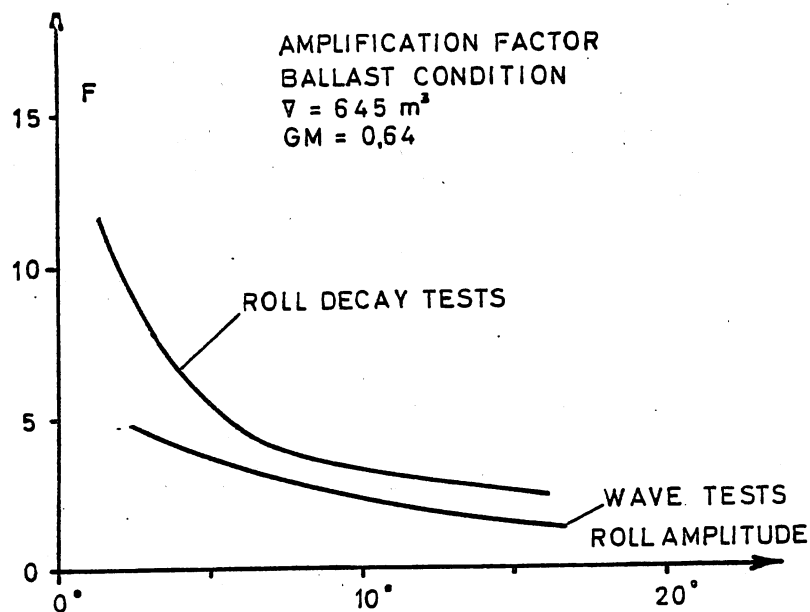


Figure 16 The Roll Damping is Expressed by the Amplification Factor

FIG. 17a

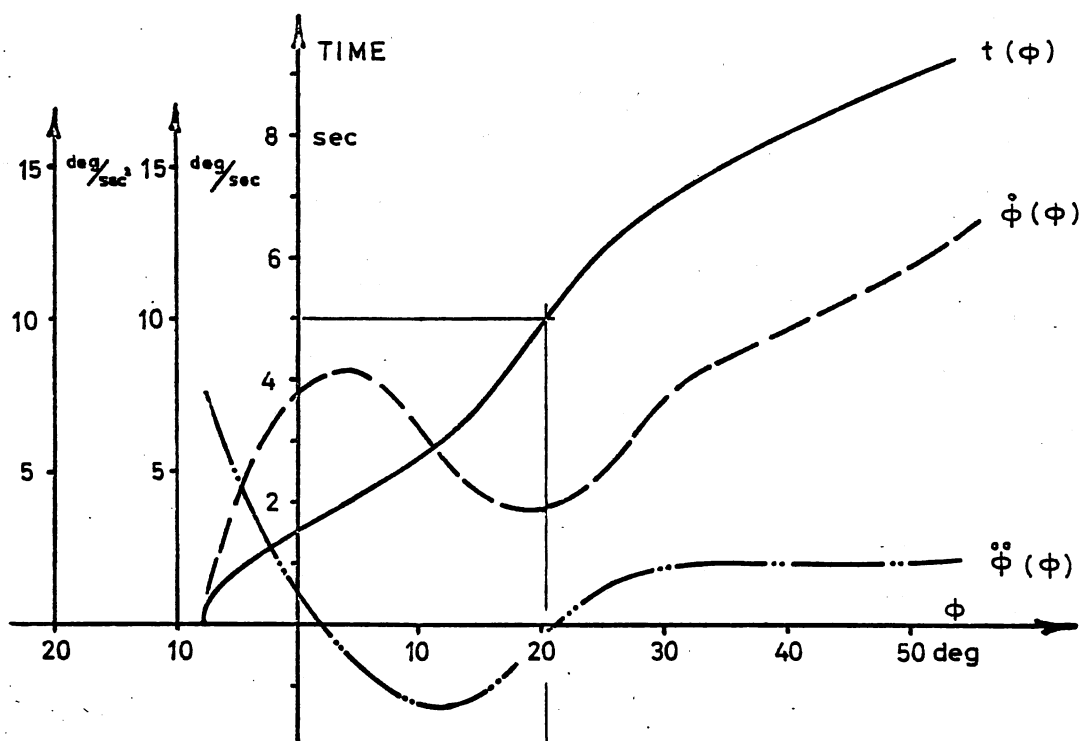


FIG. 17b

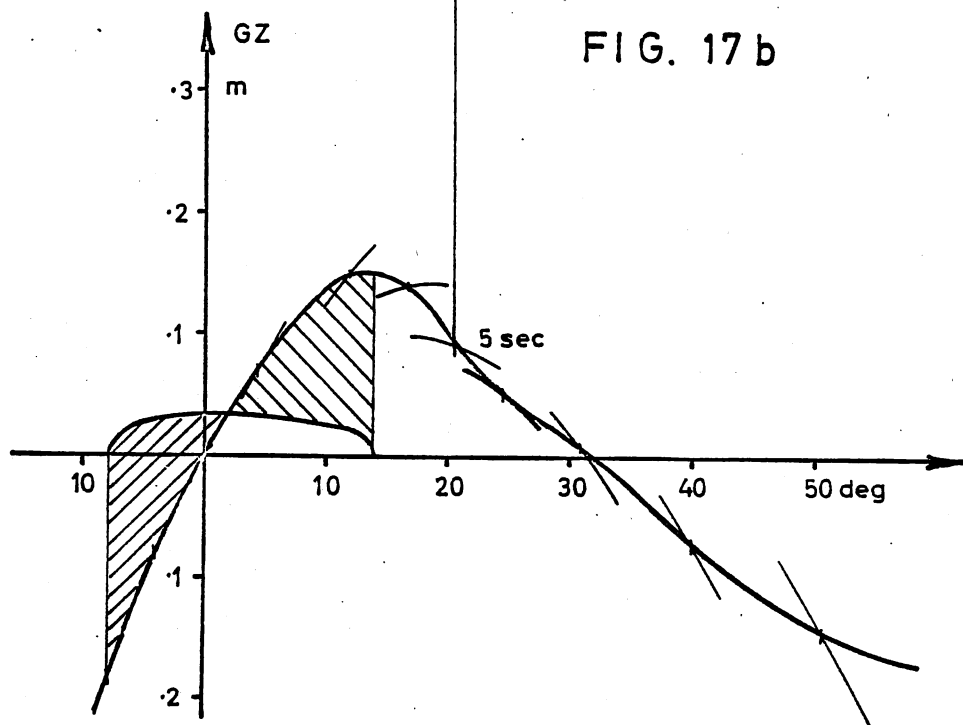


Figure 17 Analysis of the Capsize. Figure 17a Shows the Kinetics and Figure 17b Shows the Dynamics of the Capsize

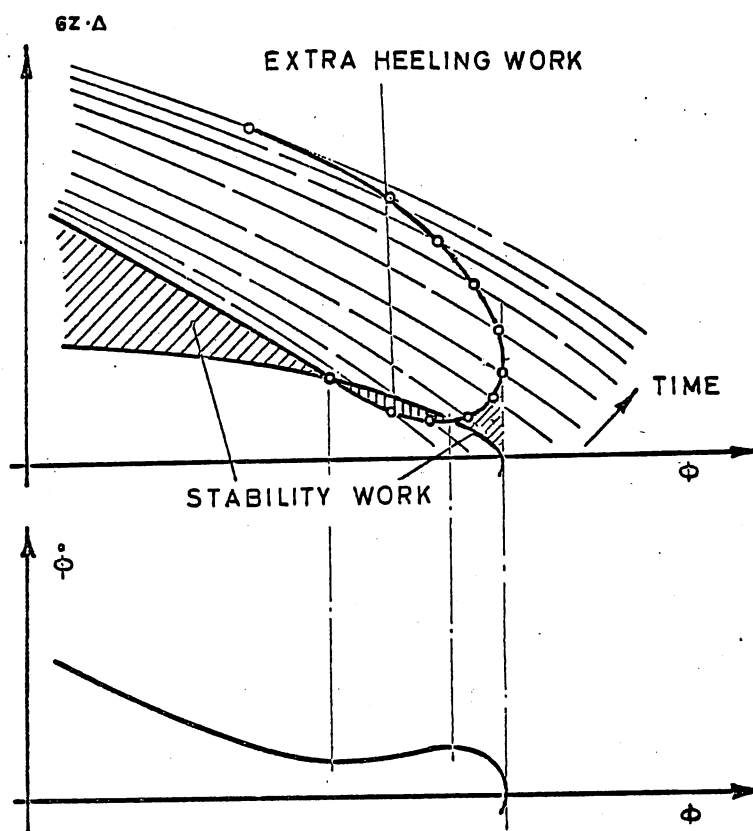


Figure 13 The Variation of the GZ-Curve Towards Higher Values can Save the Ship from Capsize

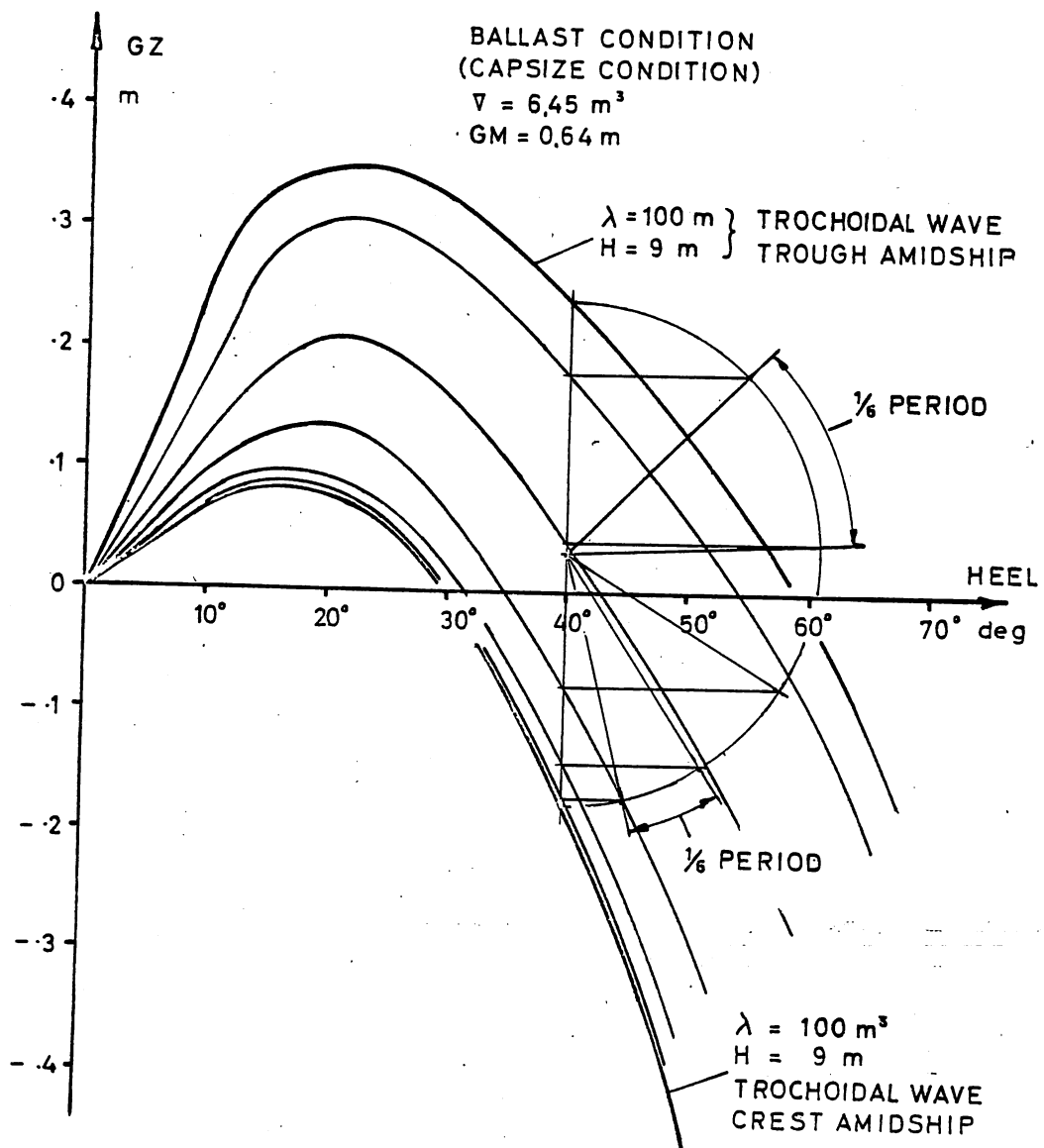


Figure 19 The Variation of the GZ-Curve is Non-Harmonic in the Ballast Condition

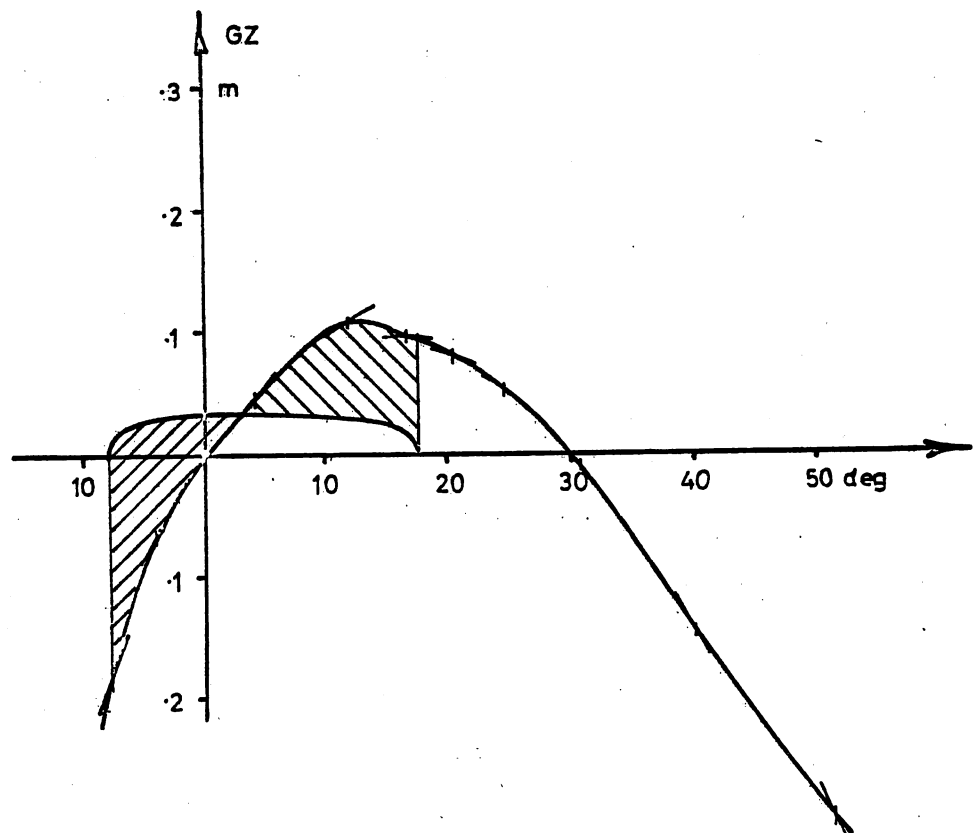


Figure 20 Analysis of the Capsize for Non-Harmonic GZ-Variations

FIG. 21 a

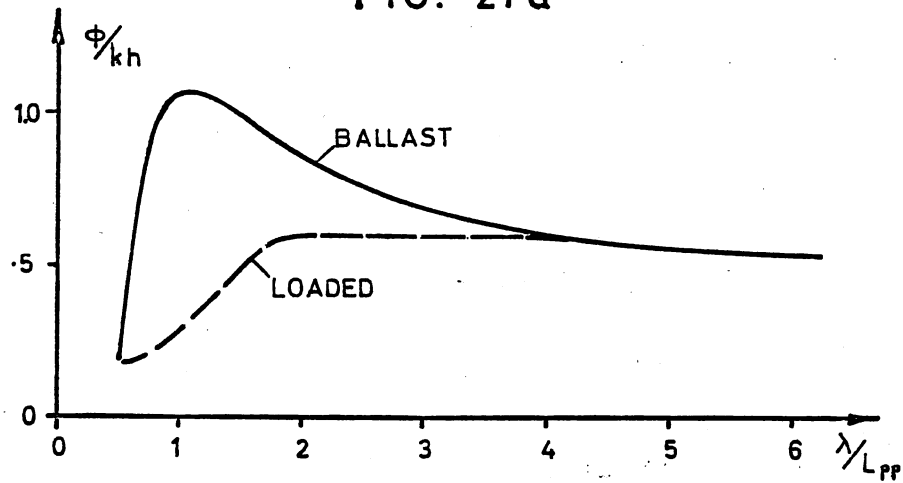


FIG. 21 b

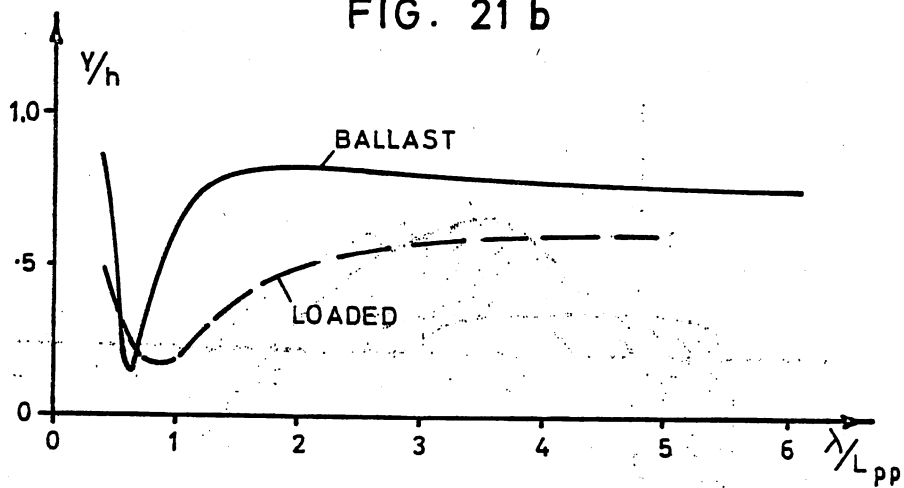


FIG. 21 c

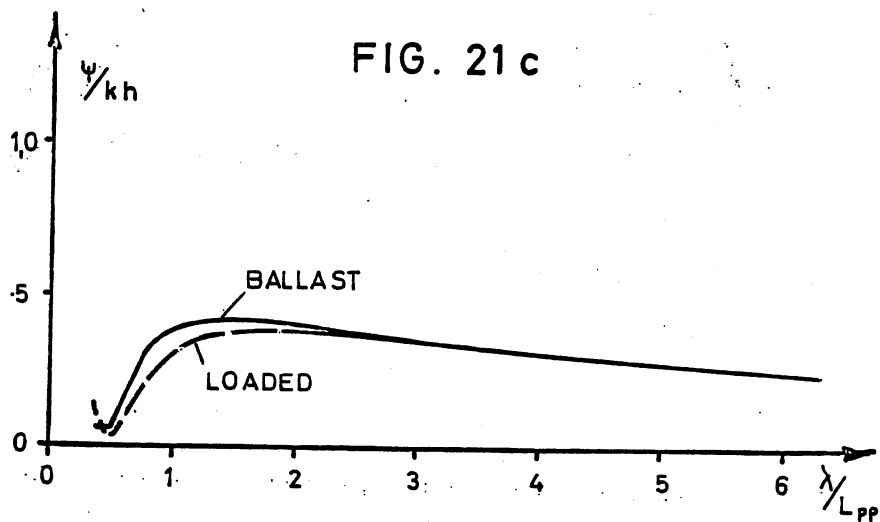


Figure 21 The Transfer Functions are Motion Characteristics Determined by Strip Theory

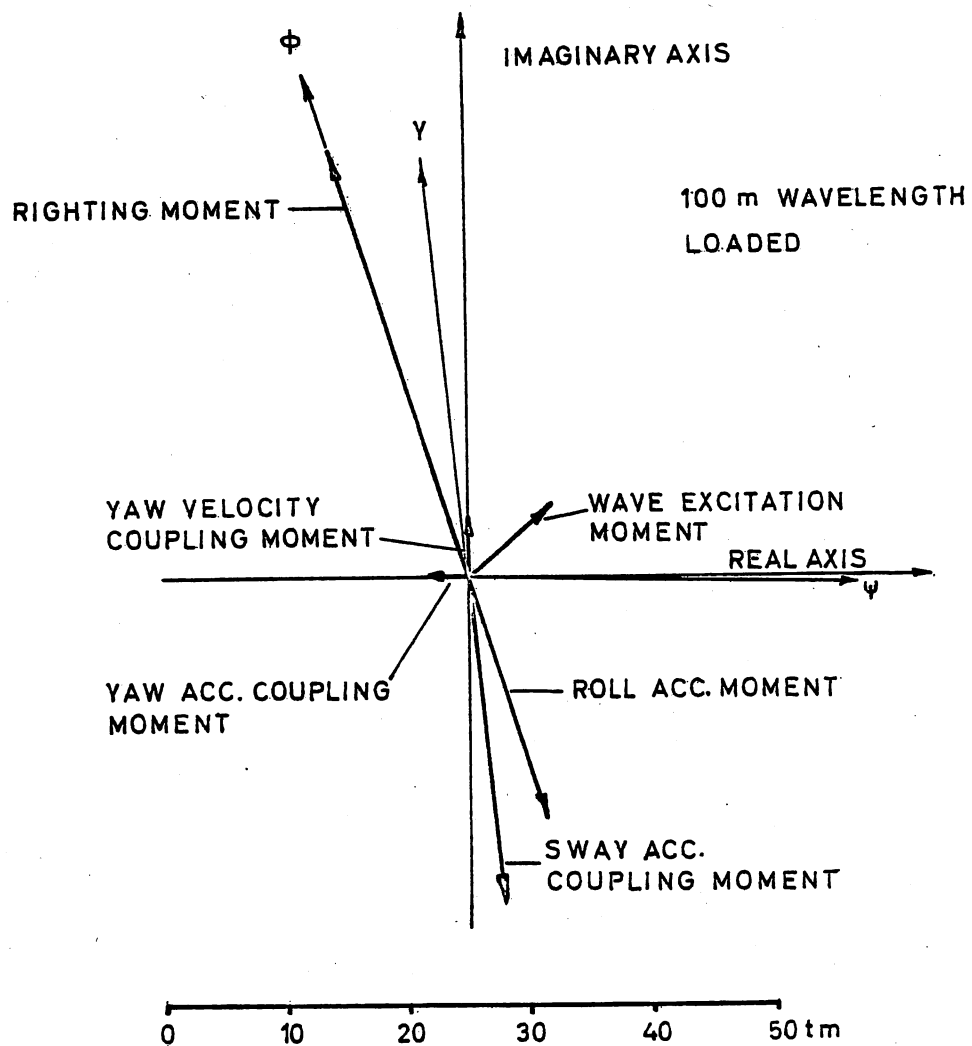


Figure 22 The Individual Terms of the Equation of Motion is Described by this Vector Diagram for Ballast Condition

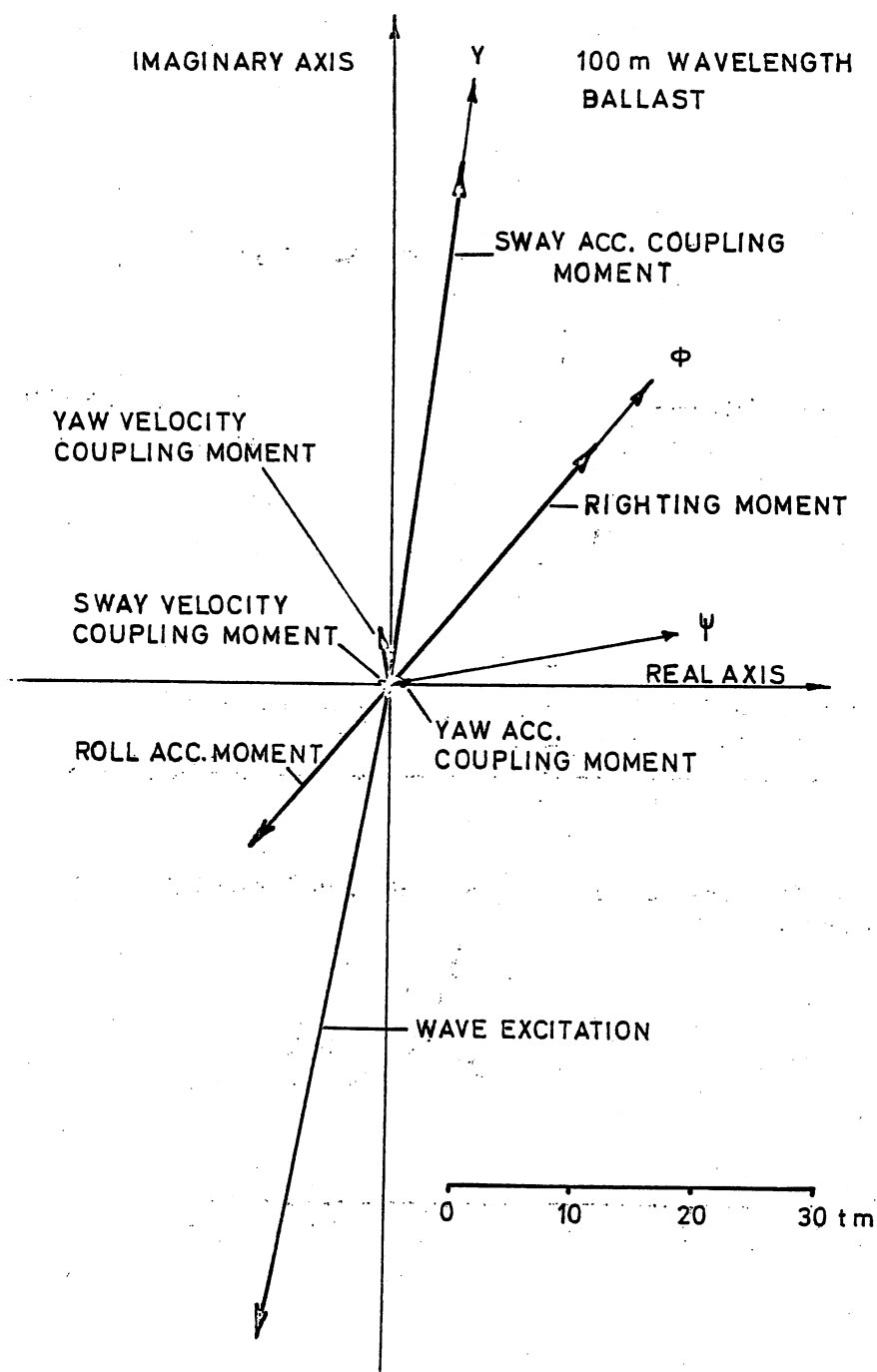


Figure 23 The Individual Terms of the Equation of Motion is Described by this Vector Diagram for Loaded Condition

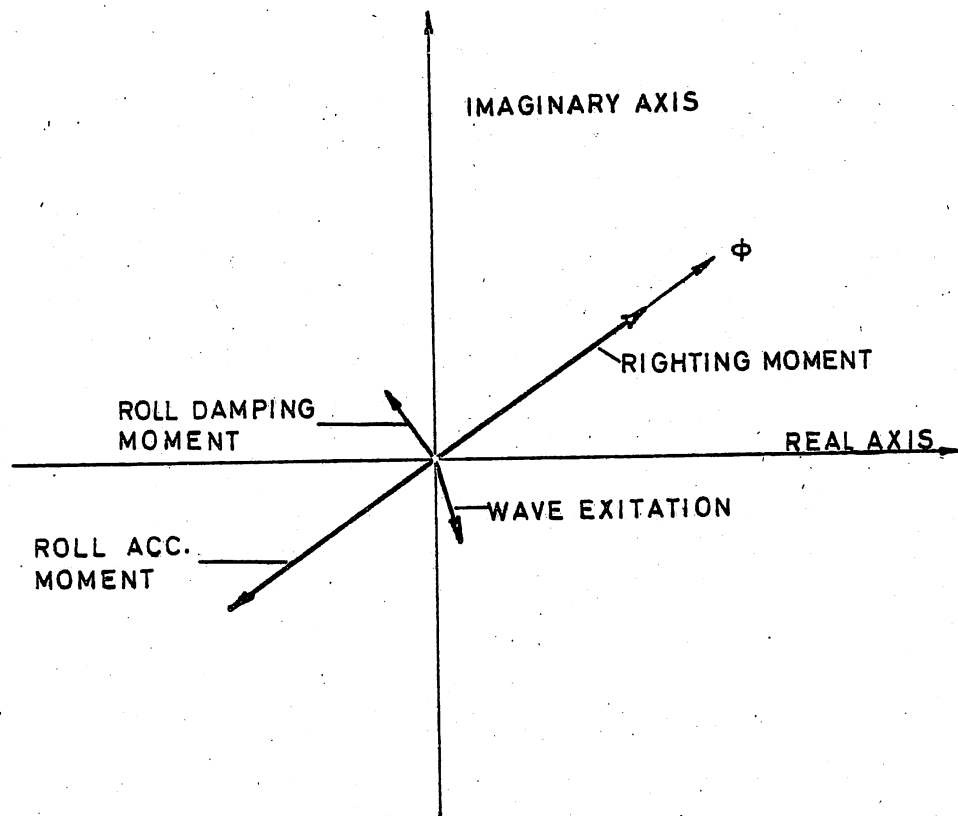


Figure 24 A Simple Roll Motion is Described by this Vector Diagram

CAPSIZE EXPERIMENT OF BOX-SHAPED VESSELS

by

SHIN TAMIYA

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1. INTRODUCTION

In a general treatment of the capsizing of ships, a finite angular displacement must be considered. Dimensions and arrangement of deck houses, hatch coamings, bulwarks and freeing ports, etc., are also relevant to the actual ship safety problem. However, taking all of these ship aspects into consideration makes the problem too cumbersome for investigating the basic mechanisms in the dynamics of ship capsizing.

In this paper the author describes briefly the capsize experiments of simple box-shaped models (1), (2) under the combined actions of transverse waves and wind. Combinations of critical wind force and wave height were determined for the models restrained only in yaw and surge with the other motions unrestrained. Some theoretical considerations are also described for understanding these experimental results.

2. MODEL EXPERIMENTS

Box-Shaped Models

Two nearly box-shaped models, A and B were tested. In Table 1 their principal particulars are summarized. They resemble standard two-dimensional test models both having flat decks and differing in the extent of the bilge radius. Four blocks of airex buoy (45.24 cm³ in total volume) were installed over the upper deck to limit excessive heel as well as actual capsizing. Bilge keels (60 cm x 0.8 cm) were also fitted. The statical

stability curves for these models are shown in Figs. (1) and (2). For model A, the following average logarithmic decrement of free rolling was obtained:-

$$\ln(A_{n+1}/A_n) = - 0.16 \quad \text{in half period}$$

Procedure and Experimental Results

The experiments were carried out in a small basin (21 m x 1.8 m) at the Institute of Industrial Science, University of Tokyo. The basin is equipped with two axial wind blowers capable of maintaining a uniform air stream over the water surface. The models were subjected to beam waves and wind, their motions other than yawing and surging were unrestrained.

Keeping the period and height of the waves constant, the wind velocity was gradually increased until the model capsized. In these experiments capsizing was defined by the immersion of a part of the buoy blocks. By repeating this procedure, a set of critical wave heights and wind velocity were obtained for each prescribed wave period. The results are shown in Figs. (3) and (4). Rolling amplitude, heel angle and drifting velocity are also shown in Figs. (5) to (9).

From a study of these figures, the following may be deduced:-

- a) For the same wave height, model A capsizes more easily in heaving-synchronism than rolling synchronism, Fig. (3).
- b) In the case of model B, the

capsize did not occur in waves of half the rolling period. This again confirms the danger of heaving synchronism, Fig. (4).

- c) Both cases of capsizing in shorter waves are not supposed possible from the concept of superposition of heeling action due to wind and rolling excitation due to waves.
- d) Deck edge immersion (leeside) possibly motivates the capsizing of the models. The observation of the record of heeling angle and the motion pictures taken through the transparent side window in the basin wall supports this hypothesis.
- e) Model A capsizes at a far smaller angle of heel under the wind heeling moment only compared with the angle where statical stability is maximum. Model B exhibited no such a peculiarity (from the static point of view).
- f) In the range of low wind velocity, the occurrence of capsizing appears small.

An example of the results of the rolling experiment of a small cargo ship in beam wind (3) is shown in Fig. (10) in which the mean angle of heel and the amplitude of rolling are given as functions of wind force. In the full load condition (small freeboard) the mean angle of heel is affected by the addition of wave excitation. Moreover, the roll amplitude increases remarkably in both loading conditions with increasing wind velocity.

These experimental results also seem to support the importance of taking the interactions among rolling, heaving and drifting (and possibly swaying), as well as the effect of deck edge immersion into the dynamics of finite ship motions.

3. NON-LINEARITY OF GZ-CURVE AND THE EFFECT OF HEAVING

In order to grasp large amplitude ship rolling quantitative effects on ship stability, (see Ref. (4)) the following non-linear equation of rolling was used to model the ship stability:-

$$I\ddot{\phi} + N\dot{\phi} + \gamma W.GZ(\phi_a) = M_0 + M_1 \quad (1)$$

where

ϕ = absolute angle of heel

$\phi_a = \phi - \phi_w$ = relative angle of heel

ϕ_w = surface slope of wave = $\bar{\phi} \sin \omega t$
 $= \bar{\phi} \sin \tau \quad (2)$

I = total moment of inertia

W = displacement of ship

N = coefficient of resistive moment

γ = coefficient of effective wave slope

M_0 ,

M_1 = heeling moment other than wave excitation

For simplicity we hereafter assume that I and N are constant and set $\gamma = 1.0$. M_0 is a moment due to constant wind velocity and M_1 is an additional moment due to heaving of the ship with heel. Fig. 11 shows the principle of generation of this moment. M_1 for model A with 2 cm freeboard is calculated and shown in Fig. (12). ζ refers to the relative wave height, positive ζ corresponding to ship dipping.

In equation (1) GZ is non-linear with respect to ϕ_a , also M_1 is non-linear with respect to ζ . However, we can suppose that ϕ_a and ζ will vary in an oscillatory fashion with the wave encounter period, so that GZ and M_1 may be expanded in a Fourier time series and approximated using the first and second terms only.

Assuming a solution in the form:-

$$\phi_a = X_0 + X \sin(\tau - \epsilon) = X_0 + X \sin \tau_1 \quad (3)$$

and using approximation for GZ

$$GZ = g_0/2 + h_1 \sin \tau_1 \quad (4)$$

and for M_1

$$M_1 = a_0/2 + b_1 \sin(\tau - \delta) \quad (5)$$

with the expression

$$\tau = Z \sin(\tau - \delta) \quad (6)$$

we obtain the following three equations:-

$$G_0 = 2f_0 + A_0 \quad (7)$$

$$-X + bmH_1 = \phi \cos \xi + bmB_1 \cos(\xi - \delta) \quad (8)$$

$$aX = \phi \sin \xi + bmB_1 \sin(\xi - \delta) \quad (9)$$

where

$$G_0 = \varepsilon_0 / GZ_m, \quad H_1 = h_1 / GZ_m$$

$$f_0 = M_0 / W \cdot GZ_m, \quad b = \lambda^2 / \omega^2$$

$$\lambda^2 = W \cdot GM / I, \quad m = GZ_m / GM$$

$$a = N / I \omega, \quad A_0 = a_0 / W \cdot GZ_m$$

$$B_1 = b_1 / W \cdot GZ_m$$

$$GZ_m = \text{maximum of } GZ, \quad GM = dGZ/d\phi \text{ at } \phi=0$$

which is sufficient to determine X_0 , X and ξ .

The calculation for model A with 2 cm freeboard was completed and the results are shown in Fig. (13). If we increase wave height under a constant wind force, only the roll amplitude increases at first, the mean heel X_0 remains almost constant. There is a critical wave height, above which X_0 increases suddenly, and X_0 becomes

maximum. The point $X = \text{maximum}$ was verified to correspond to the instability of the motion and the curve of critical wind velocity and wave height determined from this criterion is plotted in Fig. 3(a) showing fairly good agreement with the experiments.

For $f_0 = 0.6$ where no critical point exists, the starting point of sudden increase of X_0 was assumed to be critical.

The same calculation and procedure were carried out for model B and the results are shown in Fig. (14) and 4(b) respectively. Again there is good agreement with the experimental results.

4. UNSTABLE MOMENT

Equation (1) was used only from the practical convenience of calculation of the non-linear motions. After Professor S Motora (5) the more rigorous equation of rolling is described as follows (Fig. (15))

$$(I_x + J_x) \frac{dp}{dt} = (m_y - m_z)vw + (I_y - I_z + J_y - J_z) qr + L \quad (10)$$

Usually we can neglect the first and second terms in the right hand side because of the smallness of the motion and the same order of magnitude of m_y etc. Added masses m_y and m_z , however, are of the order of $m = W/g$ and moreover, their relative magnitude varies with ship form and the frequency of the motion. For example, m_y and m_z for models A and B are shown in Fig. (16), in which m_z is shown to become larger than m_y at high frequencies and also at the vanishing frequency. In the high frequency zone where heaving synchronism occurs, we can expect that the time average of vw has a positive value. At the vanishing frequency a constant positive value of vw increases as the steady wind drives the ship leeward and gives her a steady heel. Quantitative estimation of the heeling moment due to this term results in 1~2 kg.cm of heaving synchronism which is not of negligible order.

Lacking reliable data for the added masses of the experimental models used, we can not at present arrive at any decisive conclusions as to the actual effect of the unstable moment, however, its importance should be recognised as a new topic of future research in capsizing.

5. CONCLUDING REMARKS

The author has proposed the usefulness of adopting a non-linear finite displacement equation of rolling and presented some calculated results which include the additional heeling moment due to heaving. The importance of the so-called unstable moment is also pointed out.

Future work must be concentrated to clarify the resistive moment at finite angular displacement and to obtain a more correct expression for wave excitation.

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TABLE 1. Particulars of Box-Shaped Models

Description	Model A			Model B	
L × B × D (cm)	100 × 20 × 11.5			100 × 20 × 20.25	
Freeboard f (cm)	2.0	1.6	1.2	2.0	1.6
Displacement (kg)	17.81	18.61	19.41	36.20	36.95
Section Area Ratio	.938	.940	.942	.989	.990
KG (cm)	7.89	7.93	8.00	10.45	10.65
GM (cm)	0.831			0.590	
Rolling Period (sec)	1.52	1.51	1.50	2.80	3.10
Heaving Period (sec)	ca 0.77			1.02	1.07
Wave Period (sec)	1.54	1.49	1.55	1.40	1.55
	0.92	0.88	0.92	1.02	1.07

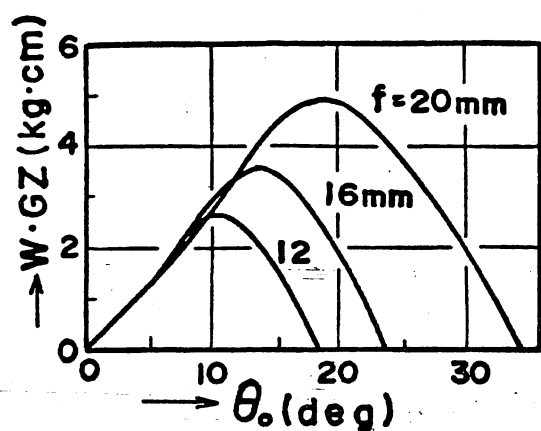


Fig. 1 Curves of Statical Stability of Model A

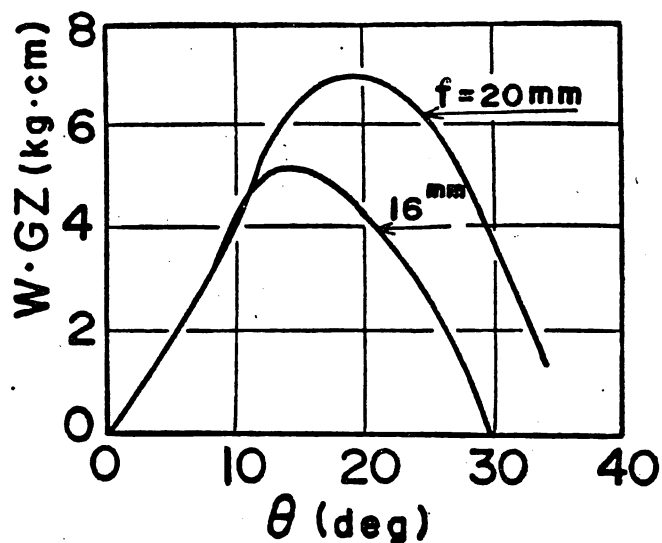


Fig. 2 Curves of Statical Stability of Model B

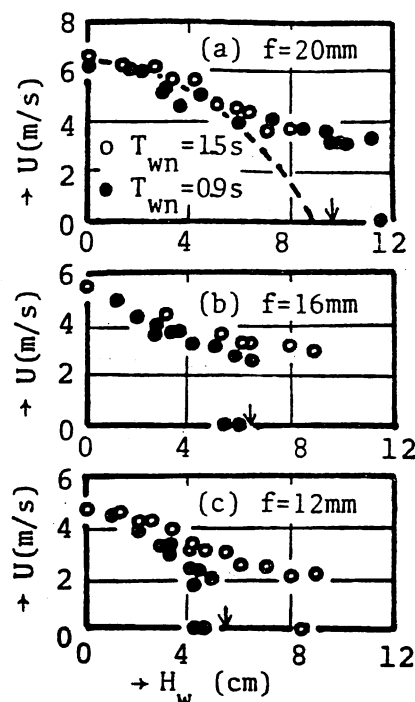


Fig. 3 Critical Wave Height and Wind Velocity

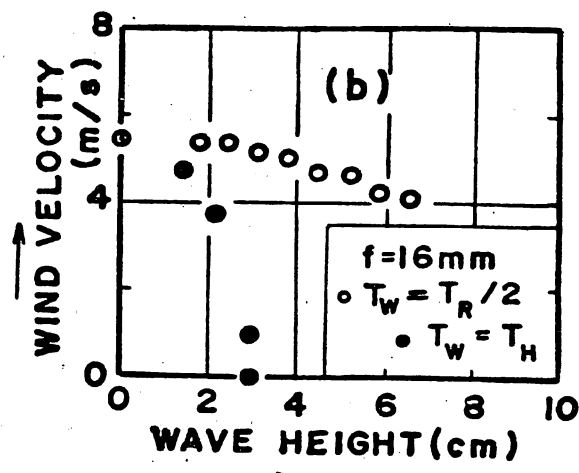
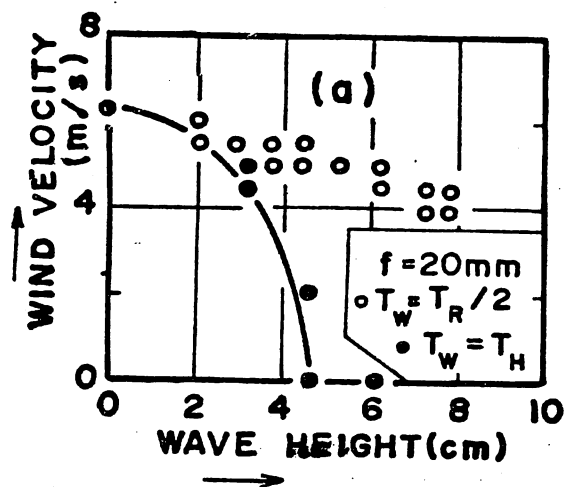


Fig. 4 Critical Wave Height and Wind Velocity

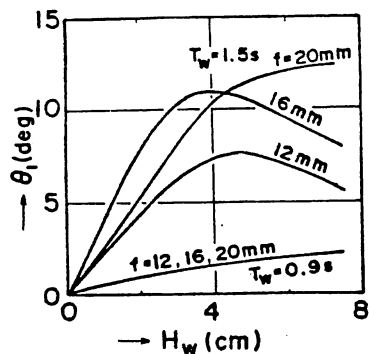


Fig. 5 Rolling Amplitude due to Wave

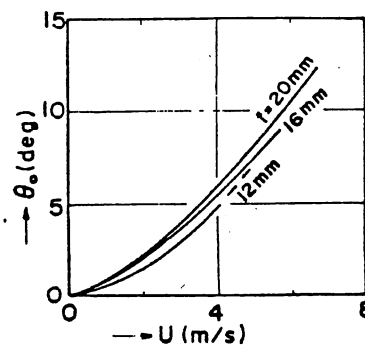


Fig. 6 Heel Angle due to Wind

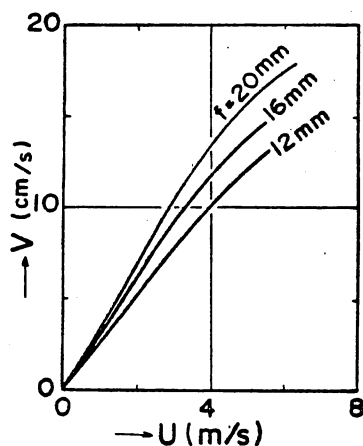


Fig. 7 Drifting Speed due to Wind

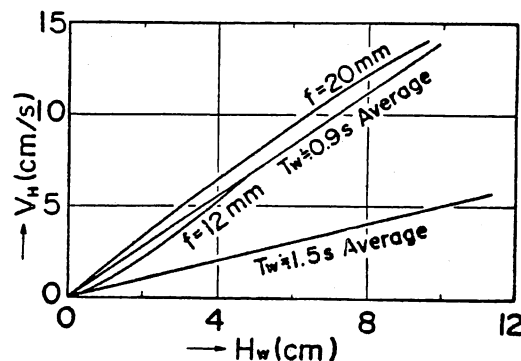


Fig. 8 Drifting Speed due to Waves

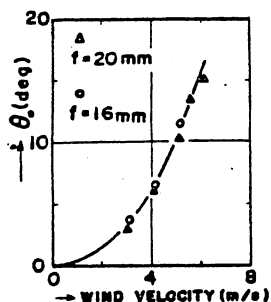


Fig. 9 Heel Angle due to Wind

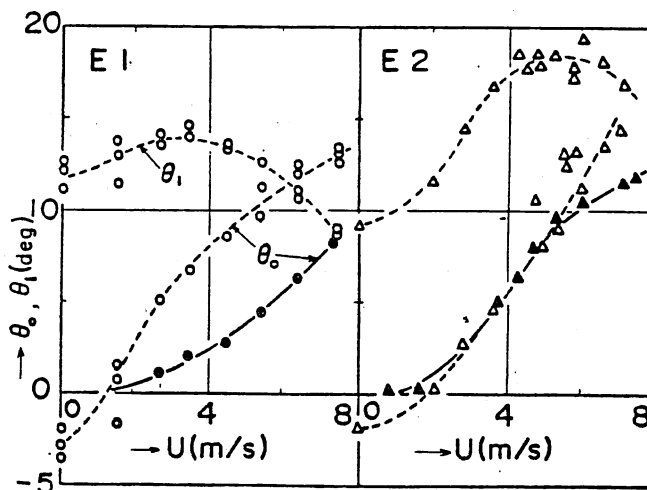


Fig. 10 Heel Angle θ_0 and Roll Amplitude θ_1
 (E_1 ; Full Load, E_2 ; Half Load)
 • without Waves

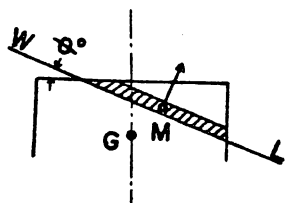


Fig. 11 Generation of Heeling Moment due to Heaving under constant ϕ_0

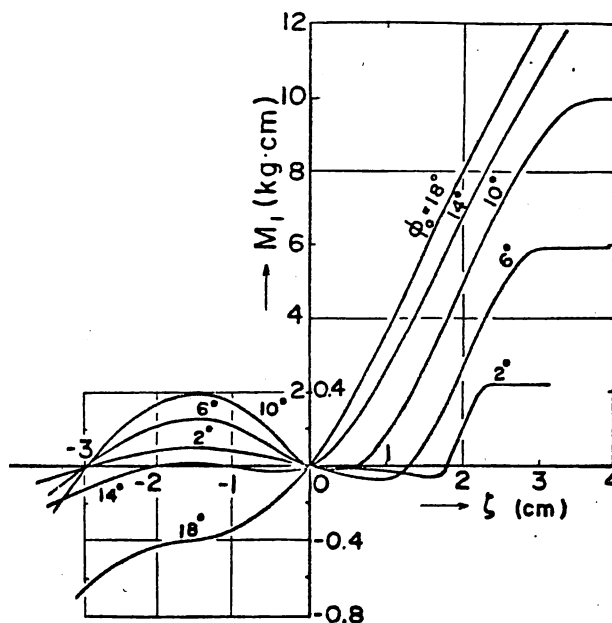


Fig. 12 M_1 for Model A

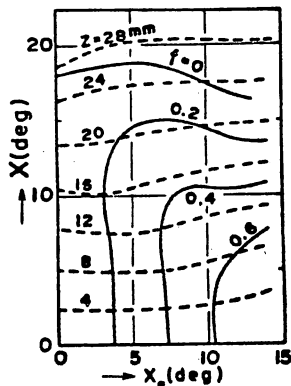


Fig. 13 Mean Heel Angle (X_0) and Rolling Amplitude (X). (Model A)

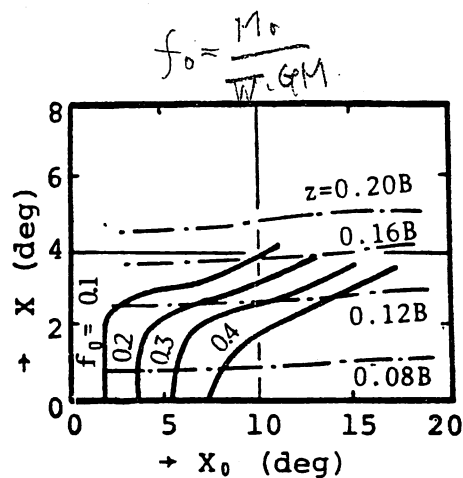


Fig. 14 Mean Heel Angle (X_0) and Rolling Amplitude (X). (Model B)

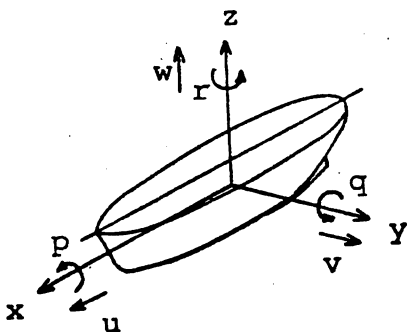


Fig. 15

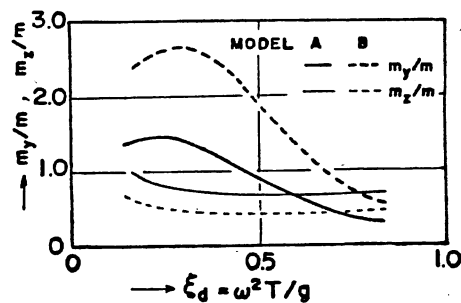


Fig. 16 m_y and m_z of the Models

SHIP CAPSIZING IN HEAVY SEAS:
THE CORRELATION OF THEORY AND EXPERIMENTS

by

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1. INTRODUCTION

For the past five years, the Naval Architecture Department of the University of California has been engaged in conducting an experimental study of the capsizing of intact ships in heavy seas. The principal part of this effort was devoted to model tests utilizing free running models approximately eighteen feet long in the open waters of San Francisco Bay. The two models which have been tested to date represented a conventional dry cargo ship and a large fine high speed container ship. The models were equipped with self-propulsion equipment, autopilots, motion sensing and tape recording instruments, and a radio link for control signals. The instrumentation provided for measurement of six degrees of motion freedom, rudder angle, relative water speed and shaft RPM. Wave conditions in the test area were monitored by means of a four gauge array which provided directional wave information.

Both model and wave data were recorded on analog magnetic tape. The signals were later digitized for purposes of time series analysis by digital computer.

Three summers of testing have yielded a staggering amount of data. Much of the daily effort was devoted to software development, data reduction, and the seemingly endless number of cross checks in an effort to insure the data validity. Despite the amount of data collected, it was not possible to obtain a sufficient number of

capsize events for an accurate long-term statistical prediction as a result of the great variety of sea conditions and ship parameters, such as speed, loading, and GM. The data are somewhat deficient for comparison with some of the theoretical developments that were concurrent with and subsequent to the experimental work. These shortcomings are all the more obvious when viewed with the benefit of hindsight. Yet it should be remembered that the desire for something other than pure speculation was sufficient to override the often discussed inadequacy of the proposed experimental program.

Nevertheless, the project represented one of the most ambitious and complete capsize studies to date and has yielded significant observations and correlations.

2. MODELING OF INTACT CAPSIZING

Intuitively it seems reasonable to assume that an important component of actual ship capsizing, rather than an idealized version, is the directionality of the seaway. Short crestedness can lead to dynamic conditions both more and less severe than a corresponding long crested situation. Open water testing therefore appears to be not only very desirable, but essential in light of currently available laboratory testing facilities. Considerable effort was directed initially toward assessing the appropriateness of testing with approximately twenty foot models in San Francisco Bay and later in quantifying the directional

* Now at M. I. T.

character of those waves.

The significant wave height measured in the test area typically ranged from 0.3 ft to 1.5 ft. Using a prototype vessel of 520 ft and a scale ratio of 30, which yields a seventeen foot model, provides a reasonable full scale range of wave heights and an attractive model for the type of testing under consideration. The shallow water depth had only a minimal effect on the longest waves ever encountered and the test site can be considered to be effectively deep water. The next question is whether the wave profiles and spectra scale properly? The point spectra, though varying gradually as the wind conditions changed, resembled the fetch limited spectra reported by JONSWAP (3). These are characterized by spectral peaks that overshoot their final or fully developed values as they shift to lower frequencies while growing. The four point spectra measured on one day are shown in Figure (1). The following figure is a representative collection of the larger spectral shapes encountered. Despite the fact that they tend to be more sharply peaked than the Pierson-Moskowitz fully developed model, they look remarkably like selected full scale storm spectra recorded in the winter in Lake Michigan, the Atlantic, and the Pacific and in hurricane conditions in the Gulf of Mexico, c.f. Figure (3). The Pierson-Moskowitz sea spectra, plotted to the same scale, are given in Figure (4) for comparison. Until additional full scale storm spectra and time-series data become available for comparison, the experimental spectra seem to represent eminantly reasonable "extreme" sea conditions.

The directional character of the seaway is effectively given by the directional spectrum $S(f,k)$ which was computed from the digitized time-series data of the four gage wave array. The directional spectra were obtained using the Fast Fourier Transform and the Maximum Likelihood (or data adaptive) spectral techniques. The directional spectrum for wave run number (4) of Figure (1) is given as a contour plot in Figure (5). The radial distance represents frequency and the angle is the compass direction. Note the narrowness of the spectral peak. Figure (6) shows the directional spectrum of an unusual combination of a recently developed wind sea traveling North superimposed on a swell (i.e. a non-local wave system) traveling East.

The result was a classic example of "cross-seas". The connecting arc between the two peaks is probably due to the mathematical smooth process and therefore spurious.

Unfortunately this data exists at only one point along the model run, which could be very long if the waves were small, and was recorded only between the actual experiments. The model runs were selected so that, as far as possible, similar wave conditions existed along the course. The fact that the wave system was only measured intermittently is mitigated somewhat by the high correlation of observed wave conditions and the computed significant wave height. This meant that a critical change in the observed wind and wave conditions during a run could be noted and direct interpolation between measured spectra to the time of the run would be valid when no discontinuous changes were observed.

Since some variation in water depth and current velocity occurred along the model course, the principal wave direction also varied. In an effort to keep the model on the same relative course, the autopilot was adjusted a few degrees at a time during the period of the run.

Reports of extreme rolling in following seas often describe a significant wind sea superimposed on a large swell. Some of the wave spectra exhibit two peaks, and occasionally the distinct combination of a wind sea and swell coming from different directions. Usually, however, the directional wave spectrum was fairly broad for the lower seastates and tended to become more narrow for the larger. Swell has received scant attention in the ship motions literature. Since such low frequency excitation is important in following seas, the measured spectra should be compared with oceanographic swell data.

3. OBSERVATIONS

The observed wind and wave conditions and certain aspects of the motions have been briefly discussed in the previous section. The characteristics of the capsize event have been described in some detail in earlier reports (2), (4). However, the important aspects of the capsize motions will be summarized here to facilitate the correlation discussion.

The experiments were conducted in waves ranging up to a full-scale significant height of 45 feet with a maximum Froude number of 0.33. These appear to be rather extreme conditions, yet very little statistical data exists concerning the frequency and geographical distribution of such wave conditions. It has recently become apparent that extreme conditions can and do occur outside of the "roaring 40's", given the proper conditions of wind, current and bottom topography. Ship owners and naval architects should take heed of the fact that high speed ship operation may be imperiled in certain very limited geographical locations with a far higher frequency than the occurrence of a hurricane or a one hundred year winter gale along the same route might suggest. This discussion is directed at moderate and large size vessels. However, smaller craft such as tugs and trawlers run a greater risk of encountering short, steep, and chaotic wave systems and operate at higher relative speeds. They, therefore, require relatively greater attention to the question of stability in extreme seas.

For simplicity, the observed modes of capsizing have been labeled as pure loss of stability, broaching, and low-cycle resonance.

Pure loss of stability was the most frequently observed mode of capsizing. The optimum conditions for this mode are a high ship speed and very steep waves whose lengths probably vary between one-half and twice the ship length. If the ship finds itself centred on the crest of a wave for a long enough period of time, the attendant reduction in righting arm can be sufficient to result in a capsize. The vessel need not travel at the speed of the wave, i.e., a Froude number of 0.4, but the wave heights must become increasingly larger as the speed is reduced.

Capsizing due to broaching was the most dynamic mode observed. Broaching is caused by directional instability of a vessel accelerated on the face of a wave. The situation can be either caused by or compounded by any loss of effectiveness of the rudder due to wave orbital velocity and emergence. Again, the principal ingredients are relatively short, steep, following seas and high vessel speeds. These situations are more likely to be met by high speed naval vessels and small commercial craft rather than large contemporary

merchant ships. Broaching and loss of directional control can also occur in a cumulative manner as a result of the repeated encounter and overwhelming of the vessel by steep, breaking seas. The vessel is violently forced off course with a significant centrifugal force adding to the large roll moment applied at both the bow and stern. Capsizes due to "pure" broaching were rarely identified in the model experiments on the Bay. Large vessels that run at very high speeds usually have sufficient directional control to minimize the likelihood of a broach. However, some degree of loss of directional control was regularly observed in conjunction with large wave induced variations in static stability. In the case of the container ship model especially, capsizing usually appeared to result from the combined effects of broaching, wave reduction of stability and motion instability.

The third mode has been called low-cycle resonance or auto parametrically induced rolling. In this case the sinusoidal oscillation of the righting arm leads to a resonant build-up of roll motion at one half of the encounter frequency. This occurs when the increased righting arm (trough amidships) results in a large roll velocity timed to meet a crest amidships one half an encounter period later. This mode was regularly observed with the first model tested, the conventional cargo ship. A clear example of this mode was observed only once with the second model representing a fine container ship.

The division of capsizings into one of these three categories was not always possible, however. Some capsize sequences seemingly started out as a resonant build-up, but the actual capsize then appeared to be caused largely by a drastic loss of stability. The final part of the fatal roll would also include some yawing which probably could be called something of a broach. Nevertheless, two factors appeared to be present in virtually all of the capsizes: the successive encounter of a group of large waves with significant reduction in static stability at the critical moment. Any theory attempting to predict the occurrence of capsizing must address the latter fact and will be simplified if it effectively considers the former.

4. COMPARISON OF EXPERIMENTAL RESULTS WITH THEORY

An attempt to explain some of the results of the experiments has been made by means of three different theoretical procedures: linear ship motion theory, nonlinear two-dimensional ship motion theory, and numerical simulation. A linear theoretical procedure similar to that developed by Salvesen, Tuck and Faltinsen (1970) (7) has been used in combination with some of the experimental wave spectra to produce motion predictions for comparison with experimental results. By including a linear autopilot and experimentally determined equivalent linear damping in the theoretical model reasonably good agreement was obtained in some cases between experiment and prediction. Inexplicably, the correlation was quite poor in other cases. Examples of two such comparisons are shown in Figures (7) and (8). The theoretical predictions are shown for the spectra measured before (1) and after (2) the model run.

The explanations for the mediocre agreement are not clear. Most published comparisons of motion theory and experiment have been based on experiments conducted in towing tanks in low head seas. In the present case, we have steep following or quartering seas, and, of course, less control over experimental conditions and measurement precision. Nevertheless, the erratic nature of the comparison leaves many questions unanswered.

Nonlinear two-dimensional motion theory has also been used, not so much for direct comparison with experiments, but to explain observed phenomena. The effects of higher order terms in the roll damping and restoring moment representation has been explored by earlier writers. In general, the effects are to merely modify the results which might be obtained with a linear model.

A particularly important result is obtained, however, if we consider the effect of waves on the roll righting moment curve. In general, the presence of a wave crest near amidships results in a decreased righting moment and a wave trough amidships results in increased righting moment. As following or quartering seas overtake the ship, the resulting periodic variations in stability, if of the proper frequency, can result in "autoparametric" excitation of an unstable rolling

motion. In this situation, the unstable motion is identified with unstable solutions of Mathieu's equation. Such motion has, in fact, been observed during the experiments described here, and appears to play an important role in some capsize as noted earlier.

The effect of waves of varying steepness on the righting arm curves for two models is shown in Figure (9). This figure also illustrates the difference in such effects attributable to differences in hull form. In particular, note that the wave induced stability variations are nearly symmetrical about the still water curve for hull (a) while they are quite nonsymmetrical for (b). This symmetry or lack of symmetry has a pronounced effect on the relationship of the unstable frequency of wave encounter to the calm water natural frequency of roll.

Model (a) will experience autoparametrically excited unstable motion in following seas at a frequency of encounter of twice the still water natural frequency of roll. Model (b), however, has a mean righting arm curve in waves appreciably higher than the calm water curve and, as a result, will experience autoparametric instability at an encounter frequency somewhat higher than twice the still water roll frequency. Again, this difference in the apparent frequency of occurrence of unstable roll motion was detected during the experiments.

The third theoretical procedure used to study the large motion and capsizing was a numerical, time-domain simulation. This procedure is discussed in some detail in the next section.

5. NONLINEAR TIME-DOMAIN DIFFERENTIAL EQUATION MODEL

Solutions of the hydrodynamic problems described above have been obtained under the assumption of small motion amplitudes, in which case the forces acting on the ship are computed as though the instantaneous position of the ship differs but little from its mean position. Such an assumption cannot be used in the present case where large deviations in position from the mean are an essential feature of the phenomenon. Instead, we observe that at high speed in following and quartering seas the frequency of wave encounter will be low and the ship motion will be determined largely by the

hydrostatic forces. This enables us to retreat from the necessity of determining the hydrodynamic forces with great accuracy but to concentrate instead on the hydrostatic forces which may be accurately computed for the exact position of ship and waves at any instant of time. These forces, plus additional external forces representing the steering and controls, plus a simplified approximation to the relatively unimportant hydrodynamic terms then are used as the right hand side of the six degree of freedom rigid body equations of motion. These differential equations are integrated numerically resulting in a step-by-step time history of the vessels motion.

There can be significant variations in the roll restoring moment as a wave progresses along the ship's length as well as the change in this moment caused by large amplitude roll angles. This means that the hydrostatic restoring and coupling coefficients computed for the equilibrium position cannot be used when finite amplitude motions are assumed. The hydrostatic (or Froude-Krylov) force that is computed at each time step includes both the motion exciting force and the restoring force and moment that result from the situation of the ship in the system of waves. The sea surface is represented by the sum of a finite number of sinusoidal waves.

Approximations are used for the hydrodynamics forces resulting from the diffraction of the incident waves and motion of the ship. These forces are computed using constant two-dimensional added mass and linear damping coefficients for each station combined with averages of the water acceleration and velocity relative to the stations. The hydrodynamic approximations are not expected to lead to serious errors if the Froude-Krylov force is dominant. This, as noted, is expected to be the case in the most severe capsizing situations in following or quartering seas.

A numerical model of the steering system of a typical ship consisting of an autopilot, steering machinery and a rudder is included. As an option, surge damping may be obtained by interpolation in a table of resistance versus speed data.

Two examples of motions simulated by the numerical time domain integration are presented here. These compu-

tations were performed for the American Challenger class of cargo ship. These calculations were carried out at ship scale rather than at the scale of the model used for the experiments. The ratio of ship length to model length is 30.189, and the time scale ratio for Froude number similarity is about 5.5. Metacentric heights corresponding to model ballast weight positions 2 and 3 were used for the simulations. The righting arm curves for the Challenger are shown in Figure (10). These examples are for quartering seas which consist of the sum of two sinusoidal waves with circular frequencies of 0.433 and 0.601 radians per second, corresponding to wave lengths of 1076 and 558 feet. Each wave component has an amplitude of 10 ft, and they approach the ship from 35 degrees to starboard of dead astern. At the beginning of the runs the two wave components are in phase.

The first example is for a metacentric height of 0.257 ft (GM position 2). The wave record for this run is shown in Figure (11). To decrease the effect of starting transients, all forces are multiplied by a ramp function which increases linearly with time from zero to one at the beginning of the run. The starting ramp was 60 seconds in length, and ended near the end of the first wave group. Roll and pitch records are in Figures (12a) and (12b). At about forty seconds from the start a wave crest comes amidship and the vessel rolls 14 degrees to port. The next wave crest comes amidships at about sixty seconds and a starboard roll into the wave of 4 degrees is reached. The next roll is 22 degrees to port just after the next crest passes. The ship rolls to starboard as the waves begin to build in the second wave group. The maximum starboard roll of 23 degrees is reached as the next crest moves away. The roll momentum imparted the ship by the increased righting arm of the next trough coming amidship together with the reduced stability in the crest that follows causes the ship to capsize to port - clearly an example of low-cycle resonance. Figure (13) shows the last two and a half minutes of a model run corresponding to the same speed, heading relative to the waves and GM condition. The experiment also ended with a capsize to port caused by low-cycle resonance.

An example of pure loss of stability is shown in Figures (14) and (15). The metacentric height was 0.557 ft. (GM position 3). The wave amplitude at the centre of gravity of the ship is shown in Figure (14). Roll and pitch are in Figures (15a) and (15b). Rolling is at one half the frequency of wave encounter between 40 and 110 seconds into the run. After that the wave amplitude and the roll moment are so large that the ship is forced to roll at the encounter frequency. The vessel finally capsizes with the combination of a large port roll and a wave crest amidships.

6. CONCLUSION

A more detailed account of the above work can be found in references (1), (2), (4), (5) and the bibliographies therein. The observations and insights gained through the extensive experimental programme have proven to have rather good qualitative correlation with the linear and non-linear modelings investigated. The lack of quantitative agreement with the five degree linear theory has revealed possible gaps in the experimental modeling techniques or the attempted extension of the linear theory to a region outside of its validity. Further work in this area is needed. The nonlinear time-domain simulation, on the other hand, has exhibited clearly the very complicated dynamics caused by the changes in geometry and hydrostatics. With experience, the simulation should prove to be a useful tool in the study of extreme motions.

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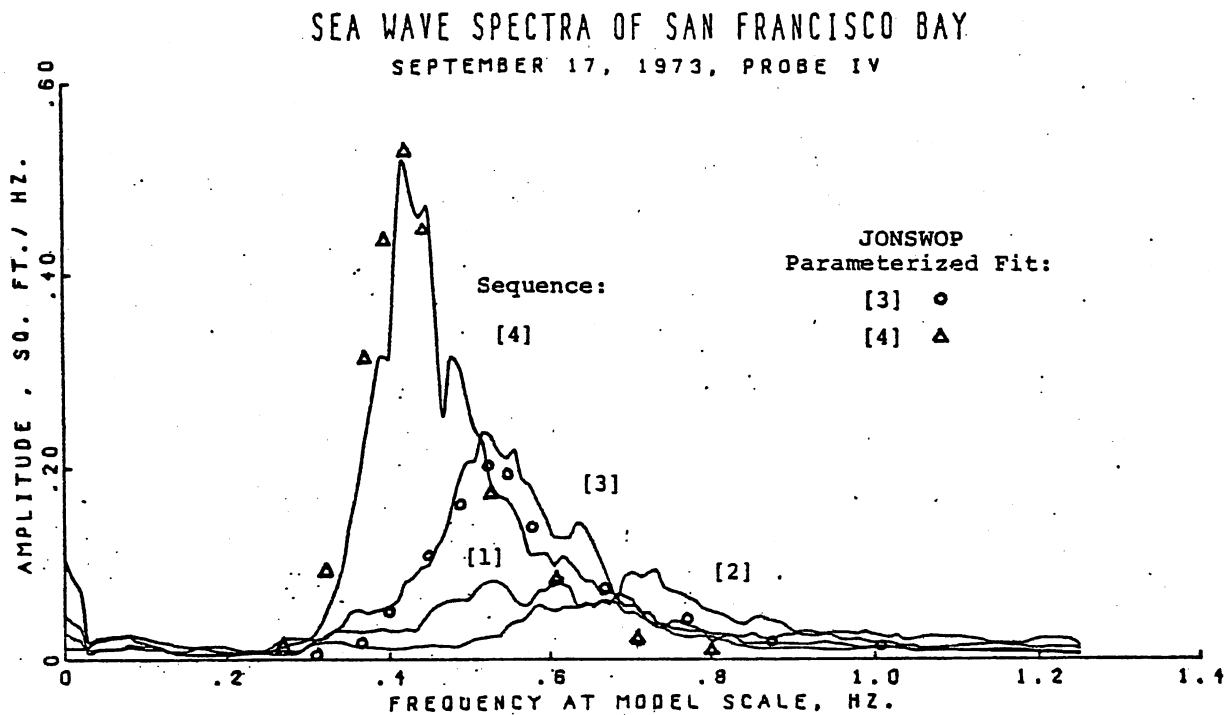


Figure (1)

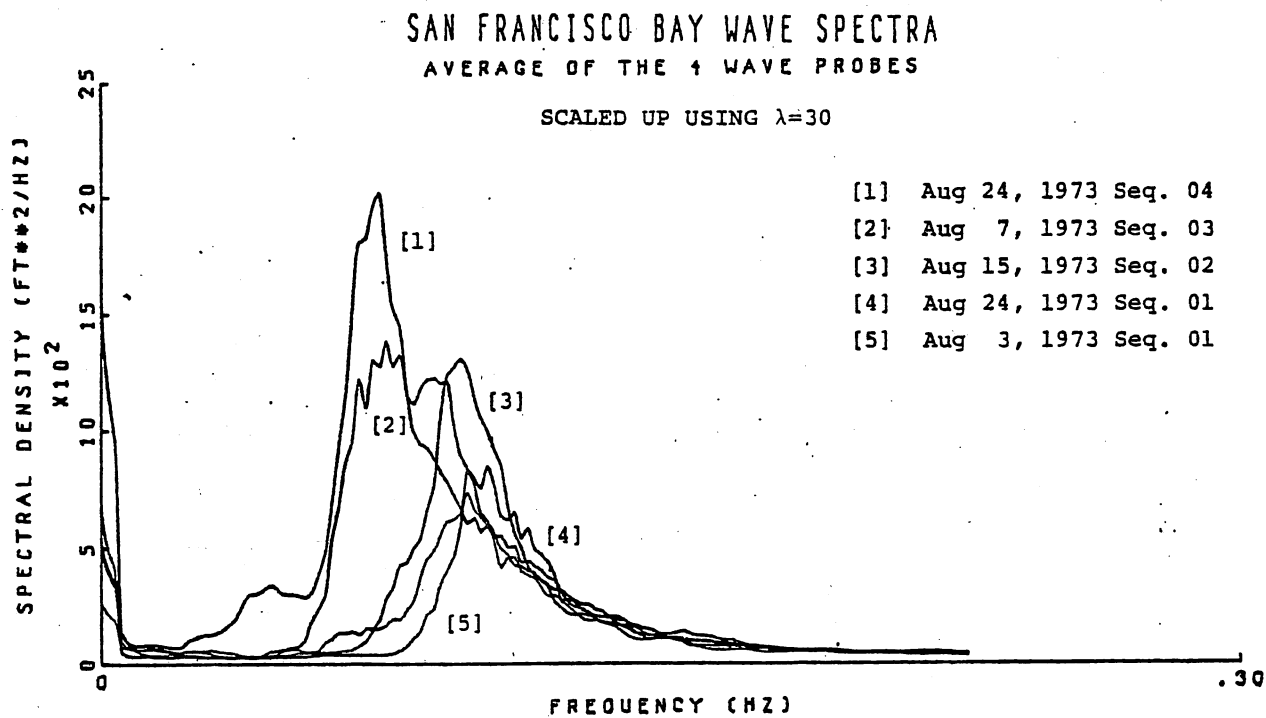


Figure (2)

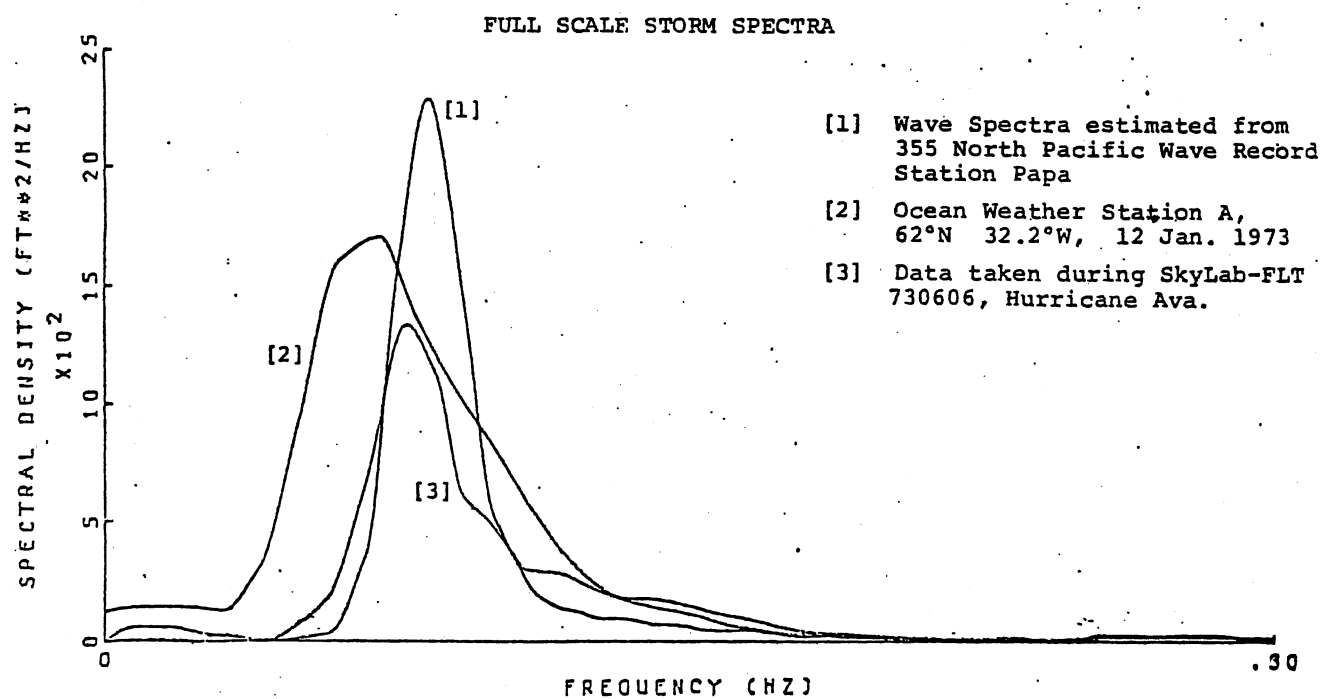


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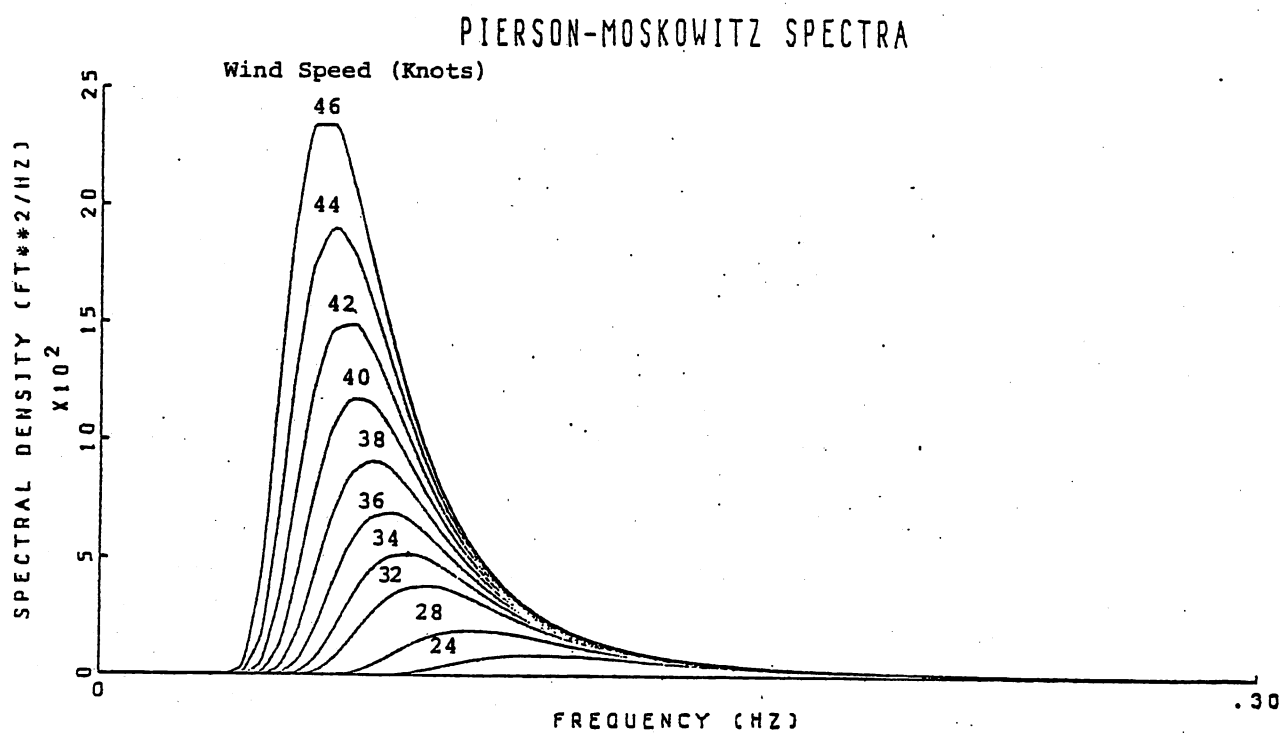


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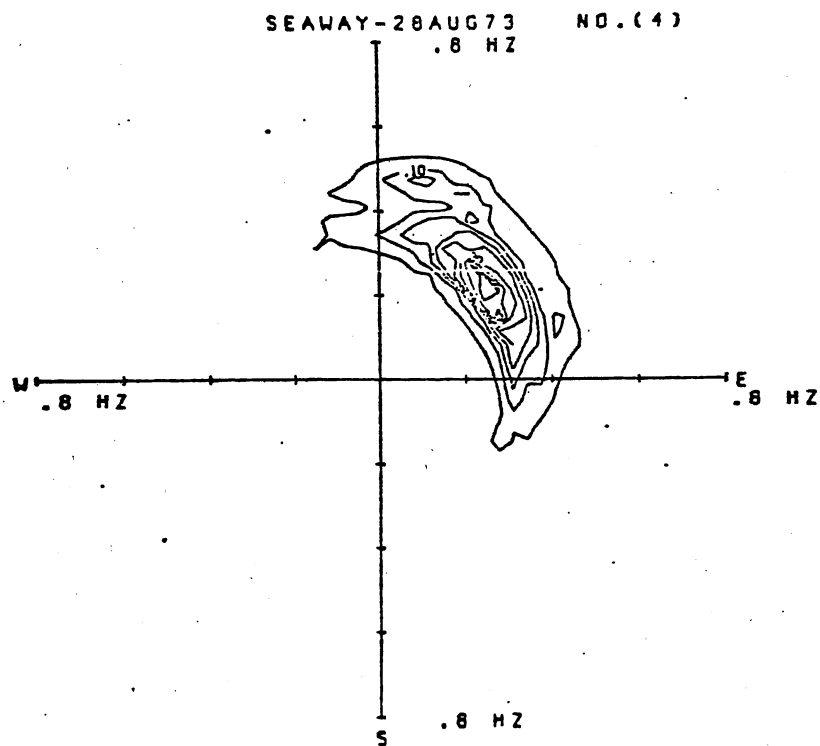


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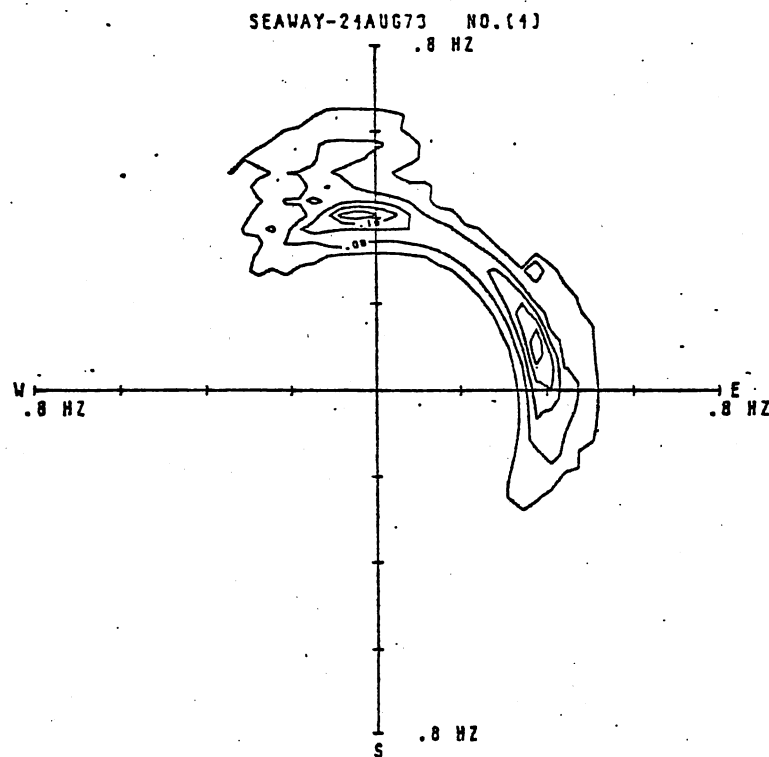
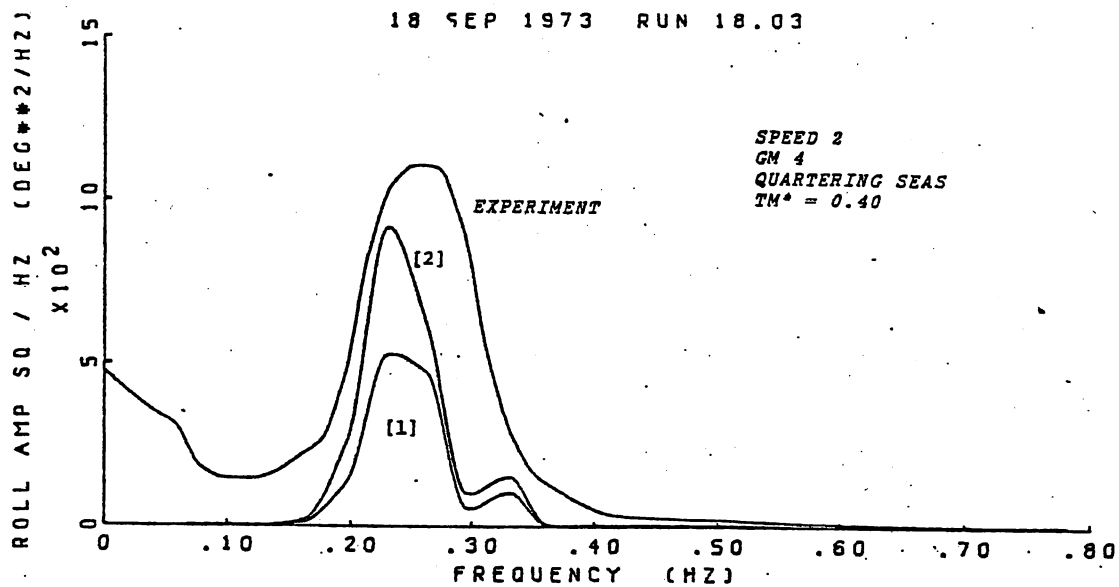


Figure (6)

SL-7 MODEL ROLL SPECTRA

18 SEP 1973 RUN 18.03

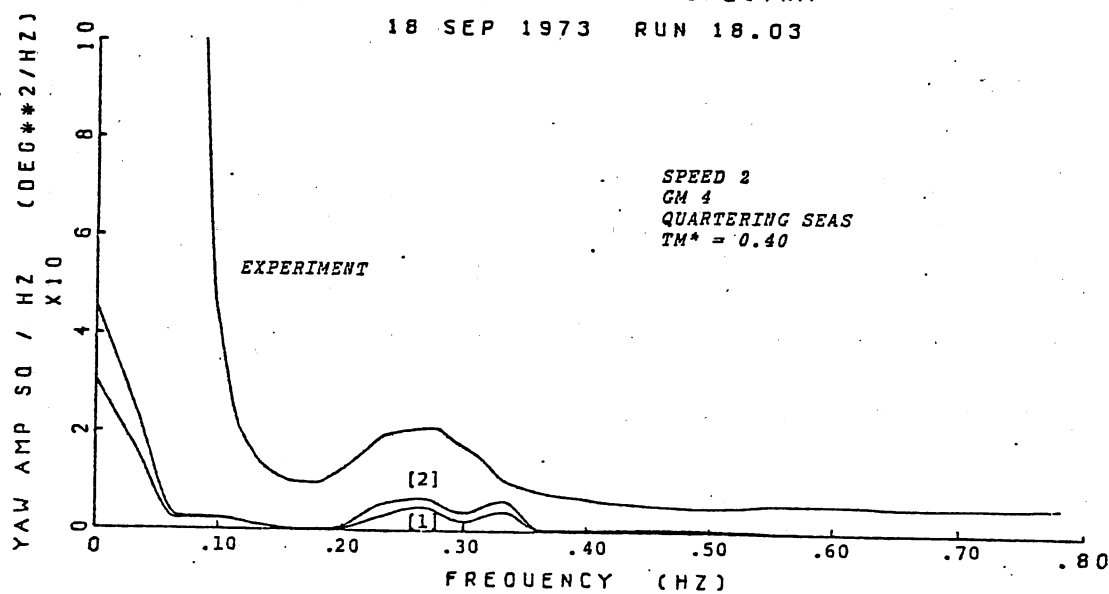


* TM = Time of the model run as a percentage of the time between seaway records.

Figure (7a)

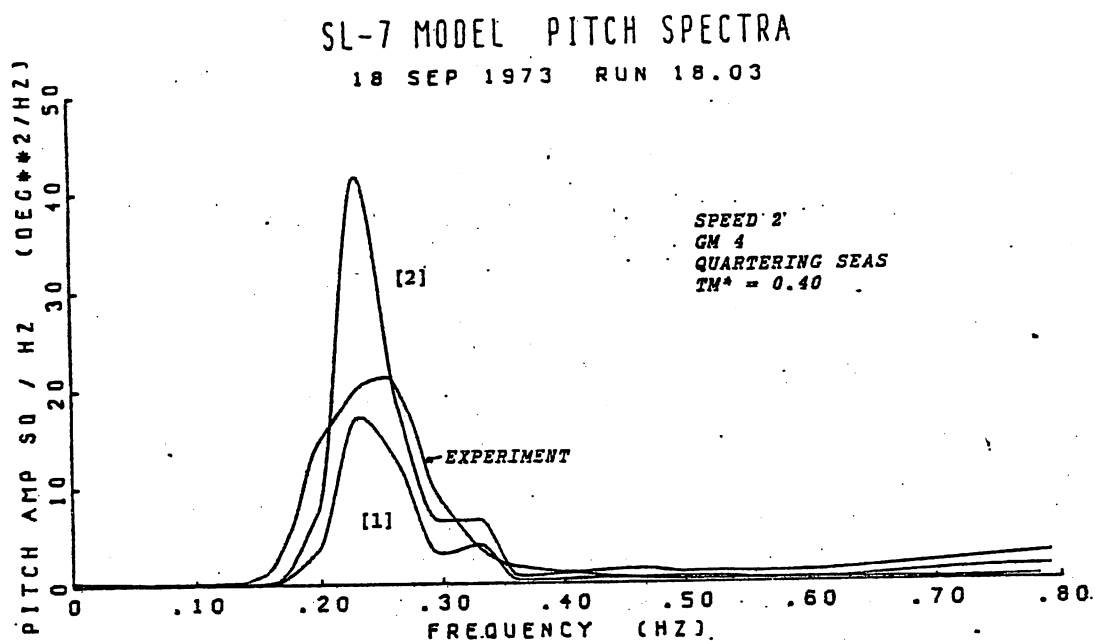
SL-7 MODEL YAW SPECTRA

18 SEP 1973 RUN 18.03



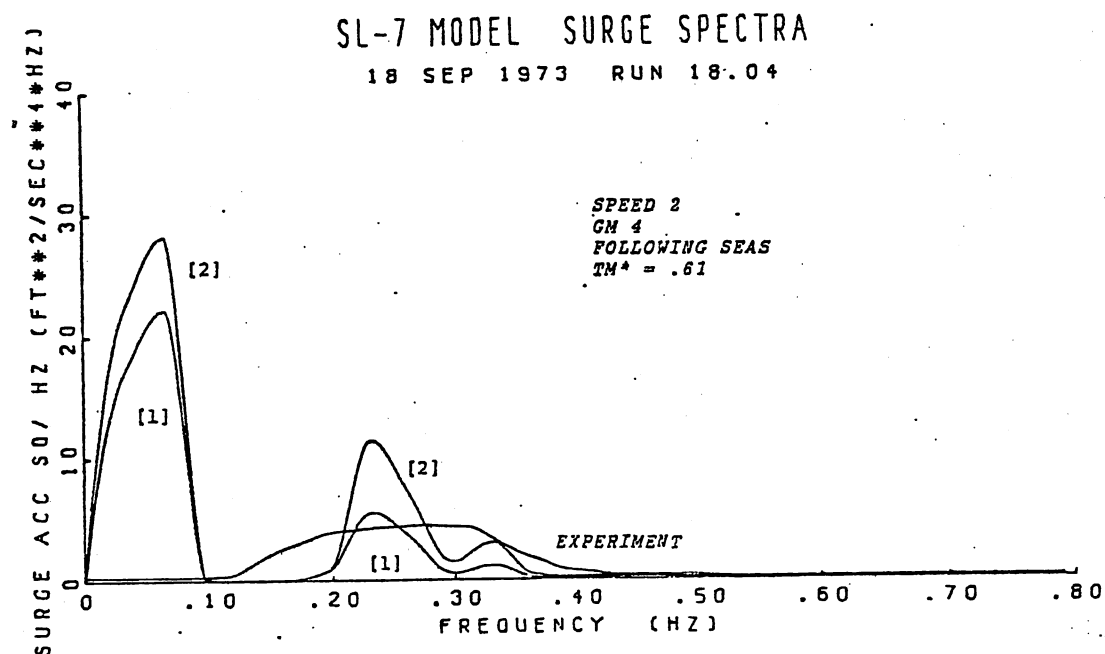
* TM = Time of the model run as a percentage of the time between seaway records.

Figure (7b)



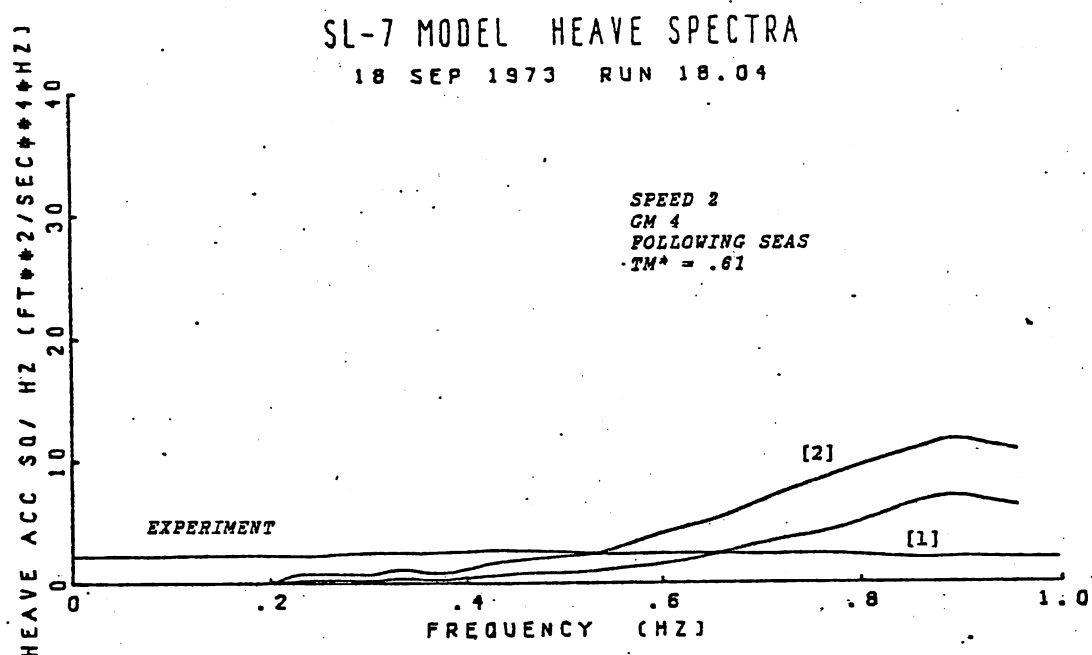
* TM = Time of the model run as a percentage of the time between seaway records.

Figure (7c)



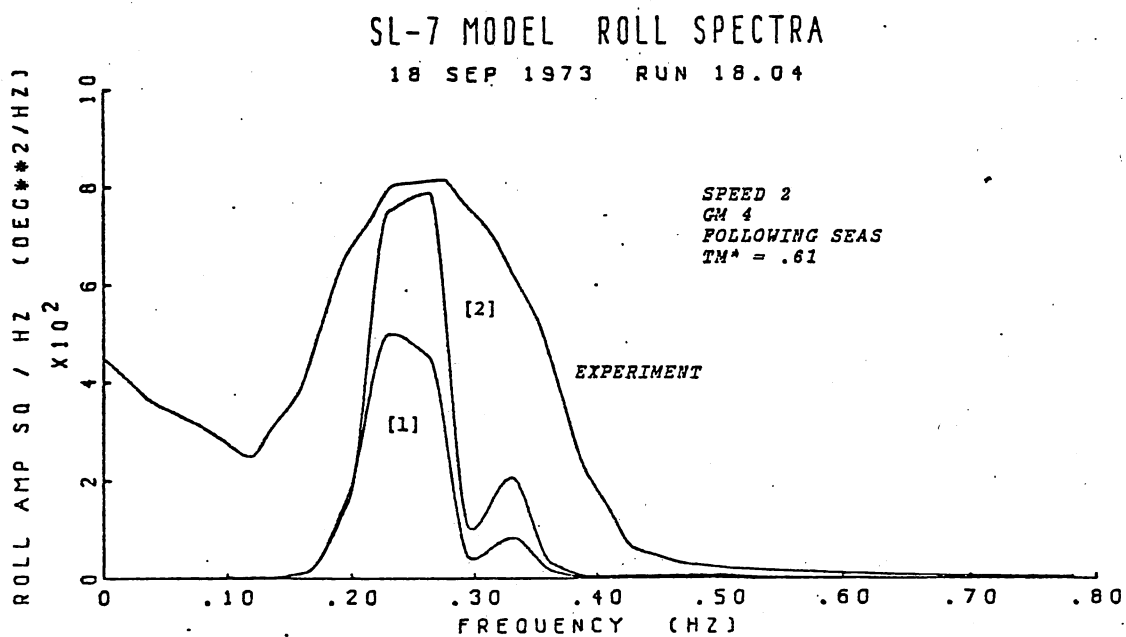
* TM = Time of the model run as a percentage of the time between seaway records.

Figure (8a)



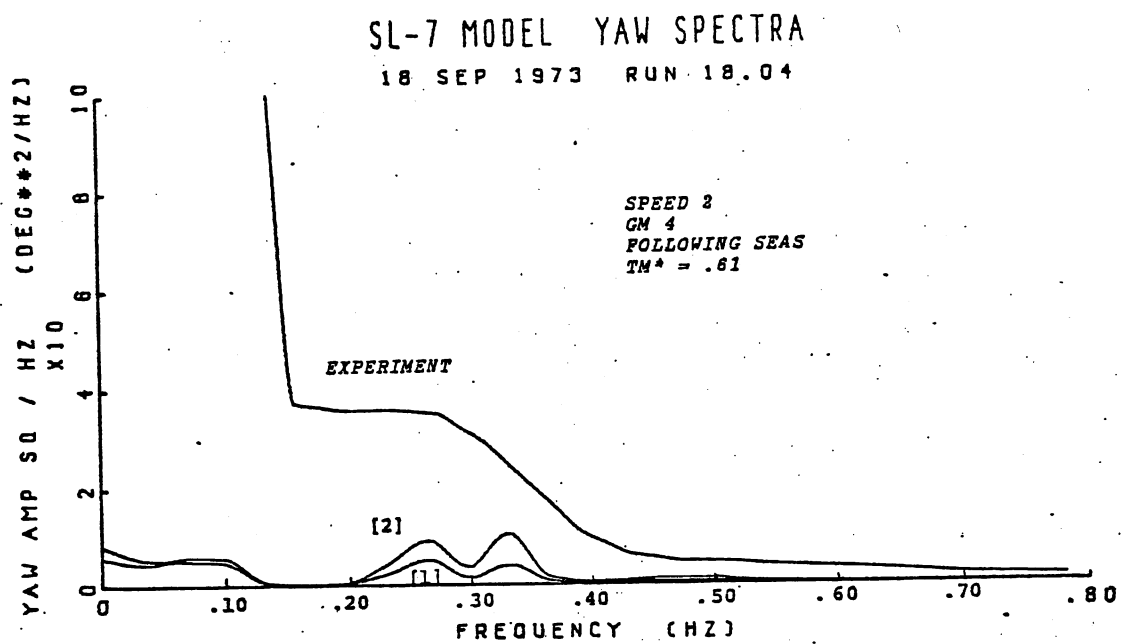
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Figure (8b)



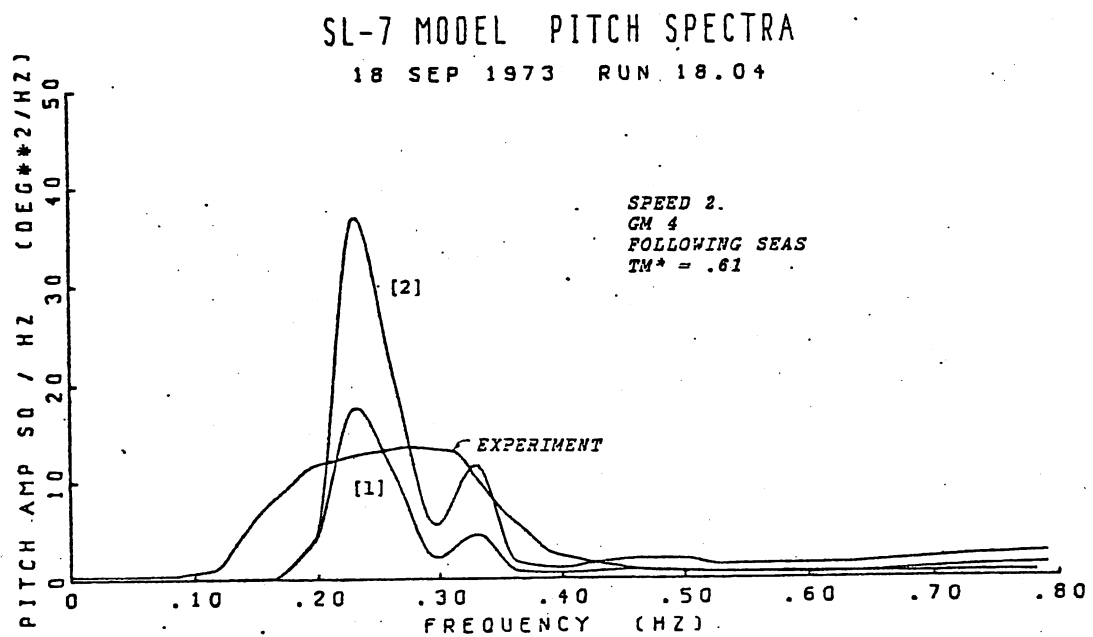
* TM = Time of the model run as a percentage of the time between seaway records.

Figure (8c)



* TM = Time of the model run as a percentage of the time between seaway records.

Figure (8d)



* TM = Time of the model run as a percentage of the time between seaway records.

Figure (8e)

Note that the righting arms, measured in feet, have been plotted for $\overline{GM}=0$; See Figure (10)

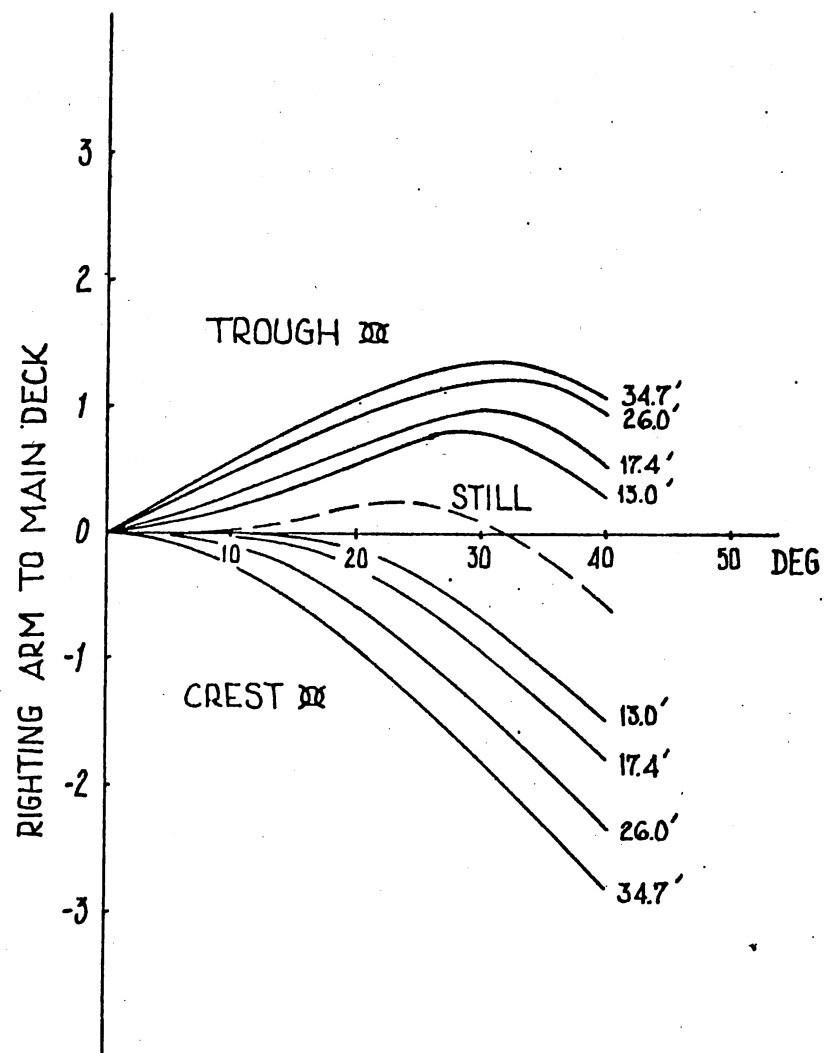


Figure (9a)

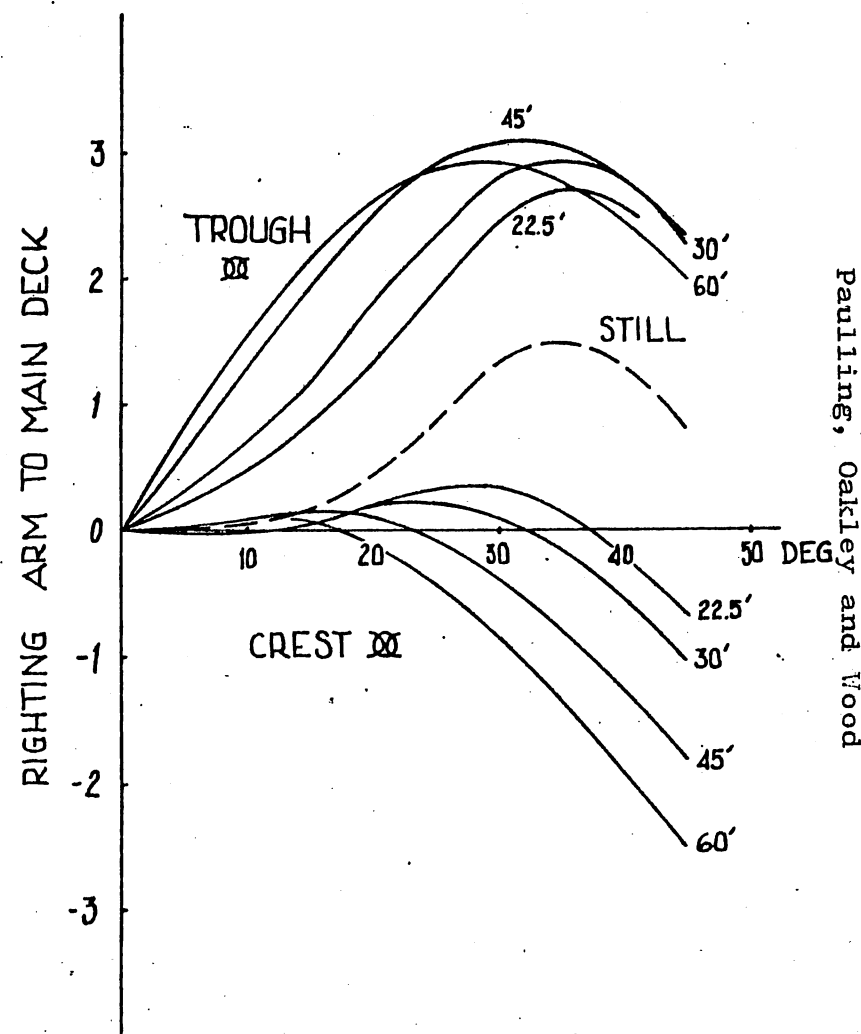


Figure (9b)

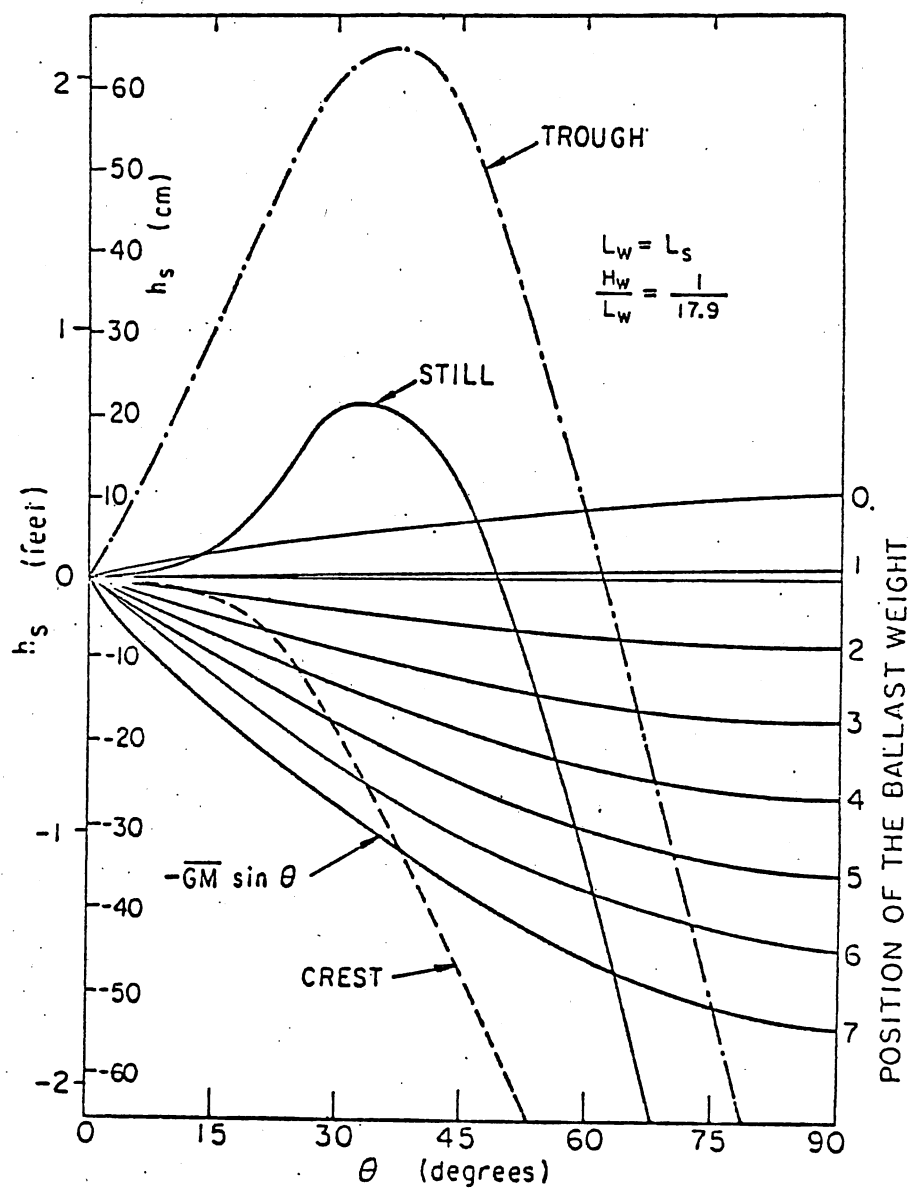


Figure (10) RIGHTING ARM CURVES, SHIP "CHALLENGER"
FOR HEAVY MODEL CONDITION

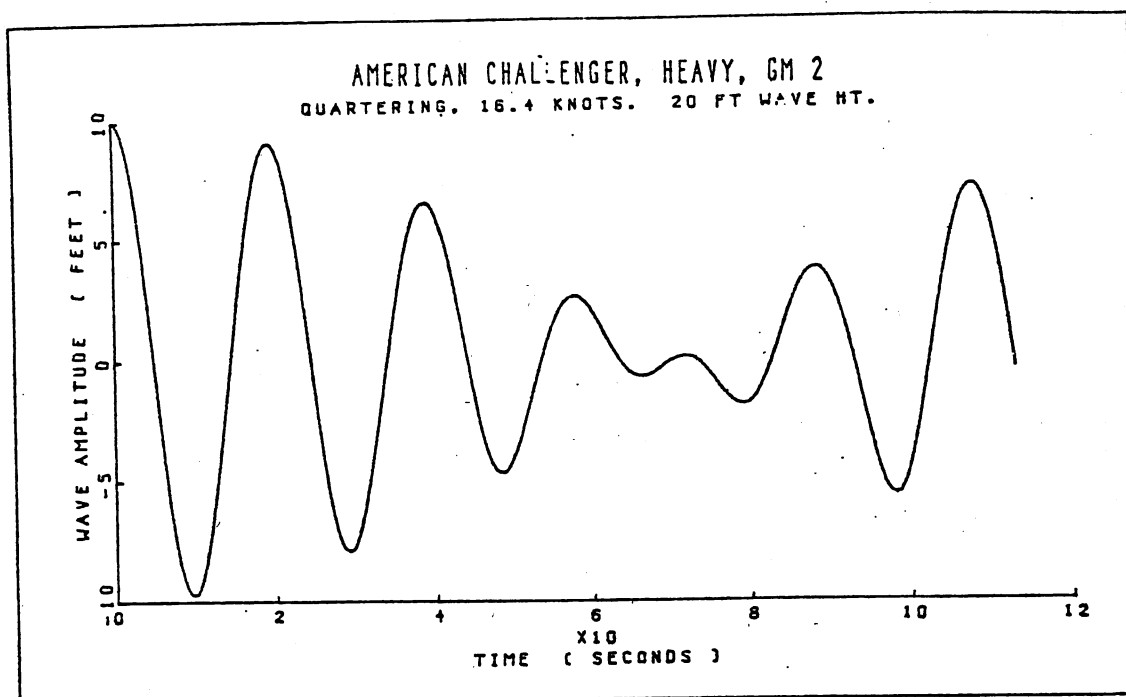


Figure (11)

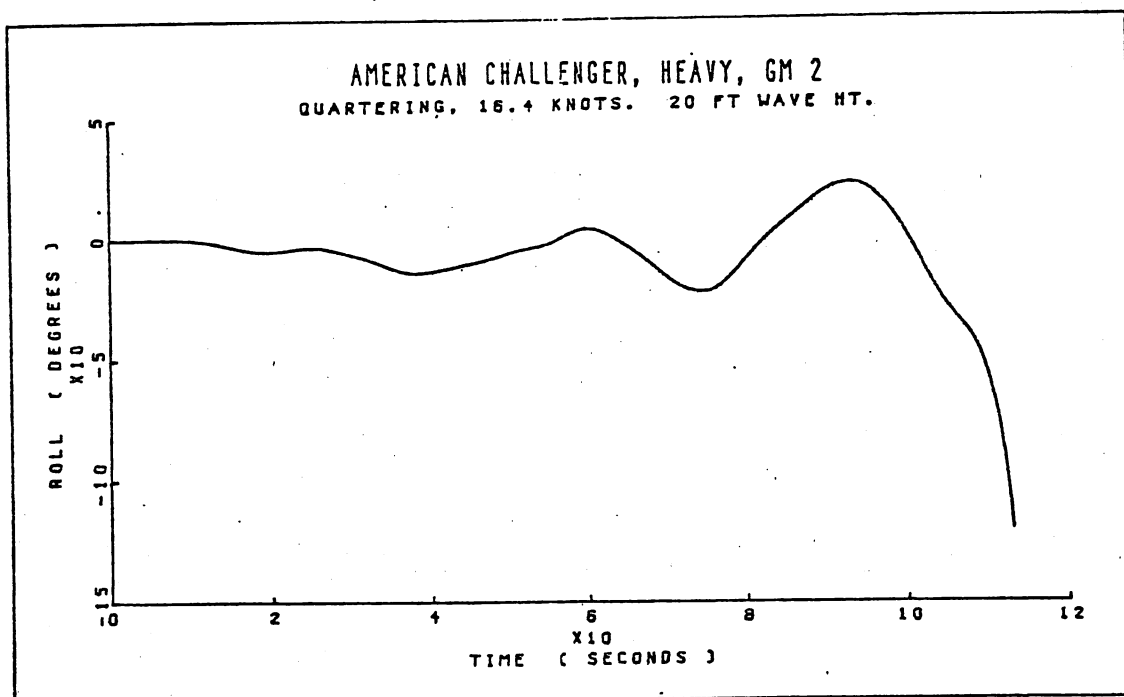


Figure (12a)

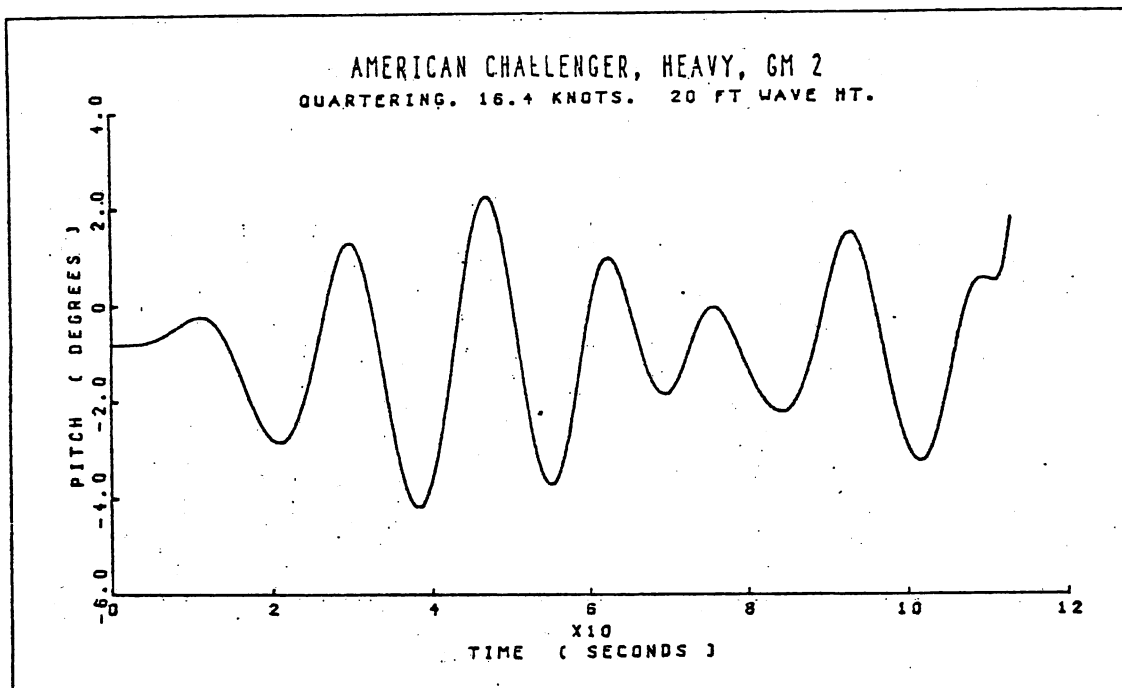


Figure (12b)

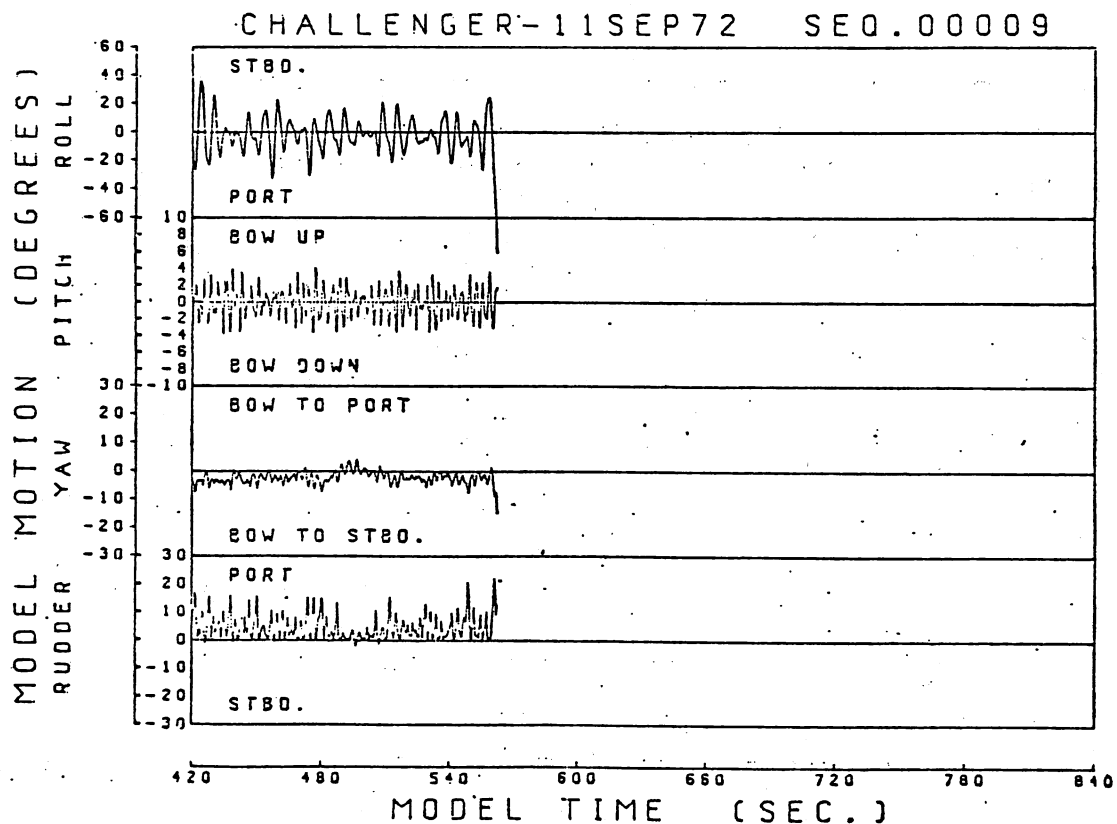


Figure (13)

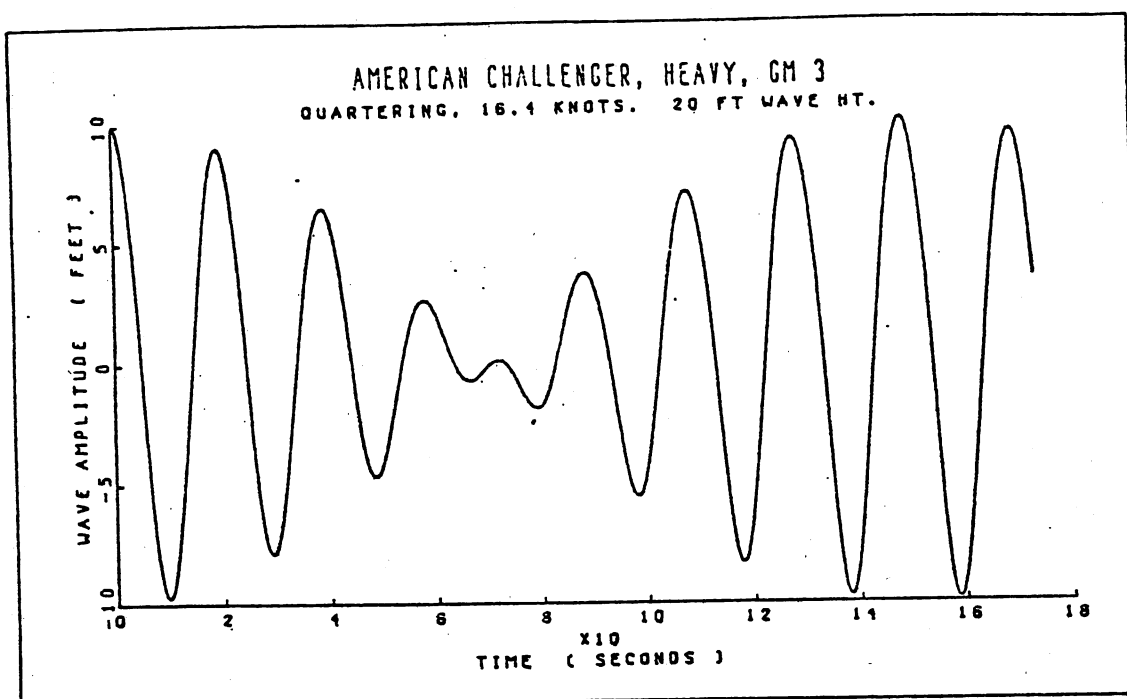


Figure (14)

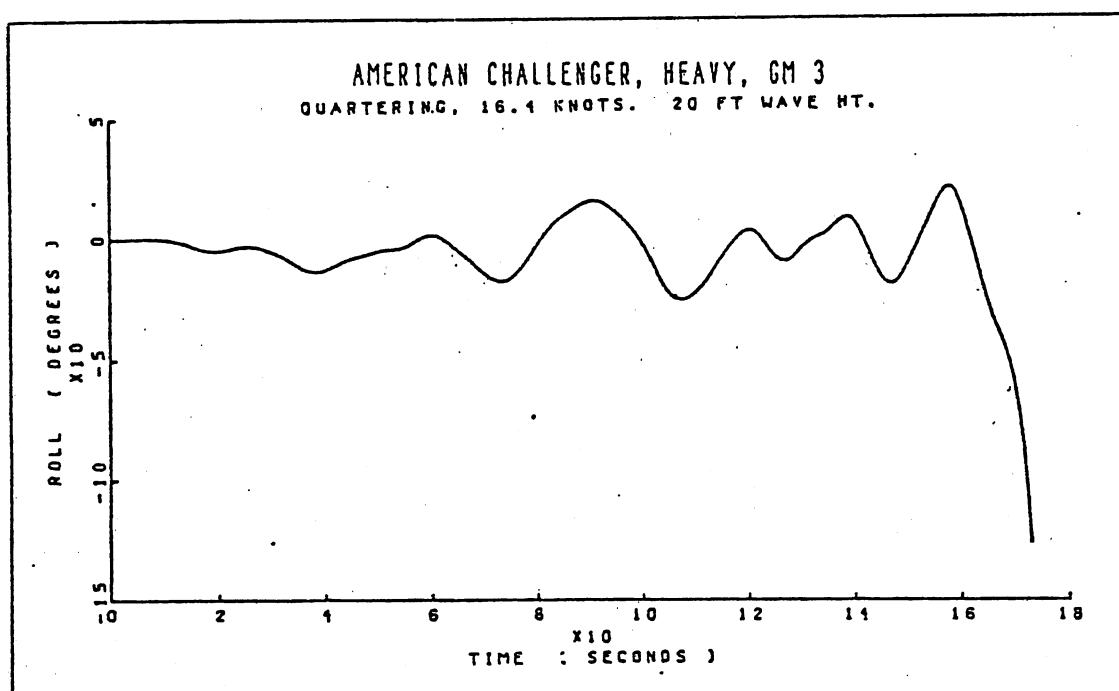


Figure (15a)

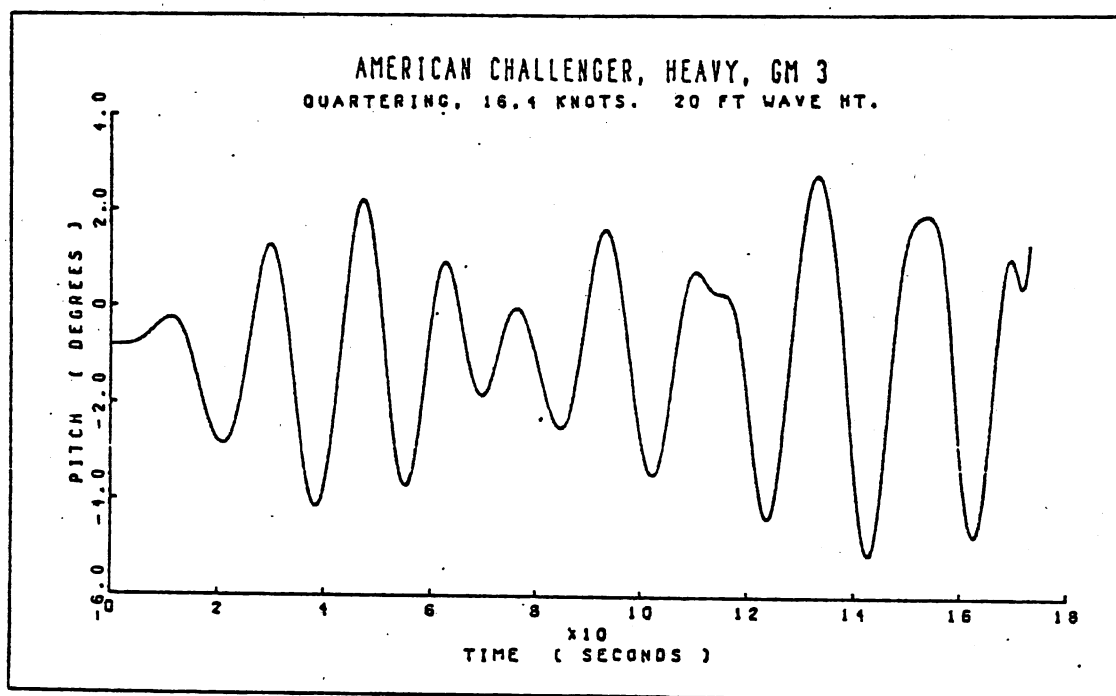


Figure (15b)

HYDRODYNAMIC FORCES AND MOMENTS ACTING ON
TWO-DIMENSIONAL ASYMMETRICAL BODIES

by

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SUMMARY

Many studies have been published on the hydrodynamic forces and moments acting on two-dimensional oscillating bodies. Most of them, however, have been concerned with the symmetrical bodies oscillating in the surface of a fluid.

In this paper, the author tried to propose a method to calculate the hydrodynamic forces and moments acting on the asymmetrical two-dimensional oscillating bodies, distributing the singularities on their surfaces. For two kinds of models, the hydrodynamic characteristics were computed by this method and some of them were compared with the experimental results of the same section.

As a development of this method, the coupled motions of a ship in beam seas in its heeled conditions under the effect of the strong winds were calculated using these hydrodynamic forces and moments.

The calculated two-dimensional hydrodynamic characteristics were confirmed by the known relation to be accurate in numerical values and checked very well the results obtained by the model experiments.

Through the numerical calculations of the coupled motions, it was also found that the effect of the heaving motion on the rolling motion can not be ignored for the inclined ship as was originally expected.

1. INTRODUCTION

It is important in investigating the

stability of a ship, to predict the motions of the ship in regular waves. Generally, these motions have been predicted applying the so-called strip method, which uses the two-dimensional characteristics of each cross section of the ship. Two-dimensional hydrodynamic forces and moments acting on the cross sections of the ship for the upright condition can be calculated employing Tasai-Ursell Method (1) - (2) and Source Method (3) - (4).

When the ship is in the heeled condition under the effect of the strong winds or by damage, there is an uneasy feeling that the capsizing moment induced by the heaving motion might become greater. It is accordingly important to estimate the motions of the inclined ship in regular waves in considering the stability of the ship. As the cross sections of the ship are asymmetrical in these cases, if the strip method is to be used to predict the motions of the ship in its heeled condition, it is necessary to estimate the two-dimensional hydrodynamic characteristics for the asymmetrical cross sections.

In this paper, various hydrodynamic coefficients were theoretically computed on the two kinds of asymmetrical configurations and some of them were compared with the results obtained by model experiments. The ship's coupled motions of heaving, swaying and rolling in beam seas were calculated when she is under the effect of steady inclining moment.

2. FORMULATION OF THE PROBLEM

2.1 Integral equation

Consider a coordinate system with its origin in the plane of the undisturbed free surface (see Figure 1). Viscous effects are neglected and the fluid is assumed to be incompressible. The depth of water is infinite and the free surface extends infinitely in the direction of x-axis.

The body boundary conditions and free surface condition can be assumed to be linear as the motion of the body and fluid are supposed to be small.

When a two-dimensional body is periodically oscillating, the velocity potentials ϕ_j , $j = 1, 2, 4$, must satisfy

$$\frac{\partial^2 \phi_j}{\partial x^2} + \frac{\partial^2 \phi_j}{\partial y^2} = 0$$

in the fluid domain,

$$\omega^2 \phi_j + g \frac{\partial \phi_j}{\partial y}$$

on $y=0$,

$$\frac{\partial \phi_j}{\partial y} = 0$$

on $y=\infty$

Further, the velocity potentials satisfy the radiation conditions and the following body boundary conditions on the mean position of the wetted surface of the body oscillating in the fluid.

$$\frac{\partial \phi_1(x,y)}{\partial n} = -\frac{\partial y}{\partial n},$$

$$\frac{\partial \phi_2(x,y)}{\partial n} = -\frac{\partial x}{\partial n},$$

$$\frac{\partial \phi_3(x,y)}{\partial n} = -y \frac{\partial x}{\partial n} + x \frac{\partial y}{\partial n},$$

$$\frac{\partial \phi_4(x,y)}{\partial n} = -\frac{\partial \phi_0(x,y)}{\partial n}.$$

When we introduce the stream function corresponding to the velocity potential, we can write the body boundary conditions (5) as follows,

$$\psi_1(x,y) = x + C_1,$$

$$\psi_2(x,y) = -y + C_2,$$

$$\psi_3(x,y) = -0.5(x^2 + y^2) + C_3. \quad (7)$$

Introducing Green's function, the velocity potential and the stream function can be written as

$$\phi_j(x,y) = \int_C \sigma_j(x',y') G(x,y;x',y') ds,$$

$$\psi_j(x,y) = \int_C \sigma_j(x',y') S(x,y;x',y') ds,$$

(8)

where

$$G(x,y;x',y') = \log r_1 - \log r_2 - 2 \lim_{\mu \rightarrow 0} \int_0^\infty \frac{e^{-k(y+y')} \cos k(x-x')}{k - K + i\mu} dk,$$

$$S(x,y;x',y') = \theta_1 - \theta_2 - 2 \lim_{\mu \rightarrow 0} \int_0^\infty \frac{e^{-k(y+y')} \sin k(x-x')}{k - K + i\mu} dk$$

$$r_1^2 = (x-x')^2 + (y-y')^2, \quad r_2^2 = (x-x')^2 + (y+y')^2,$$

$$\theta_1 = \tan^{-1} \frac{y-y'}{x-x'}, \quad \theta_2 = \tan^{-1} \frac{y+y'}{x-x'}.$$

Integration in equation (8) is calculated along the wetted contour C of the body, x' and y' being the co-ordinates of a point on C.

Since ϕ_j, ψ_j, G and S are complex functions,

$$\phi_j = \phi_{jc} + i \phi_{js}, \quad \psi_j = \psi_{jc} + i \psi_{js}, \quad \sigma_j = \sigma_{jc} + i \sigma_{js}$$

$$G = G_c + i G_s, \quad S = S_c + i S_s \quad (10)$$

The boundary value problem is transformed to the integral equations using the stream functions which are derived from the equation (8), so that

$$\begin{aligned}\psi_{js}(x, y) &= \int_c \{ \sigma_{jc}(x', y') \cdot S_c(x, y; x', y') \\ &\quad - \sigma_{js}(x', y') \cdot S_s(x, y; x', y') \} ds, \\ \psi_{js}(x, y) &= \int_c \{ \sigma_{jc}(x', y') \cdot S_s(x, y; x', y') \\ &\quad - \sigma_{js}(x', y') \cdot S_c(x, y; x', y') \} ds. \quad (11)\end{aligned}$$

The source densities σ_{jc} and σ_{js} in the equation (11) are obtained satisfying the boundary conditions (7).

Equation (11) is solved approximating the body surface by a large number of line segments, over each of which the source density is assumed to be constant. This transforms the integral equations into a set of linear algebraic equations with the unknown values of the source density on the elements.

2.2 Forces and moments

The hydrodynamic force and moment of the k-th mode induced by the j-th mode of oscillation, defined as $F_{jk} e^{i\omega t}$, are normalized by the velocity amplitude and can be expressed as

$$\begin{aligned}f_{jk} &= \frac{F_{jk}}{\rho \omega^2 X_j} = - \int_c \phi_j \frac{\partial \phi_k}{\partial n} ds = \\ &= f_{jkc} + i f_{jks} \\ (j, k &= 1, 2, 3) \quad (12)\end{aligned}$$

where

$$\begin{aligned}\frac{\partial \phi_1}{\partial n} &= - \frac{\partial y}{\partial n}, \quad \frac{\partial \phi_2}{\partial n} = - \frac{\partial x}{\partial n}, \\ \frac{\partial \phi_3}{\partial n} &= - y \frac{\partial x}{\partial n} + x \frac{\partial y}{\partial n}\end{aligned}$$

The wave exciting force and moment

of the j-th mode is written as $E_j^+ e^{i\omega t}$, E_j^- and E_j^- being the exciting force due to the incident waves which come from the positive direction and the negative direction of the x-axis respectively, normalized by the amplitude of the incident waves, and are expressed,

$$e_j^\pm = \frac{E_j^\pm}{\rho g \zeta_0} = - \int_c (\phi_0 + \phi_4) \frac{\partial \phi_j}{\partial n} ds \quad (13)$$

The first term of the equation (13) is the Froude Kriloff force and the second term is the diffraction force. According to the Green's theorem, the equation (13) is transformed as follows,

$$e_j^\pm = - H_j^\pm(k), \quad (j = 1, 2, 3) \quad (14)$$

where

$$H_j^\pm(k) = \int_c \left(\frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial}{\partial n} \right) e^{-ky \pm ikx} ds. \quad (15)$$

The amplitude ratio of the radiation waves produced by the j-th mode of the oscillating motion is written as,

$$\bar{A}_j^\pm = \left| \frac{\eta_j^\pm}{X_j} \right|_{x \rightarrow \pm\infty} = K |H_j^\pm(k)| \quad (16)$$

Then, from equations (14) and (16),

$$|e_j^\pm| = \frac{\bar{A}_j^\pm}{K} \quad (17)$$

In the equation (14) the exciting forces and moments induced by the incident waves were represented using the radiation potential. The exciting forces and moments are also transformed into the following expression introducing the source density distribution on the body surface, so that

$$e_j^\pm = -2\pi \int_C \sigma_j(x, y) e^{-ky \pm ikx} ds, \quad (j=1, 2, 3) \quad (18)$$

Since $e_j^\pm = e_{jc}^\pm + i e_{js}^\pm$, $\sigma_j^\pm = \sigma_{jc}^\pm + i \sigma_{js}^\pm$ the equation (18)

$$\begin{aligned} e_{jc}^\pm &= -2\pi \left(\int_C \sigma_{jc}^\pm e^{-ky} \cos Kx ds \right. \\ &\quad \left. + \int_C \sigma_{js}^\pm e^{-ky} \sin Kx ds \right), \\ e_{js}^\pm &= -2\pi \left(\pm \int_C \sigma_{jc}^\pm e^{-ky} \sin Kx ds \right. \\ &\quad \left. + \int_C \sigma_{js}^\pm e^{-ky} \cos Kx ds \right). \end{aligned} \quad (19)$$

The phase lag ϵ_j^\pm of the j -th mode of the exciting force and moment to the incident waves is defined by

$$\epsilon_j^\pm = \tan^{-1} \left(\frac{e_{js}^\pm}{e_{jc}^\pm} \right) \quad (20)$$

2.3 Reflection and transmission coefficients (5)-(6)

The function $H_j^\pm(k)$ in the equation (15) is complex and is expressed as

$$\begin{aligned} H_j^\pm(k) &= A_{jc}^\pm e^{\pm i \theta_{jc}} + i A_{js}^\pm e^{\pm i \theta_{js}} \\ &= H_{jc}^\pm + i H_{js}^\pm \end{aligned} \quad (j=1, 2, 3) \quad (21)$$

Then, the function $H_4^\pm(k)$ which is treated in the diffraction problem can be derived as follows according to the

reference (6),

$$\begin{aligned} H_4^+ &= 2(A_1 e^{i\alpha_1} - A_2 e^{i\alpha_2}) / [e^{-i\beta_2}(A_1 e^{i\alpha_1} + i) \\ &\quad (A_2 e^{-i\alpha_2} + i) - e^{-i\beta_1}(A_1 e^{-i\alpha_1} + i)(A_2 e^{i\alpha_2} + i)], \\ H_4^- &= 2i \{ e^{-i\beta_2} A_2 e^{-i\alpha_2} (A_1 e^{i\alpha_1} + i) \\ &\quad - e^{-i\beta_1} A_1 e^{-i\alpha_1} (A_2 e^{i\alpha_2} + i) \} / \{ e^{-i\beta_2} (A_1 e^{i\alpha_1} + i) \\ &\quad (A_2 e^{-i\alpha_2} + i) - e^{-i\beta_1} (A_1 e^{-i\alpha_1} + i)(A_2 e^{i\alpha_2} + i) \}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} A_j^\pm &= \frac{A_{js}}{A_{jc}}, \quad \alpha_j^\pm = \theta_{js} - \theta_{jc}, \\ \beta_j^\pm &= 2\theta_{jc}, \\ (j &= 1, 2). \end{aligned}$$

The progressive waves at $x \rightarrow \pm\infty$ which are induced by the diffraction potential can be expressed as

$$\eta_{\pm\infty} \sim i \zeta_a H_4^\pm e^{-ky \pm ikx} \quad (23)$$

Therefore, when the incident waves which have unit amplitude are coming from the positive direction of x -axis the progressive waves on each side of the fixed body can be shown as,

$$\begin{aligned} \eta_{+\infty} &\sim i H_4^+ e^{-ky + ikx}, \quad x > 0, \\ \eta_{-\infty} &\sim (1 + i H_4^-) e^{-ky + ikx}, \quad x < 0 \end{aligned} \quad (24)$$

The reflection and transmission coefficient may be derived from the equation (24) as follows,

$$C_R = |i H_4^+|,$$

$$C_T = |1 + i H_4^-|, \quad (25)$$

3. RESULTS OF COMPUTATIONS AND EXPERIMENTS

The computations and experiments were performed for two kinds of models as are shown in the following:

Model A; Asymmetrical Lewis section
(see Figure 2)

$$a_1 = 0.0, a_3 = 0.2 \text{ for } x \geq 0$$

$$a_1 = 0.0, a_3 = -0.1 \text{ for } x \leq 0$$

Model B; Symmetrical Lewis section
(see Figure 3)

$$a_1 = 0.0, a_3 = -0.1$$

$$\text{heel angle } \alpha = 0^\circ, 10^\circ, 20^\circ, 30^\circ$$

where a_1, a_3 are Lewis form coefficients.

The accuracy of the computation was checked numerically using the following relation, (see Reference 6).

$$f_{jjs} = \frac{|\bar{A}_j^+|^2 + |\bar{A}_j^-|^2}{2K^2},$$

$$(j=1,2,3) \quad (26)$$

It was found that the equation (26) was satisfied with enough accuracy when the contour C is divided into 40 line segments.

3.1 Radiation wave amplitude

Figure 4 shows the calculated progressive wave amplitude ratio produced by the forced heaving oscillation of Model A. In the same figure, the calculated values for the two symmetrical bodies, which are represented by the same Lewis form coefficients of each side of the x-axis respectively, are indicated. We may conclude from this figure that the tendency of the change of the wave amplitudes progressing to

the positive and negative directions of the x-axis by the frequency $\beta\alpha$ are similar to the ones for the two symmetrical bodies which are represented by the same Lewis form coefficients of each side respectively. The comparison between the calculated and measured values are shown in Figure 5. It is found from this figure that the calculated values are in good agreement with the measured ones in the model experiments.

Figures 6, 7 and 8 show $\bar{A}_1^\pm, \bar{A}_2^\pm$ and \bar{A}_3^\pm respectively for the various heeled conditions of Model B together with the ones for the unheeled condition. From these figures we may conclude that the values of \bar{A}_j^\pm ($j=1,2,3$) for the heeled conditions can not be predicted from the ones for the unheeled condition. Comparison between the calculated and measured \bar{A}_j^\pm for the heeled angle of $10^\circ, 20^\circ$ and 30° are shown in Figures 9, 10 and 11. In these cases, the calculated values are in good agreement with the ones obtained by the experimental results, too.

3.2 Forces and moments

Non-dimensional hydrodynamic forces and moments are represented as follows,

$$m_{jk} = f_{jkc} / \frac{1}{2} \pi d^2, \quad n_{jk} = -f_{jks} \cdot K^2,$$

$$(j=1,2,3; k=1,2),$$

$$m_{jk} = f_{jkc} / \frac{1}{8} \pi d^2, \quad n_{jk} = -f_{jks} \cdot K^2 / d^2,$$

$$(j=1,2,3; k=3) \quad (27)$$

The calculated hydrodynamic forces and moments induced by heaving, swaying and rolling oscillation for Model A are shown in Figures 12, 13 and 14 together with the ones for the two symmetrical bodies which are represented by the same Lewis form coefficients corresponding to each side of the x-axis. From these figures, it is made clear that the results for the asymmetrical body are very different from the ones for the symmetrical bodies. Particularly, it is found that large rolling moment acts on Model A induced by the heav-

ing motion.

When Model B is in the heeled conditions ($\alpha = 10^\circ, 20^\circ, 30^\circ$), the calculated values of the radiation forces and moments by the heaving, swaying and rolling motions are shown in Figures 15 to 23. It is shown from these figures that the rilling moments induced by the heaving motion are too large to be ignored in these cases of heeled conditions.

3.3 Reflection and transmission coefficients

The computed values of the reflection and transmission coefficients for Model A are shown in Figure 24 in comparison with the ones of the two symmetrical bodies ($a_1 = 0.0, a_3 = 0.2$, and $a_2 = 0.0, a_3 = -0.1$). It is found that the reflection and transmission coefficients for the asymmetrical body are just about the mean values of the ones for the two symmetrical bodies.

Figure 25 shows the comparison between the calculated and measured C_r for Model A. The calculated values are in good agreement with the ones obtained by the experimental results.

The comparison between the calculated and measured C_r for Model B in its heeled conditions ($\alpha = 10^\circ, 20^\circ, 30^\circ$) are shown in Figures 26, 27 and 28. From these figures, the calculated values are in good agreement with the ones obtained by model experiments, too.

4. THE COUPLED MOTIONS IN BEAM SEAS

4.1 The equations of motion

The ship is assumed to keep its balance in the condition inclining θ degrees with respect to the undisturbed free surface, when the winds blow from the positive direction of x-axis (see Figure 29).

The the heeling moment M_w due to the winds is written as follows,

$$M_w = W \cdot GM_\theta \cdot \sin \theta \quad (28)$$

Let the translatory displacement in the x and y direction with respect to the centre of gravity G be x_G and y_G and the angular displacement about G be φ , x_G being the swaying, y_G the heaving displacement and φ the rolling angle.

Under the assumptions that the responses are linear and harmonic, the three linear coupled differential equations of the motions in beam seas can be written as follows,

$$\begin{aligned} (\rho g \nabla + a_{11}) \ddot{y}_G + b_{11} \dot{y}_G + c_{11} y_G + a_{12} \ddot{x}_G \\ + b_{12} \dot{x}_G + a_{13} \ddot{\varphi} + b_{13} \dot{\varphi} + c_{13} \varphi = F_{e1}^{\pm} \end{aligned}$$

$$\begin{aligned} (\rho g \nabla + a_{22}) \ddot{x}_G + b_{22} \dot{x}_G + c_{22} x_G + a_{23} \ddot{\varphi} + b_{23} \dot{\varphi} \\ + a_{21} \ddot{y}_G + b_{21} \dot{y}_G = F_{e2}^{\pm} \end{aligned}$$

$$\begin{aligned} (\rho g \nabla \cdot k^2 + a_{33}) \ddot{\varphi} + b_{33} \dot{\varphi} + c_{33} \varphi \\ + a_{31} \ddot{y}_G + b_{31} \dot{y}_G + c_{31} y_G + a_{32} \ddot{x}_G \\ + b_{32} \dot{x}_G = M_{e3}^{\pm} \end{aligned}$$

(29)

where

$$a_{11} = M_{11}, \quad b_{11} = N_{11}, \quad c_{11} = \rho g B / \cos \theta$$

$$a_{12} = M_{12}, \quad b_{12} = N_{12},$$

$$a_{13} = M_{13} + l_{Gx} \cdot M_{11} - l_{Gy} \cdot M_{12},$$

$$b_{13} = N_{13} + l_{Gx} \cdot N_{11} - l_{Gy} \cdot N_{12}$$

$$c_{13} = \rho g l_{Gx} \cdot B / \cos \theta,$$

$$a_{22} = M_{22}, \quad b_{22} = N_{22}$$

$$a_{23} = M_{23} + l_{Gx} \cdot M_{21} - l_{Gy} \cdot M_{22}$$

$$b_{23} = N_{23} + l_{Gx} \cdot N_{21} - l_{Gy} \cdot N_{22},$$

$$a_{21} = M_{21}, \quad b_{21} = N_{21}$$

$$a_{33} = M_{33} + L_{6x} \cdot M_{31} - L_{6y} \cdot M_{32} + L_{6y}^2 \cdot M_{22} - L_{6x} \cdot L_{6y} \cdot M_{21} \\ - L_{6y} \cdot M_{23} + L_{6x}^2 \cdot M_{11} - L_{6x} \cdot L_{6y} \cdot M_{12} + L_{6x} \cdot M_{13},$$

$$b_{33} = N_{33} + L_{6x} \cdot N_{31} - L_{6y} \cdot N_{32} + L_{6y}^2 \cdot N_{22} - L_{6x} \cdot L_{6y} \cdot N_{21} \\ - L_{6y} \cdot N_{23} + L_{6x}^2 \cdot N_{11} - L_{6x} \cdot L_{6y} \cdot N_{12} + L_{6x} \cdot N_{13},$$

$$C_{33} = \rho g V \cdot G M_{\theta} \cdot \cos \theta + \rho g L_{6x}^2 \cdot B / \cos \theta,$$

$$a_{31} = M_{31} + L_{6x} \cdot M_{11} - L_{6y} \cdot M_{21},$$

$$b_{31} = N_{31} + L_{6x} \cdot N_{11} - L_{6y} \cdot N_{21},$$

$$a_{32} = M_{32} - L_{6y} \cdot M_{22} + L_{6x} \cdot M_{21}, \quad (30)$$

$$b_{32} = N_{32} - L_{6y} \cdot N_{22} + L_{6x} \cdot N_{21},$$

$$F_{e1}^{\pm} = i \frac{\rho g \bar{z}_a}{K} \cdot \bar{A}_1^{\pm} \cdot e^{i(\epsilon_1^{\pm} + \omega t)}$$

$$F_{e2}^{\pm} = i \frac{\rho g \bar{z}_a}{K} \cdot \bar{A}_2^{\pm} \cdot e^{i(\epsilon_2^{\pm} + \omega t)}$$

$$F_{e3}^{\pm} = i \frac{\rho g \bar{z}_a}{K} \cdot \bar{A}_3^{\pm} \cdot e^{i(\epsilon_3^{\pm} + \omega t)} \\ + L_{6x} \cdot F_{e1}^{\pm} - L_{6y} \cdot F_{e2}^{\pm}.$$

The hydrodynamic coefficients M_{jk} , N_{jk} in these equations can be calculated using the following expressions,

$$M_{jk} = \rho \cdot f_{jkc},$$

$$N_{jk} = -\rho \omega \cdot f_{jks}. \quad (31)$$

4.2 The results of calculation

In this section, the author wants to show the results of computation for a ship's section represented by Lewis form ($a_1 = 0.0$, $a_3 = -0.1$) which is inclined 10 degrees by the winds. The calculations of motion are performed for two cases, namely when the

incident waves are coming from the same direction with the winds and the opposite direction with them.

The normalized amplitudes of heaving motion for the heeled condition are shown in Figure 30 together with these for the unheeled condition. It is shown that the amplitude for the unheeled condition at the neighbourhood of the resonant frequency of the rolling motion, when the incident waves are coming from the same direction with the winds, and that is inversely smaller when the waves are coming from the opposite direction with the winds. It is considered that this different tendency for the waves of each direction at the resonance frequency of rolling motion is caused by the difference of the phase lag of rolling motion to the waves.

Figure 34 shows the normalized amplitudes of swaying motion. From this figure, it is found that the amplitudes of swaying motion for the heeled condition are affected by the heaving and rolling motions.

The normalized amplitudes of rolling motion are shown in Figure 32. For the incident waves coming from either side of the x-axis, the amplitude for the heeled condition is greater than that for the up-right condition at the resonant frequency. At the resonant frequency of heaving motion, when the waves are coming from the opposite direction with the winds, the rolling amplitude for the inclined model is twice as large as the one for the up-right model. Conversely, for the waves coming from the same direction with the winds, the amplitude for the heeled condition becomes very small.

5. CONCLUSIONS

The results obtained from the present work are summarized as follows:

- 1) Calculated values \bar{A}_i^{\pm} for model A and B are in good agreement with those obtained by the experiments.
- 2) When model B is in its heeled condition, the rolling moment acting on it induced by the forced heaving oscillation can not be ignored.
- 3) Calculated wave transmission coefficients show about the same value with the ones

obtained through the experimental results.

- 4) The rolling motion for the heeled condition greatly affects the heaving motion around the resonant frequency of rolling.

- 5) When the incident waves are coming from the opposite direction of the winds, the rolling amplitude for the heeled condition is twice as large as the one for the unheeled condition around the resonant frequency of the heaving motion.

6. ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

B	Beam of cross section C
d	Draught
C	Immersed part of the cross sectional contour in the rest position
ds	Line segment along the cross sectional contour
n	Unit normal to the cross sectional contour
∇	Volume displacement of the cross section with unit length
	Transverse radius of gyration
x, y	Co-ordinate system defined in Figure 1
X_j	Oscillating amplitude in the j -th mode
g	Acceleration of the gravity
ρ	Density of the fluid
ω	Circular frequency
K	Wave number ($= \omega^2/g$)
z_a	Amplitude of the incident wave
$\frac{A_j^\pm}{A_j}$	Amplitude ratio (the amplitude of radiation wave normalized by the amplitude of the j -th mode oscillation)
ϕ_j	Velocity potential normalized by the velocity amplitude in the j -th mode motion for $j = 1, 2 \text{ \& } 3$, velocity potential normalized by the amplitude of incident wave for $j = 0 \text{ \& } 4$
ψ_j	Stream function corresponding to the velocity potential ϕ_j
σ_j	Density of source singularities distributed on the sectional contour
C_j	Integral constant for the j -th mode equation
F_{jk}, f_{jk}	Hydrodynamic forces and moments of the k -th mode by the j -th mode oscillation
E_j^\pm, e_j^\pm	Wave exciting force of the j -th mode
ϵ_j^\pm	Phase lag of the j -th mode exciting force to the incident wave
C_R	Ratio of the reflected wave amplitude to the incident wave amplitude
C_T	Ratio of the transmitted wave amplitude to the incident wave amplitude
G	Centre of gravity
B_θ	Centre of the buoyancy when a ship is inclined at degrees

l_{Gx}, l_{Gy}	Co-ordinates of the centre of the gravity
y_G	Heaving displacement of G
x_G	Swaying displacement of G
ϕ	Angular displacement of the rolling motion about G

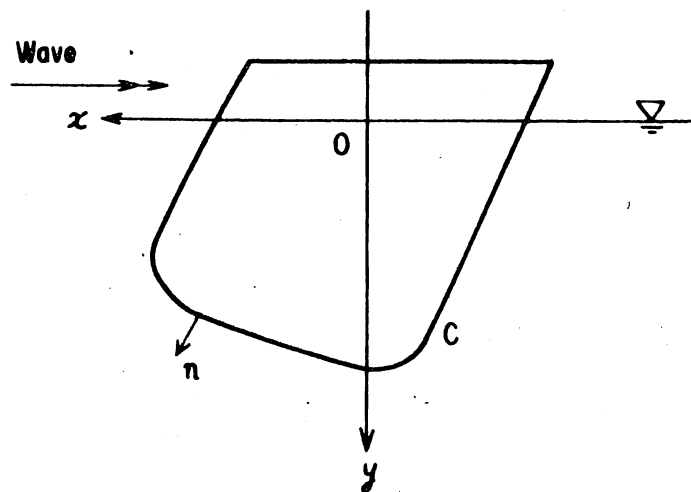


Fig. 1 Coordinate system

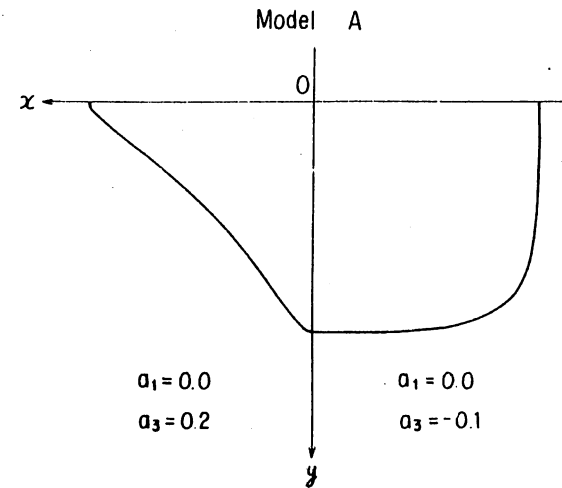


Fig. 2 Cross section of Model A

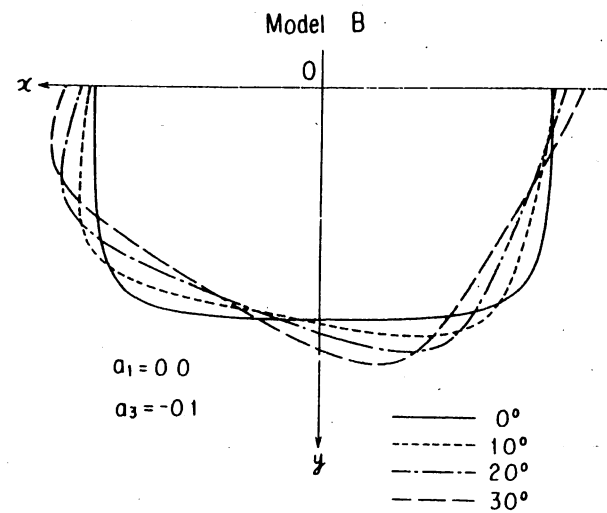
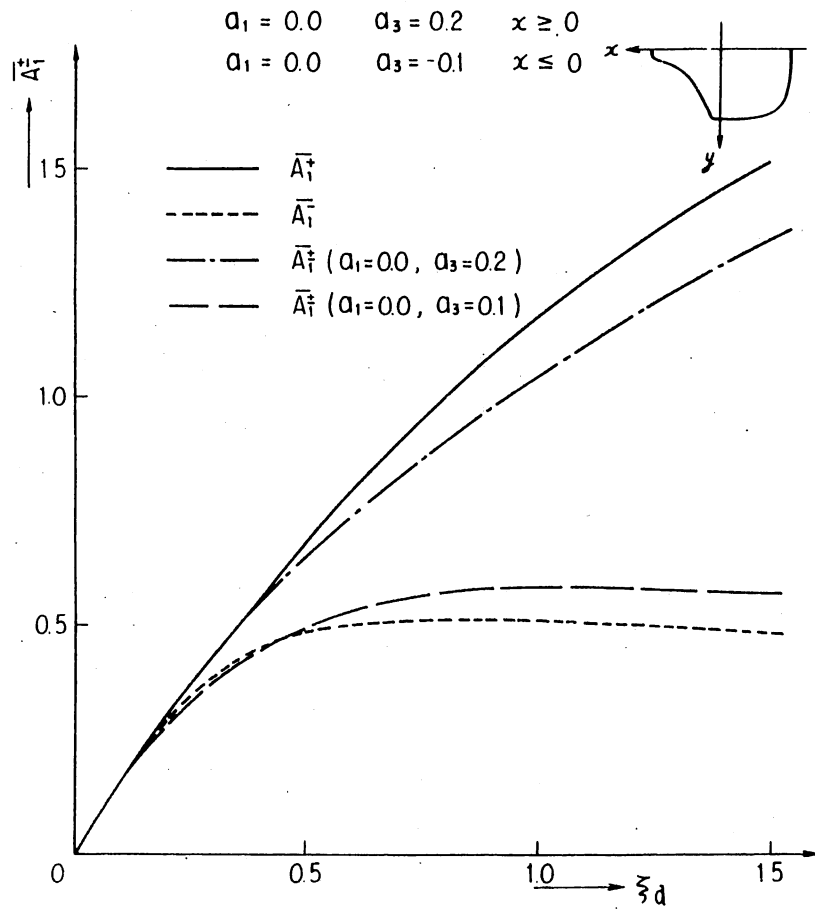
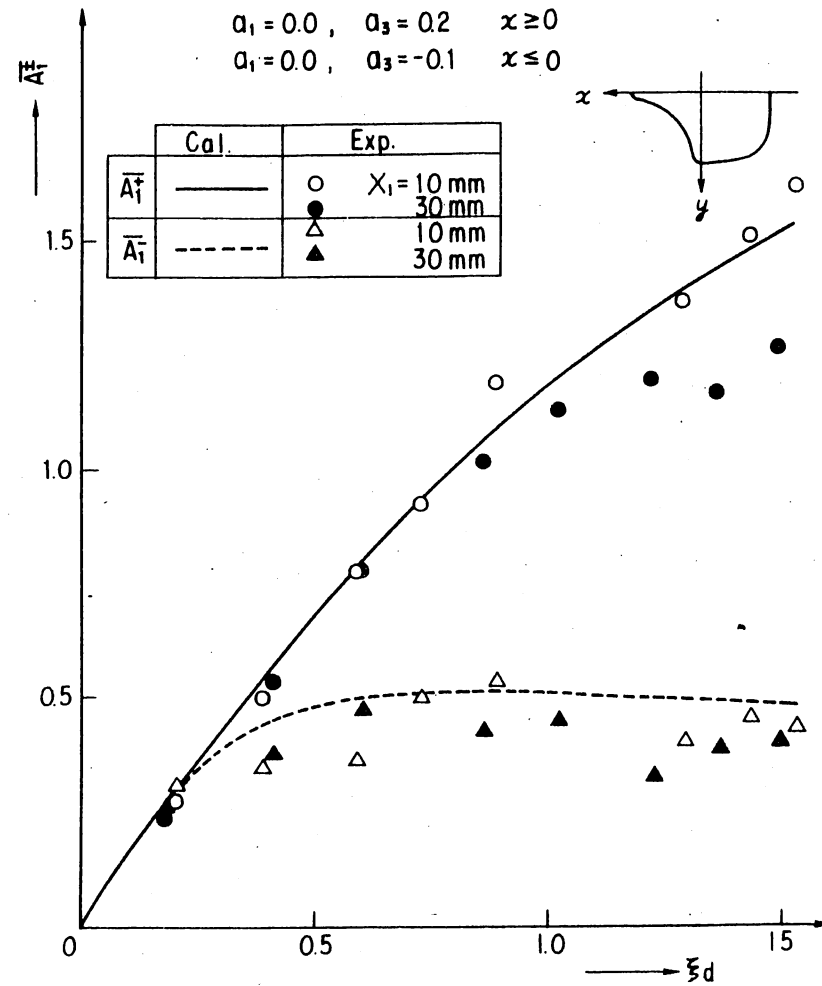


Fig. 3 Cross section of Model B

Fig.4 Calculated \bar{A}_1^+ for Model AFig.5 Comparison between calculated and measured \bar{A}_1^+ for Model A

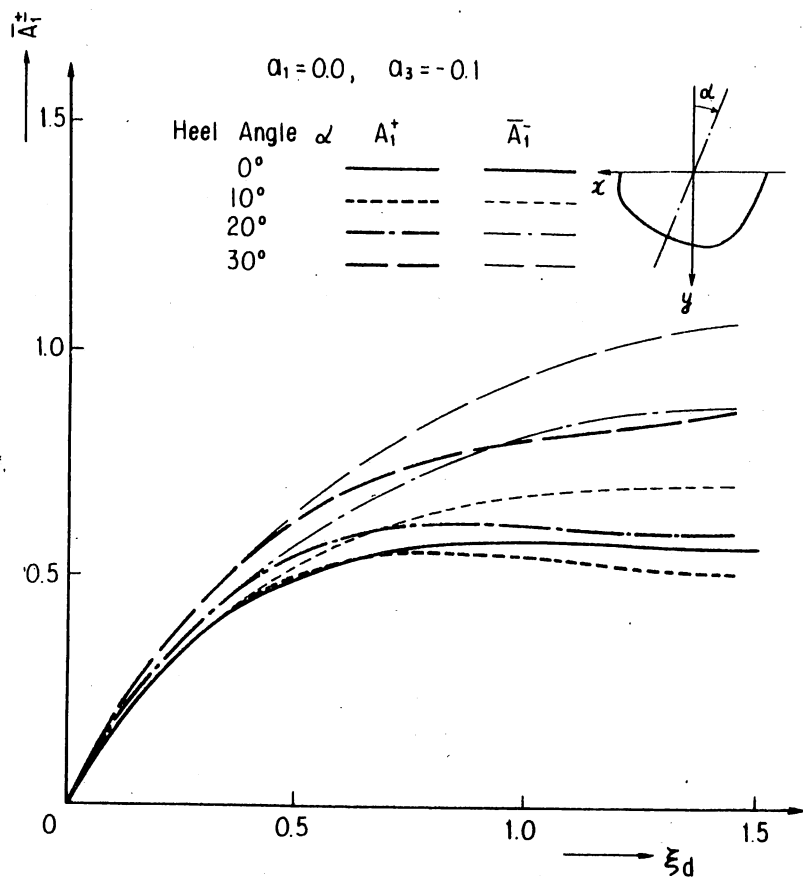


Fig.6 Calculated \bar{A}_1^+ for Model B in the heeled conditions

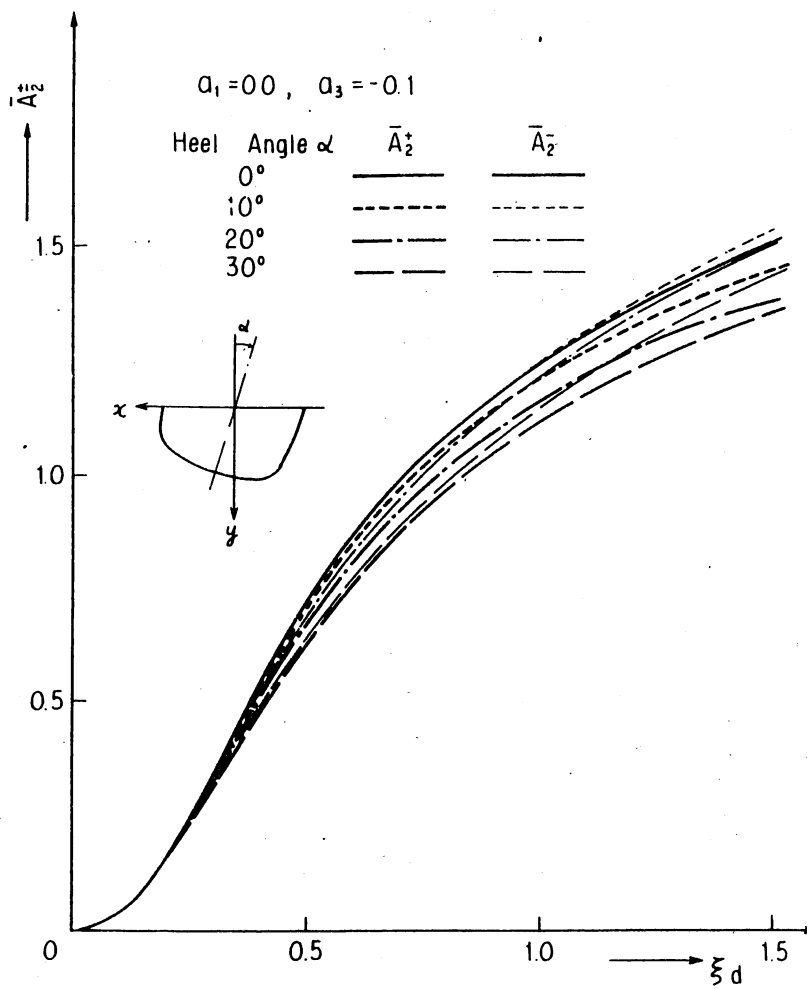


Fig.7 Calculated \bar{A}_2^+ for Model B in the heeled conditions

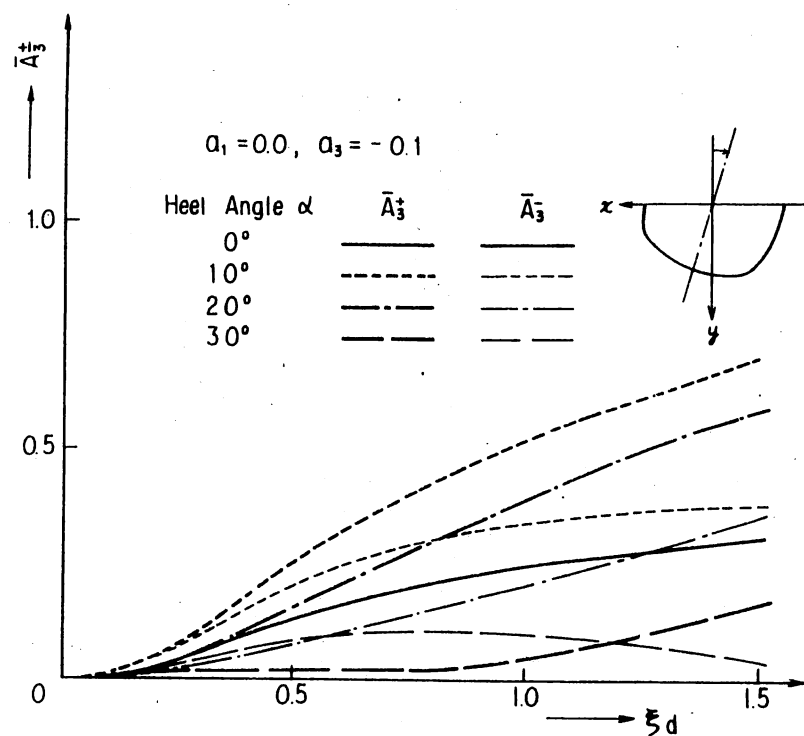


Fig.8 Calculated \bar{A} for Model B in the heeled conditions

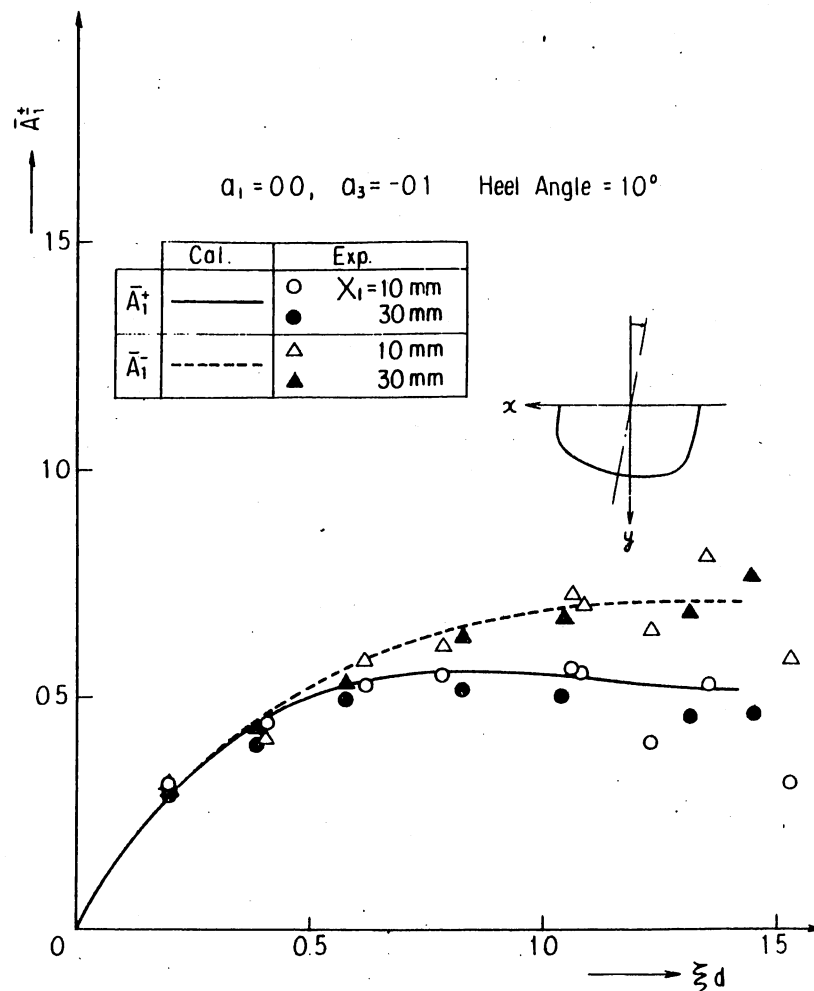


Fig.9 Comparison between calculated and measured \bar{A}_1 for Model B in the heeled condition ($\alpha = 10^\circ$)

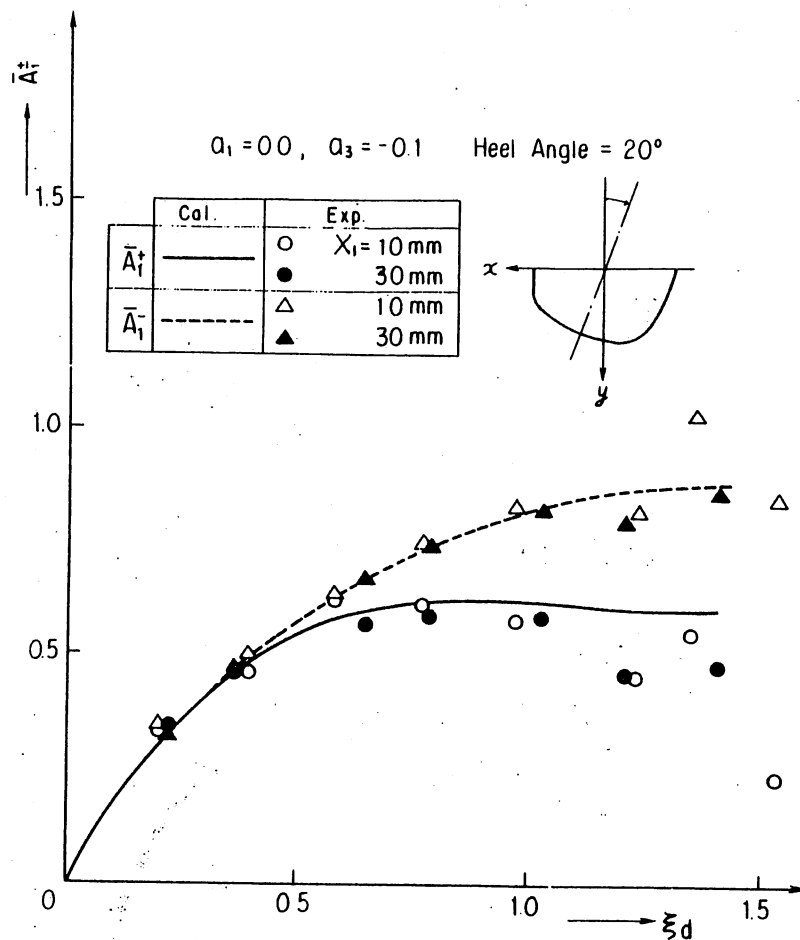


Fig.10 Comparison between calculated and measured \bar{A}_1^+ for Model B in the heeled condition ($\alpha = 20^\circ$)

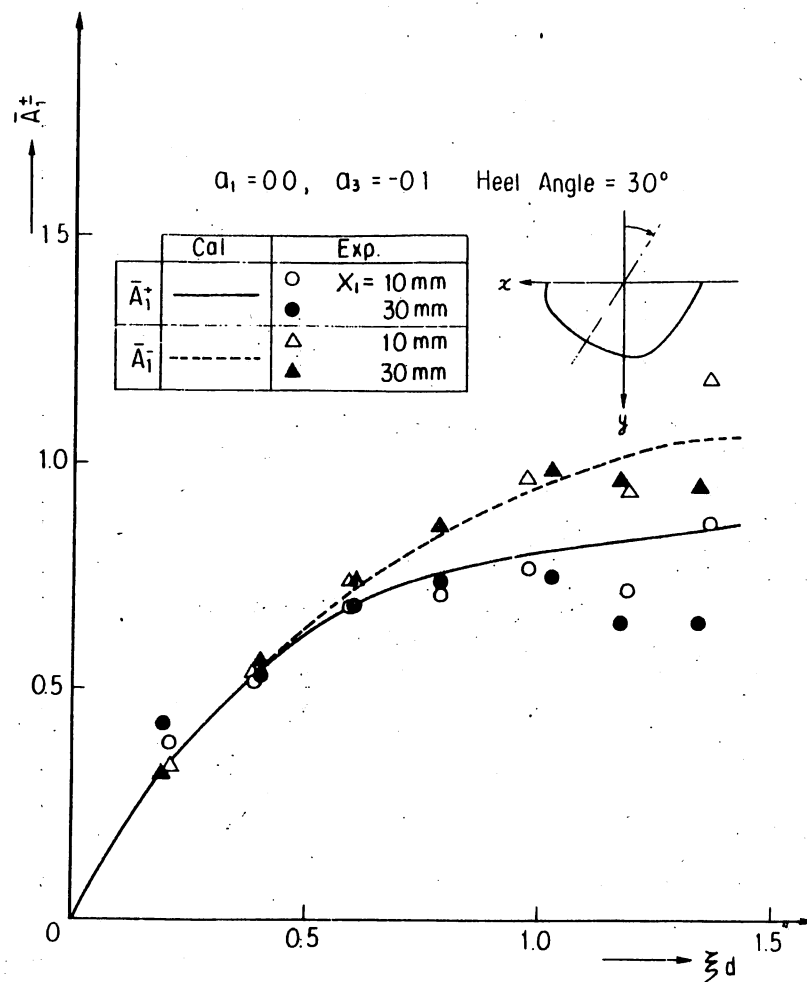


Fig.11 Comparison between calculated and measured \bar{A}_1^+ for Model B in the heeled condition ($\alpha = 30^\circ$)

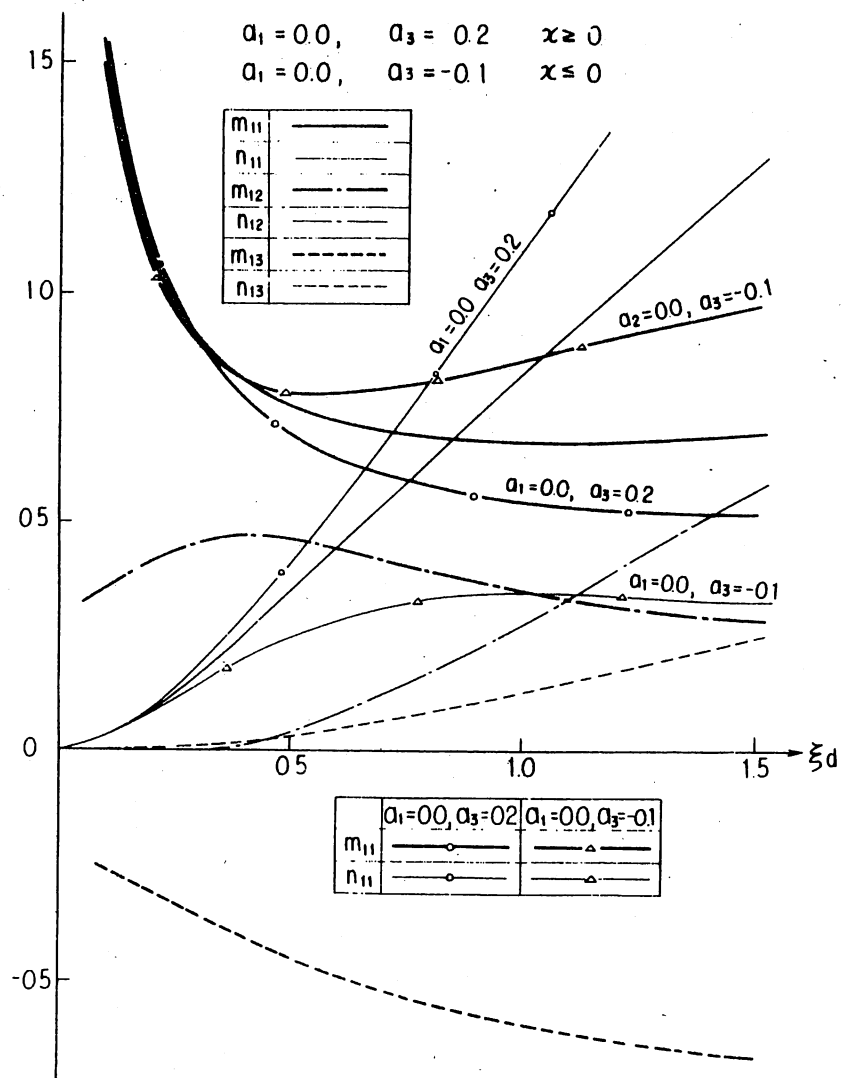


Fig.12 Radiation forces and moments due to heaving motion for Model A

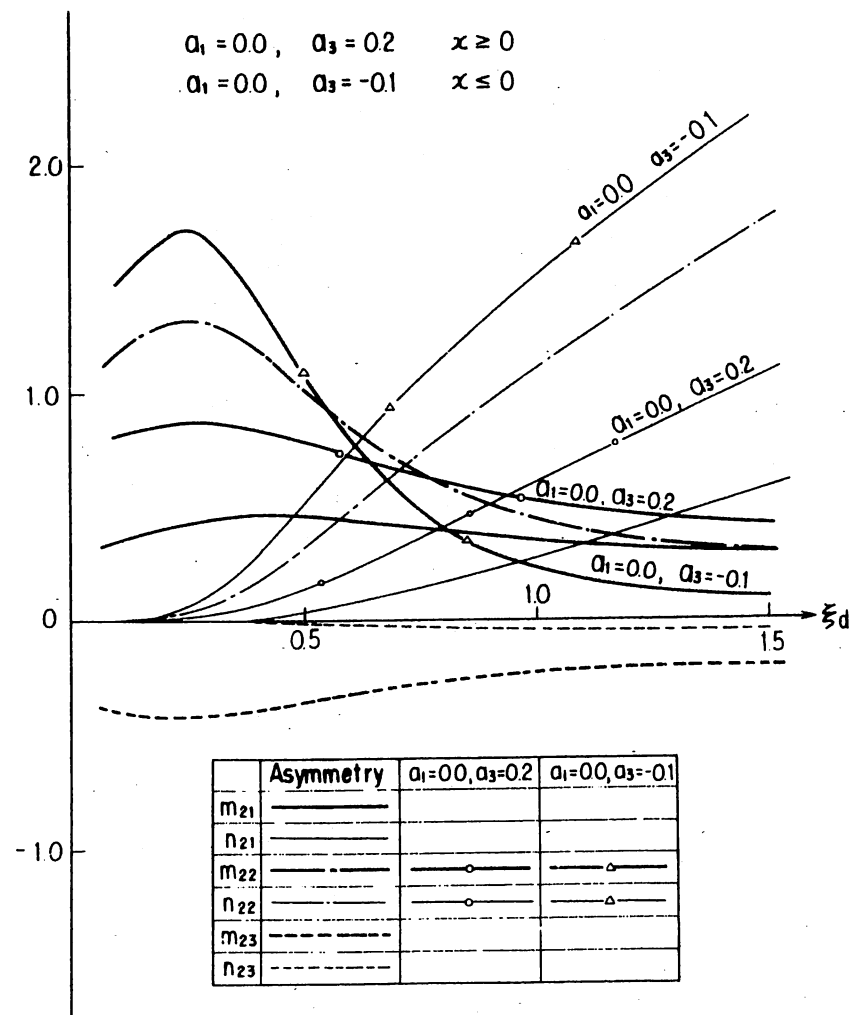


Fig.13 Radiation forces and moments due to swaying motion for Model A

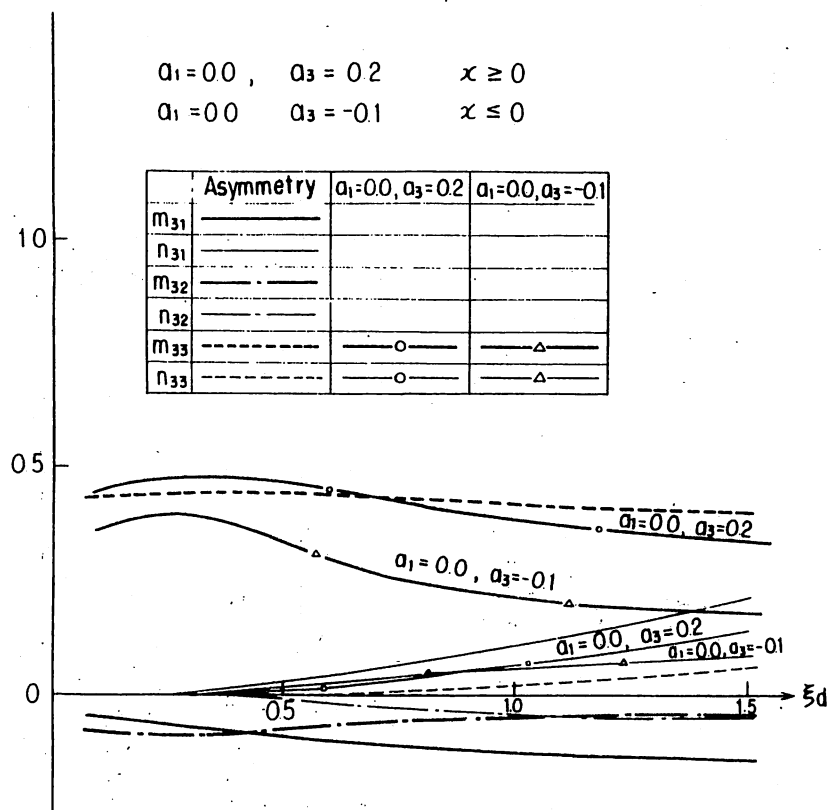


Fig.14 Radiation forces and moments due to rolling motion for Model A

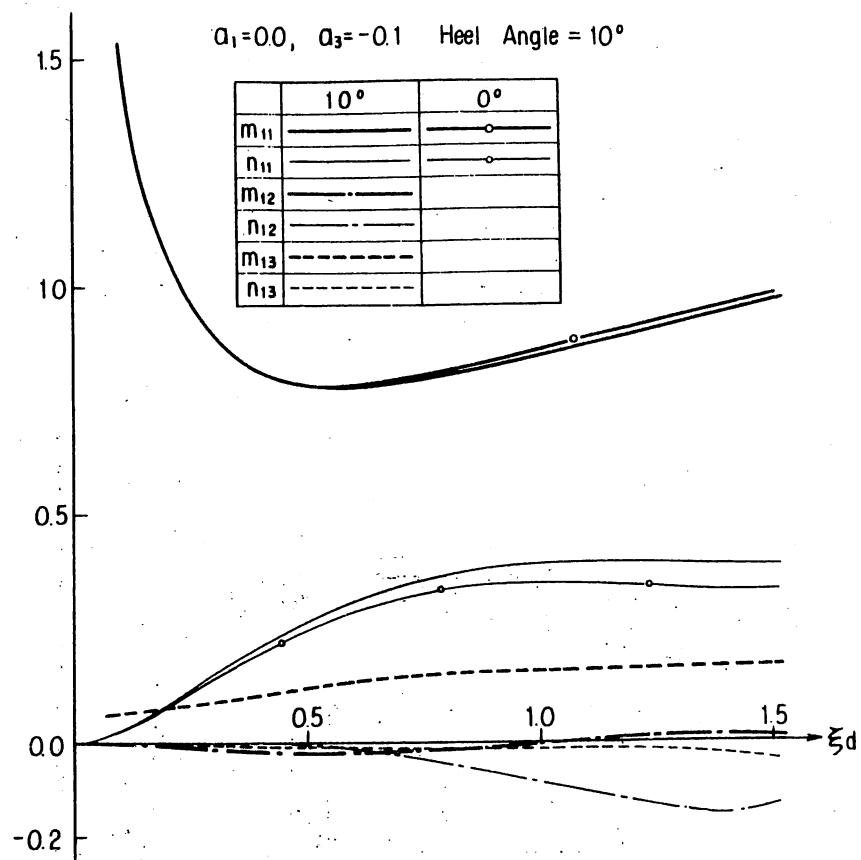


Fig.15 Radiation forces and moments due to heaving motion for Model B ($\alpha = 10^\circ$)

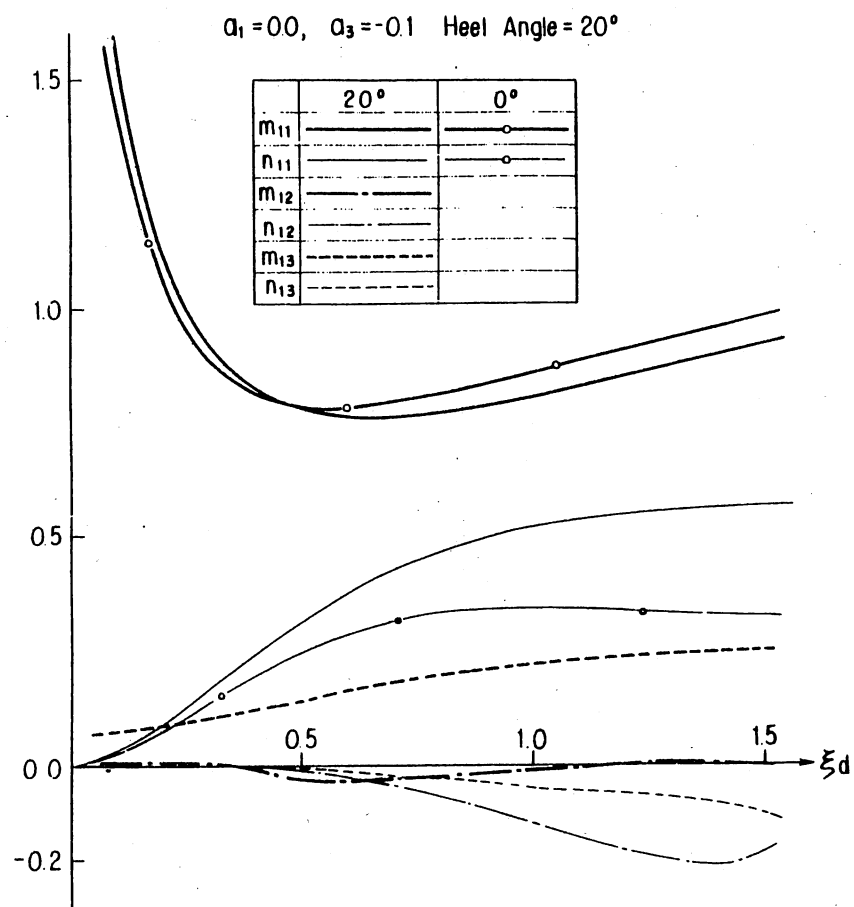


Fig.16 Radiation forces and moments due to heaving motion for Model B ($\alpha = 20^\circ$)

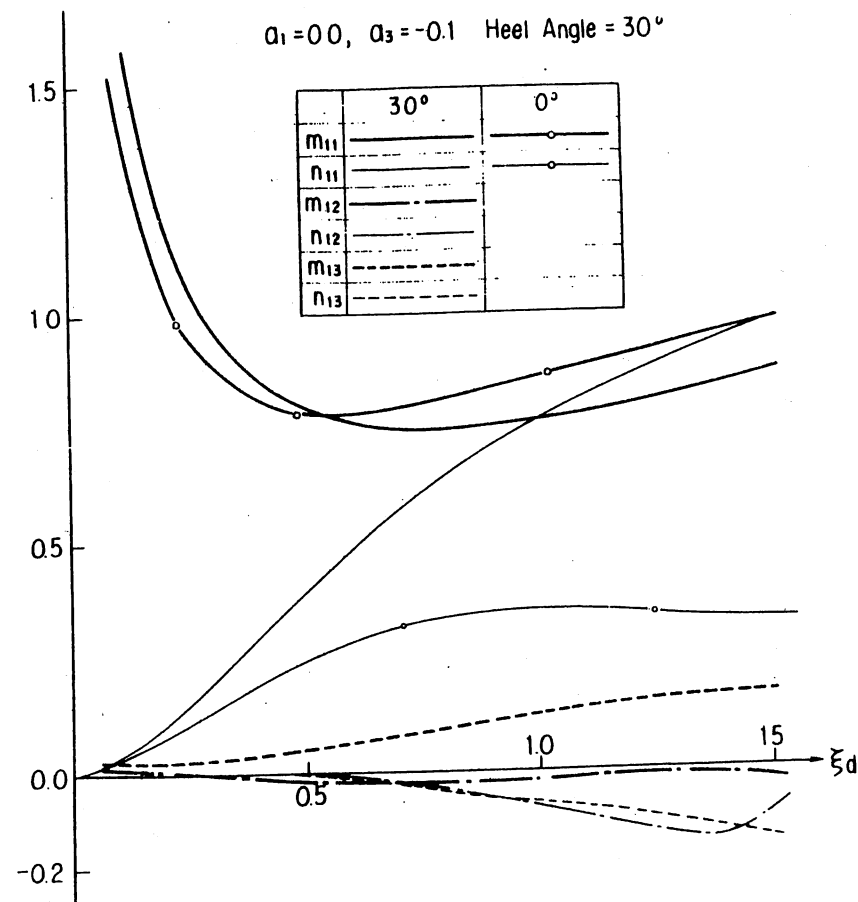


Fig.17 Radiation forces and moments due to heaving motion for Model B ($\alpha = 30^\circ$)

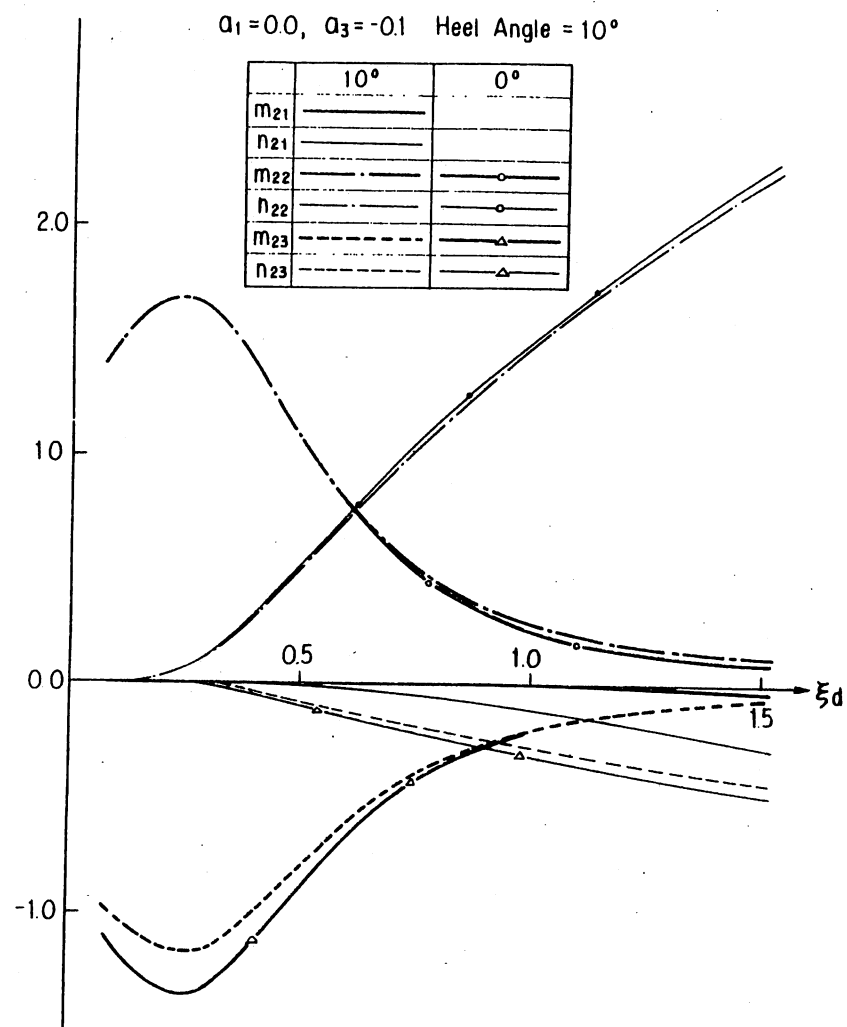


Fig.18 Radiation forces and moments due to swaying motion for Model B ($\alpha = 10^\circ$)

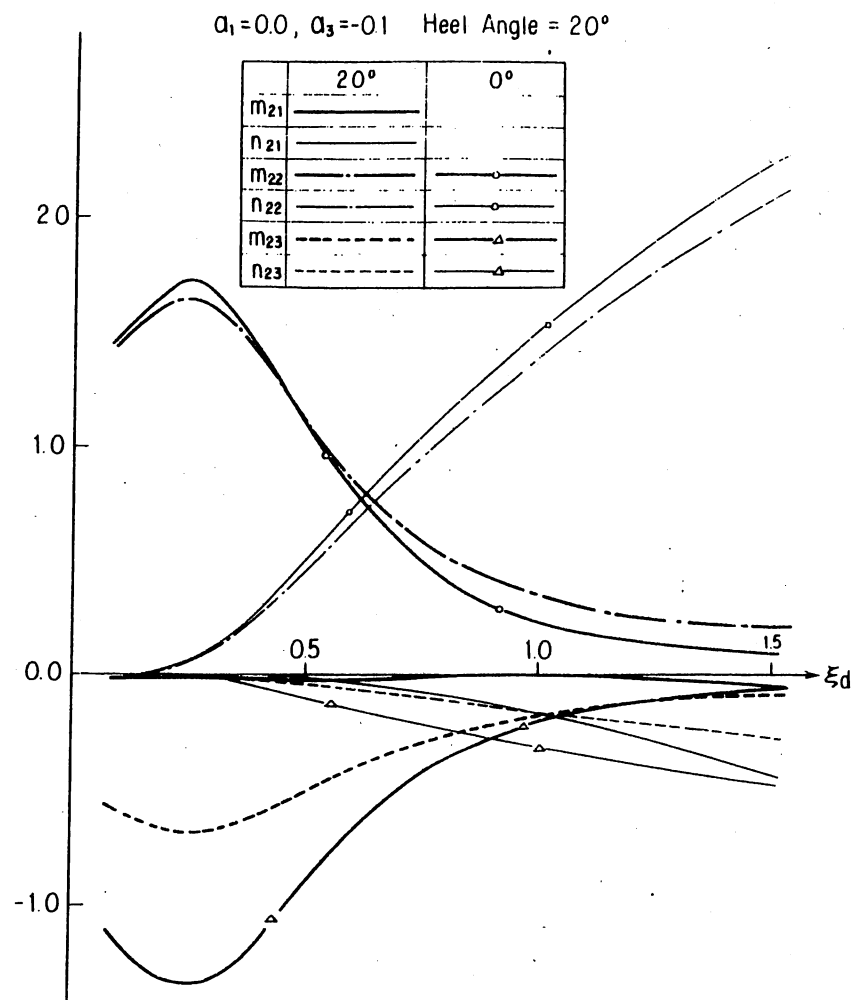


Fig.19 Radiation forces and moments due to swaying motion for Model B ($\alpha = 20^\circ$)

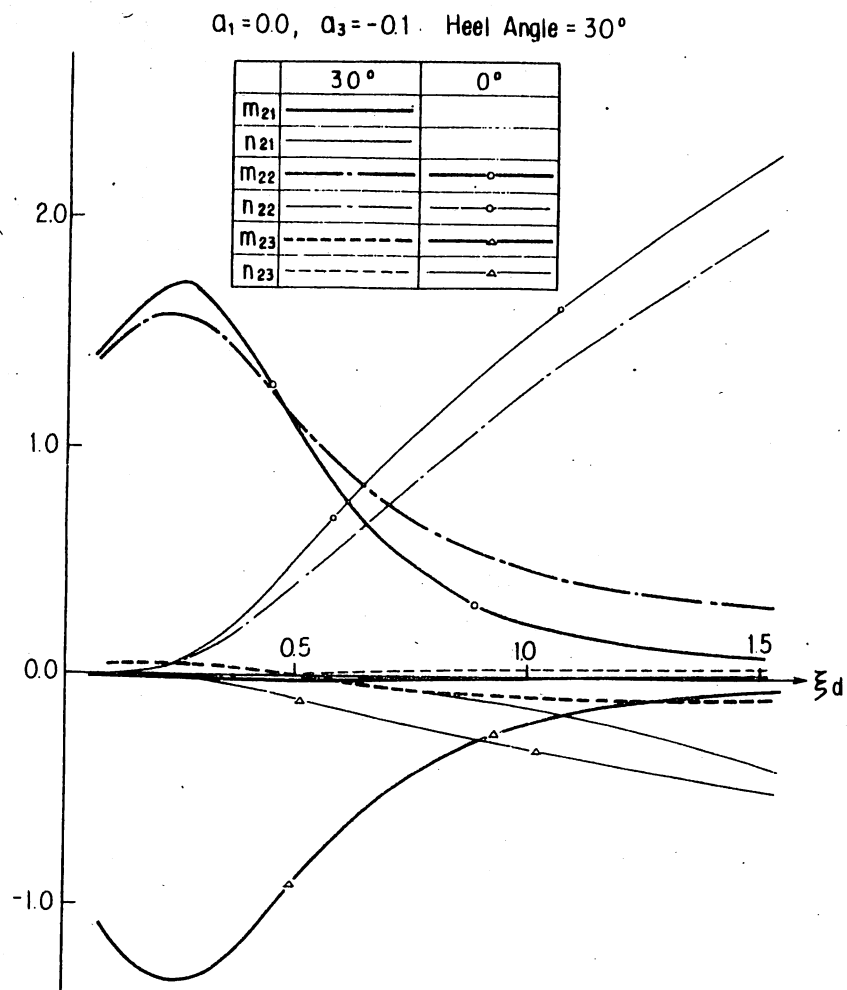


Fig.20 Radiation forces and moments due to swaying motion for Model B ($\alpha = 30^\circ$)

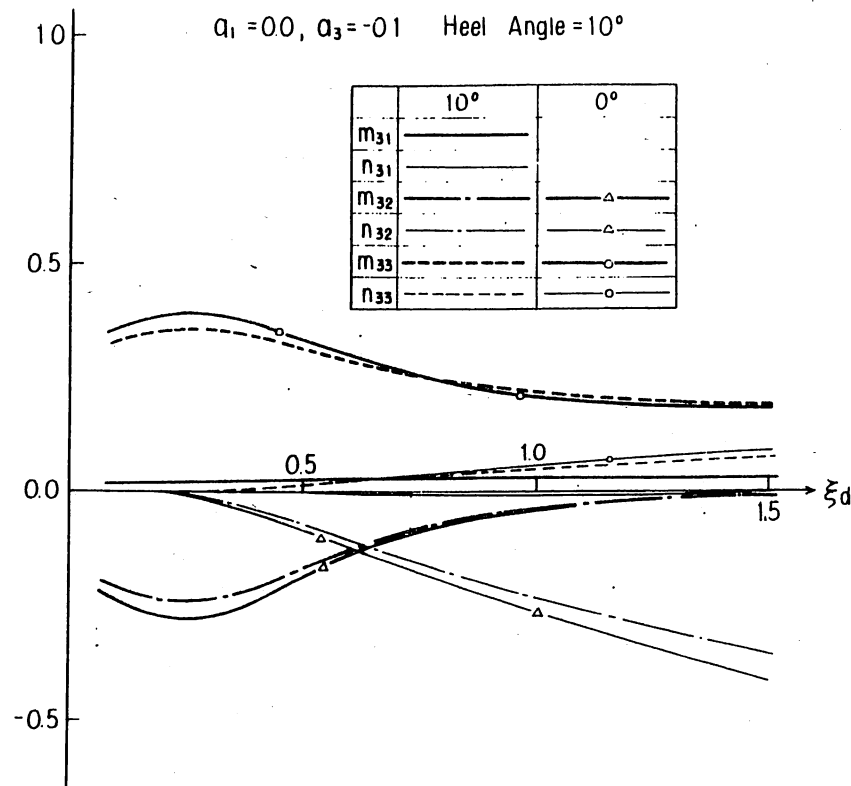


Fig.21 Radiation forces and moments due to rolling motion for Model B ($\alpha = 10^\circ$)

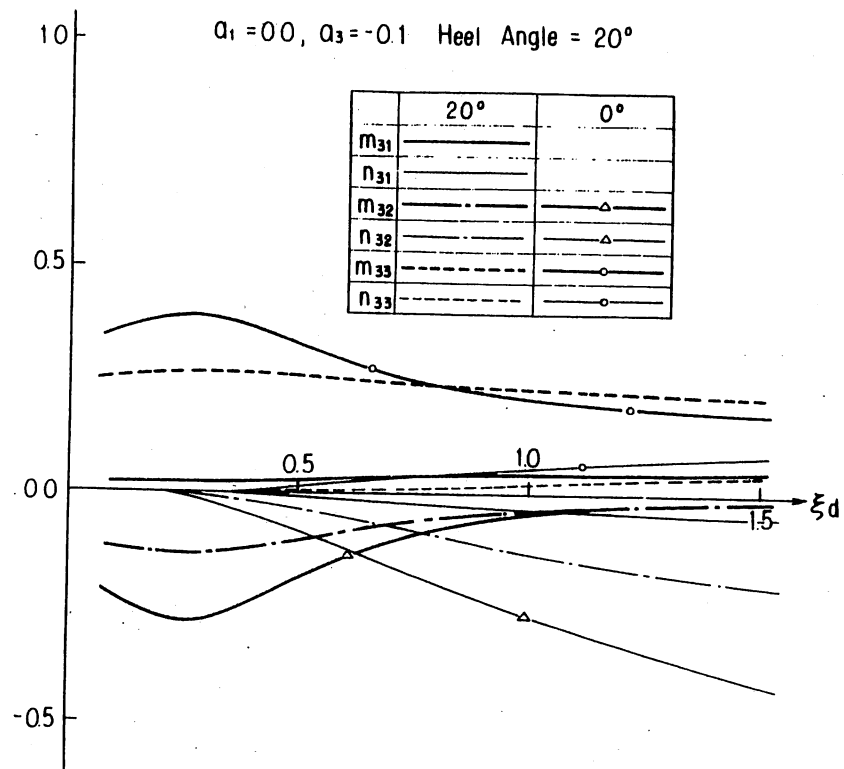


Fig.22 Radiation forces and moments due to rolling motion for Model B ($\alpha = 20^\circ$)

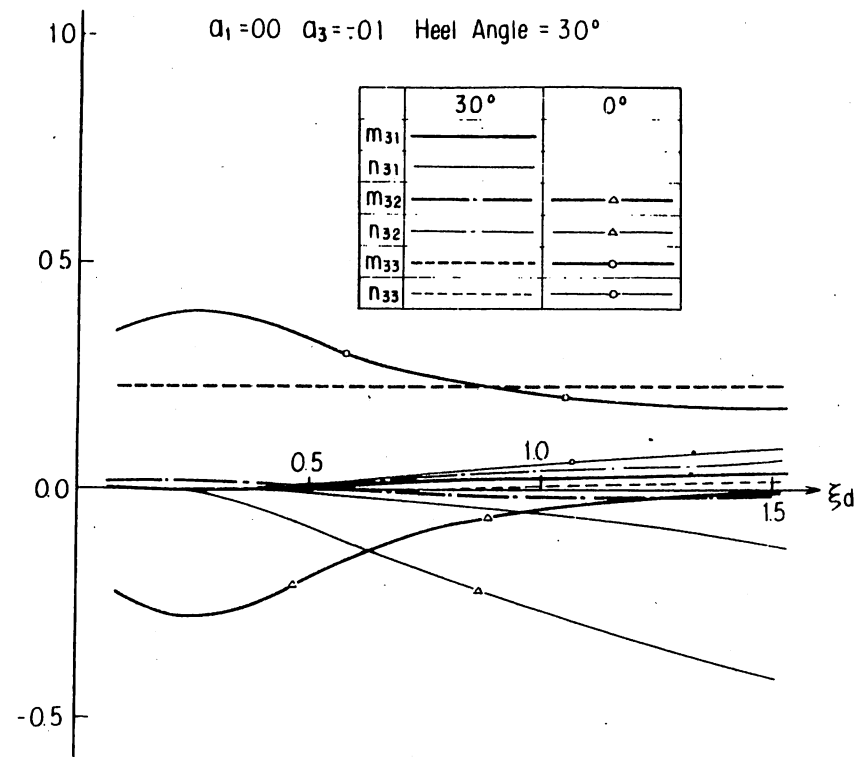
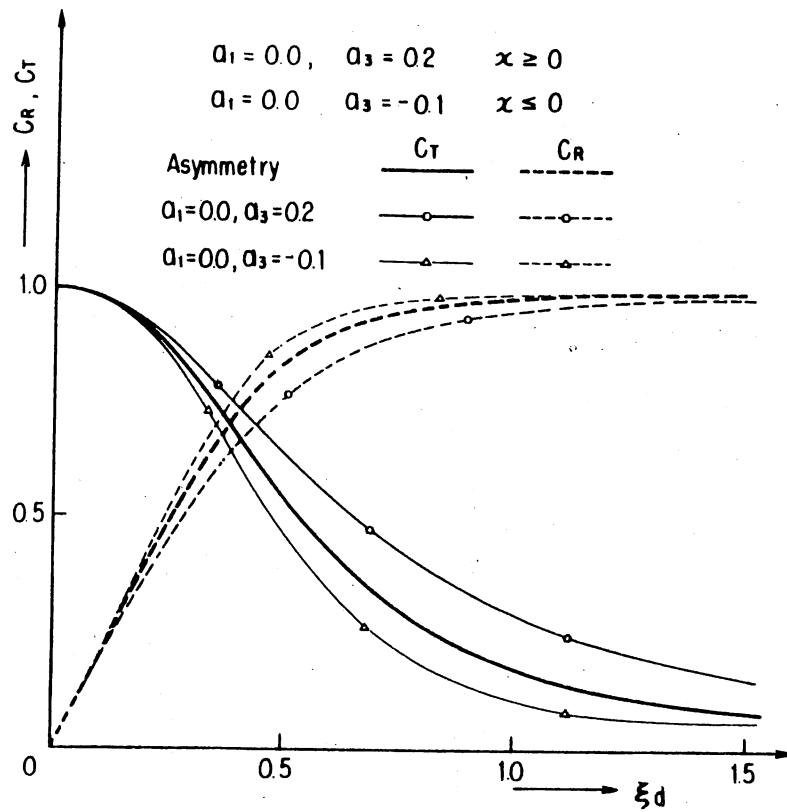
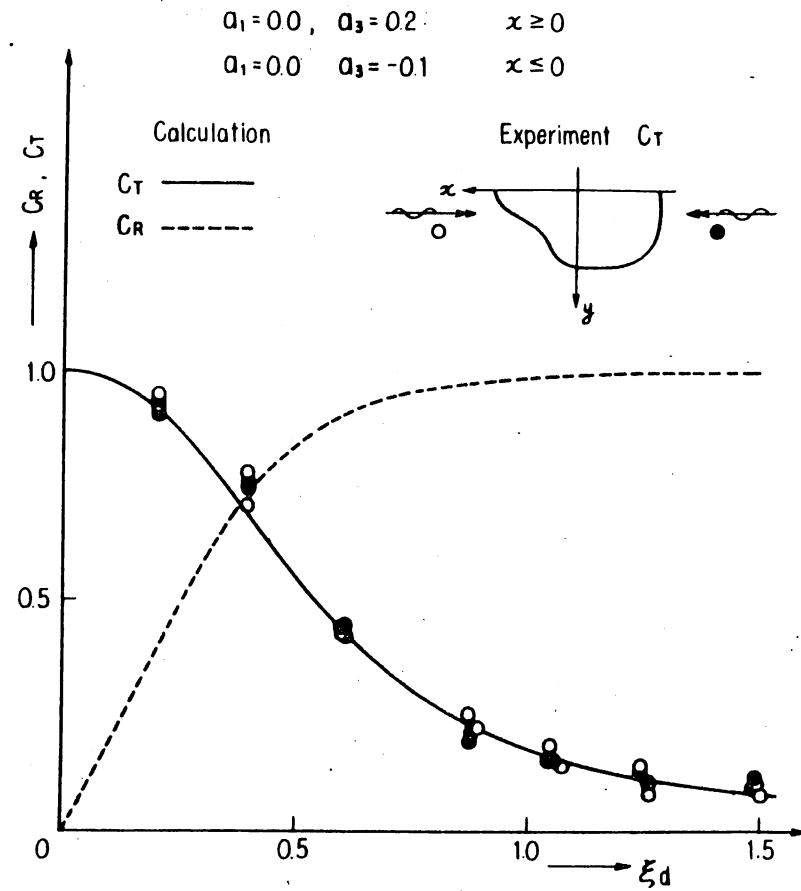


Fig.23 Radiation forces and moments due to rolling motion for Model B ($\alpha = 30^\circ$)

Fig.24 Calculated C_R and C_T for Model AFig.25 Comparison between calculated and measured C_T for Model A

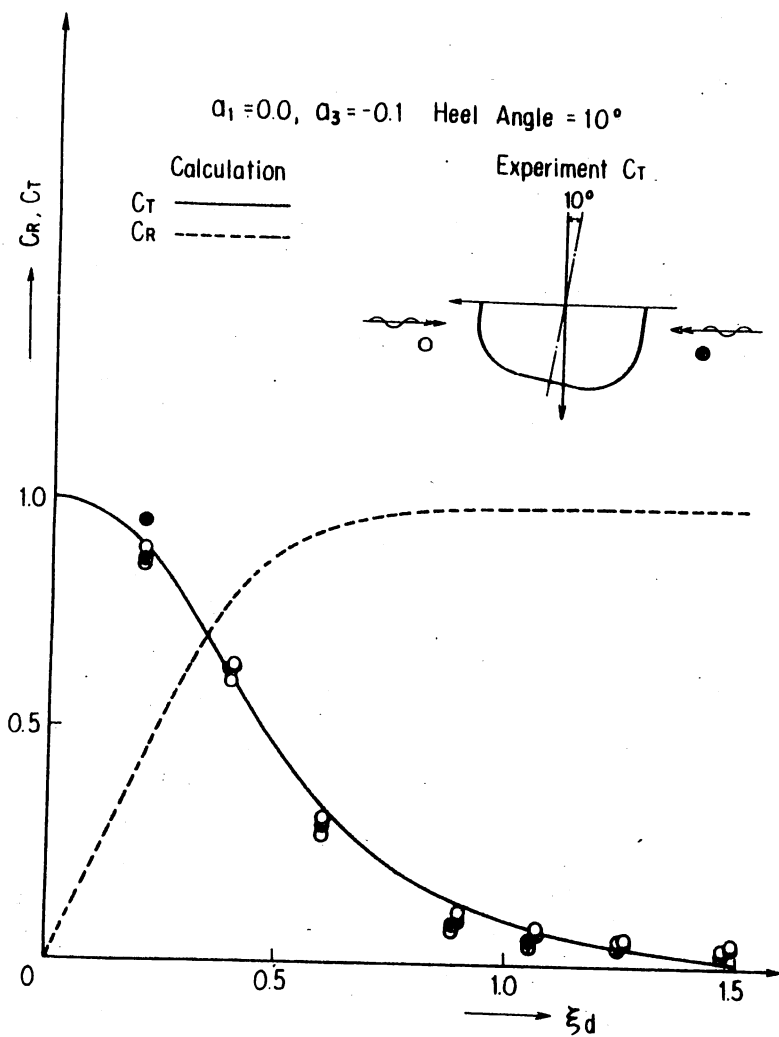


Fig.26 Comparison between calculated and measured C_T for Model B ($\alpha = 10^\circ$)

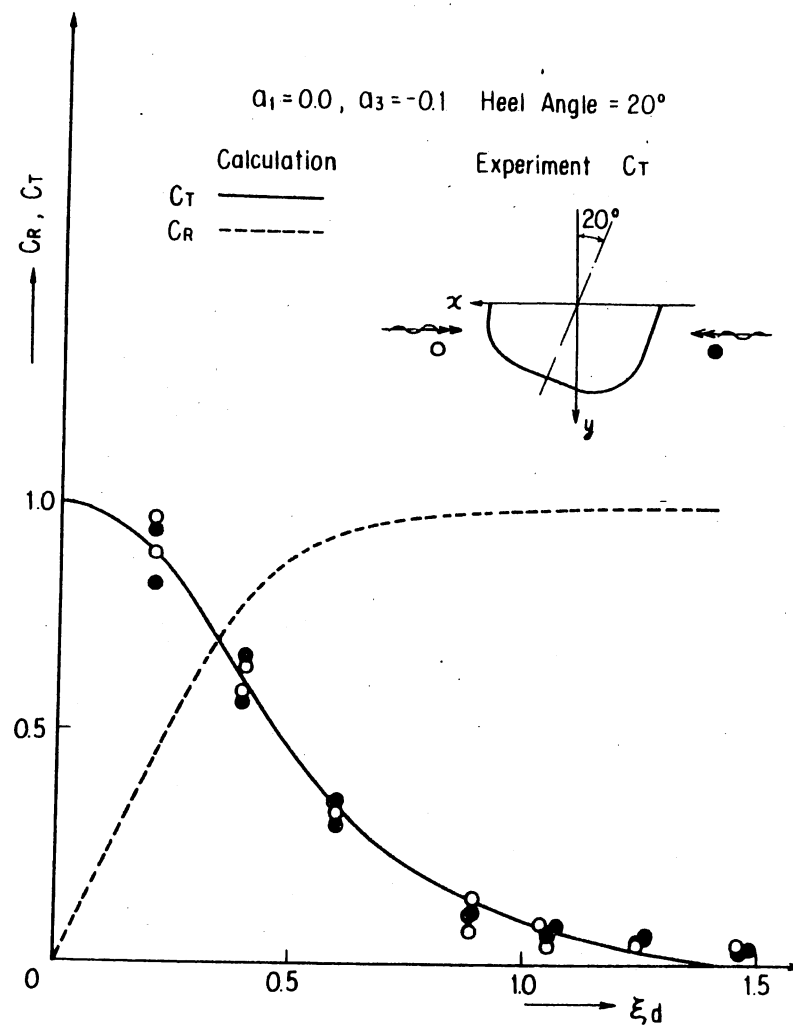


Fig.27 Comparison between calculated and measured C_T for Model B ($\alpha = 20^\circ$)

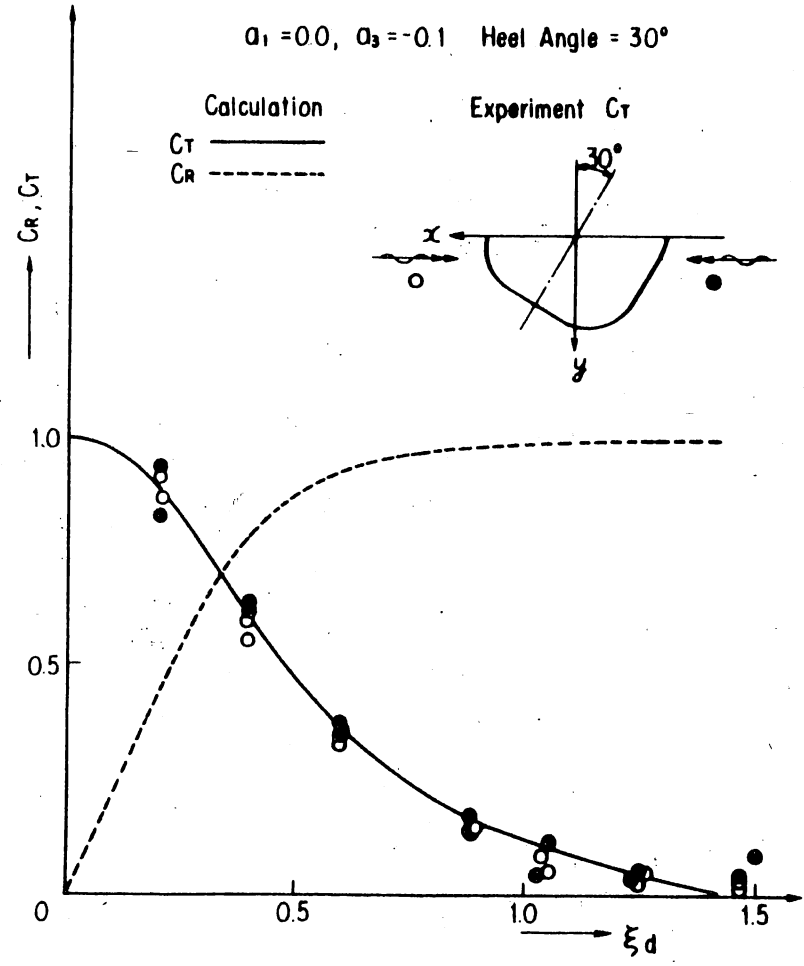


Fig.28 Comparison between calculated and measured C_T for Model B ($\alpha = 30^\circ$)

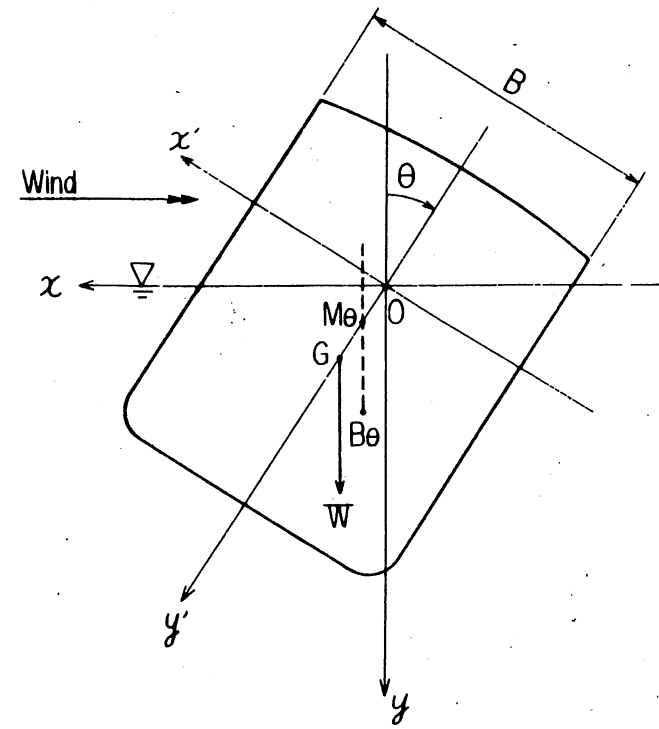


Fig.29 Coordinate system in the condition inclining θ degrees

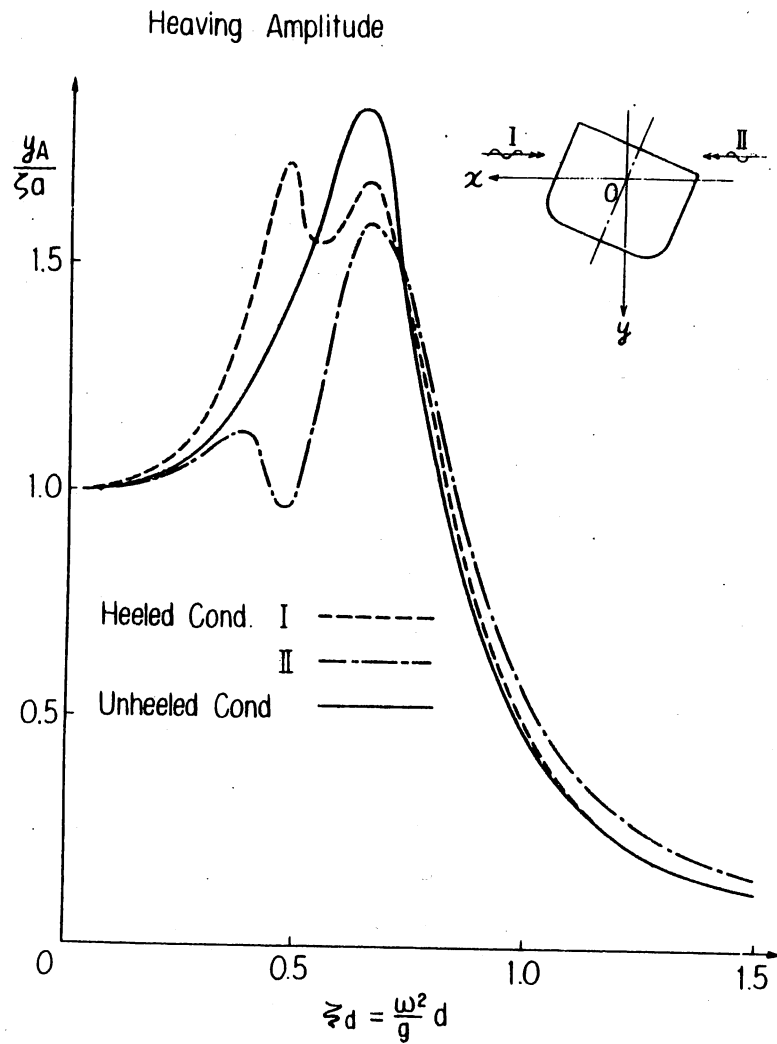


Fig.30 Heaving amplitude for the heeled condition ($\theta = 10^\circ$)

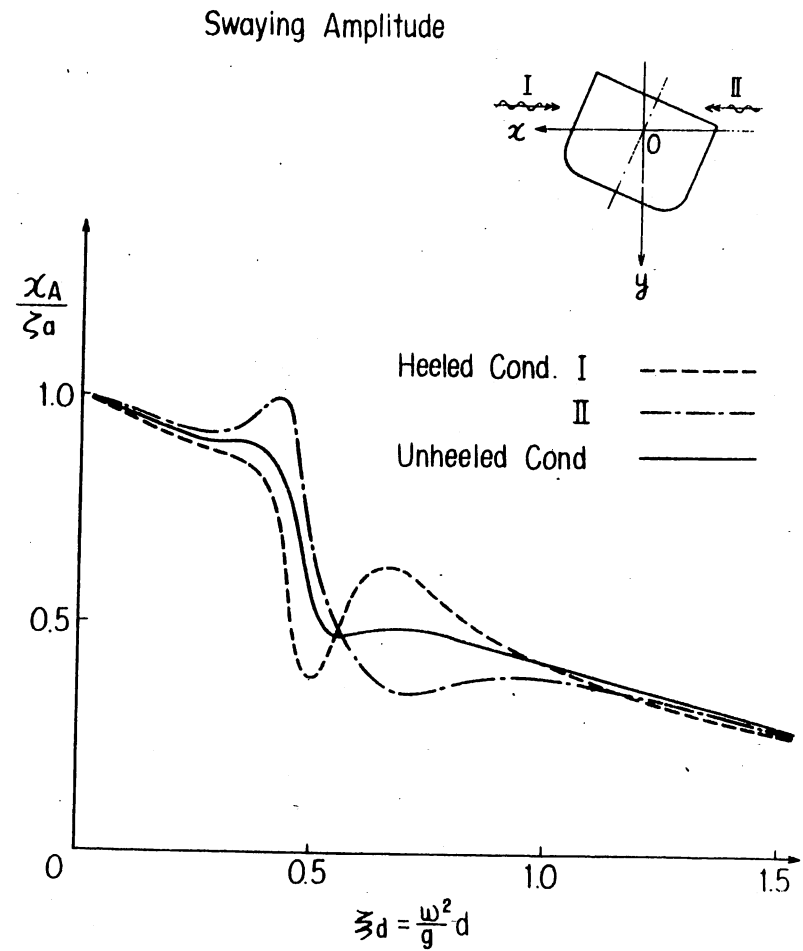


Fig.31 Swaying amplitude for the heeled condition ($\theta = 10^\circ$)

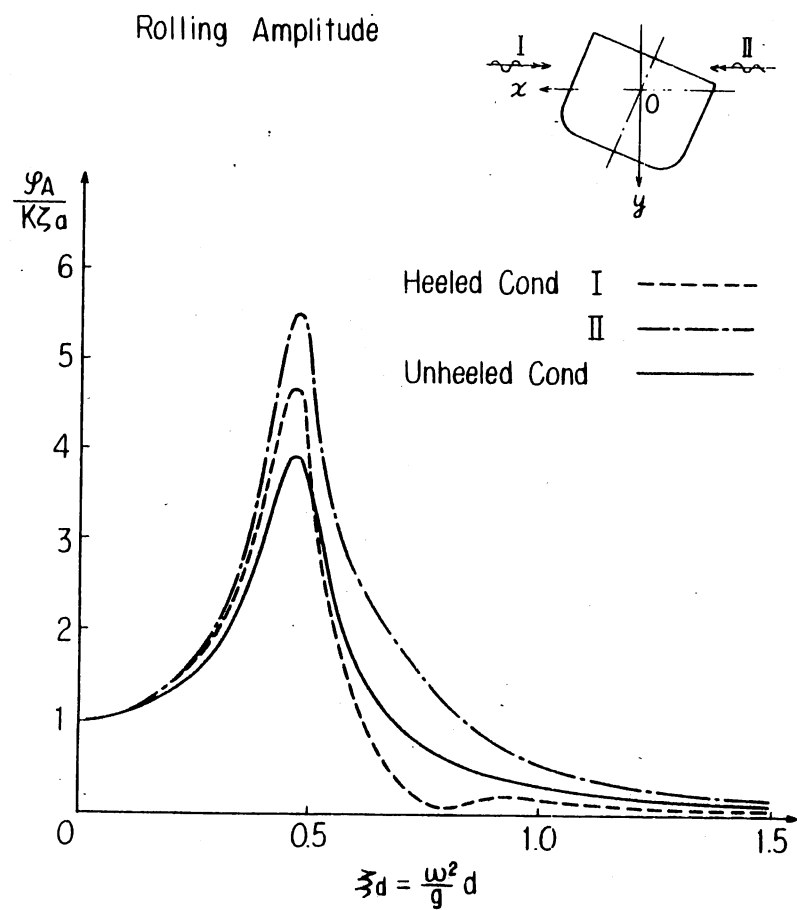


Fig.32 Rolling amplitude for the heeled condition ($\theta = 10^\circ$)

SIMULATION OF CAPSIZING IN BEAM SEAS OF A SIDE TRAWLER

by

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SUMMARY

An experimental and analytical investigation of capsizing in beam seas has been undertaken for a side trawler. Model experiments were conducted at the Ship Division, National Physical Laboratory in irregular waves corresponding to a sea state of Beaufort number 6. The results of these experiments give an indication of the conditions in which capsize in beam seas occur. Moreover, the GM values at capsize were found to be lower than those recommended by IMCO for fishing vessels.

A time-domain analysis using an analogue simulator program has been used to further the study of capsize in a seaway. This program uses a nonlinear equation of roll, with sway into roll cross-coupling, and a heave equation to describe the vessels motion in irregular seas. The results of this study compare favourably with those of the experiments and both have indicated that a number of factors contribute to capsize in beam seas.

1. INTRODUCTION

Stability is one of the most important aspects of ship safety. Small ships, and fishing vessels in particular, are often subjected to an extremely hostile environment in relation to their size and this can result in a capsize and loss of crew. It is therefore essential to design vessels with adequate stability to ensure survival against capsize in both statical and dynamical conditions. This degree of safety should be maintained in all

conditions of loading during outward and return voyages and fishing operations.

The existing IMCO recommendations on Intact Stability of Fishing Vessels Ref. (1) are based on the analysis of the trawler losses and expressed as features of the righting lever curve of statical stability (GZ curve), together with a minimum metacentric height. (GM) Although a number of external forces such as beam wind and water on the deck, are recognised as having an adverse effect on stability, only ice accretion is taken into account in the case of fishing vessels. If these external influences are considered to be of sufficient importance in relation to ship safety than it is no longer sufficient to rely on the properties of the righting lever curve, but more on the dynamics of the ship in its environment.

One of the important external forces that a vessel is subjected to is that resulting in rolling due to beam seas. This is of some importance since a fishing vessel could capsize in this situation when the vessel rolls into a wave and water is scooped onto the deck. This problem concerns the vast majority of British distant-water trawlers which are of the side trawler type. In severe weather it is normal practice on most vessels to head directly into waves at low speed, but trawlers do not do this because they wish to remain over the same spot if fish are plentiful. Instead they perform a manoeuvre called "dodging and laying" in which they first head into the waves and then turn broadside to them so as to drift to leeward. The turn and the "laying" broadside

are both dangerous from the risk of capsizing.

Of the motions of a ship in a seaway, pitch and heave can now be calculated fairly readily by means of strip theory and yaw and sway can be treated by the same means. These motions are, to a good approximation, linear in wave height and the principle of superposition is used to calculate their statistics in irregular seas. The agreement between theory and experiment is good enough for most purposes although confirmation of computer predictions by full scale trials is not quite so well documented. Roll motion is a different proposition. It is a sharply-turned resonance phenomenon, and the non-linear damping and restoring moment terms in its governing equation make it intractable to analysis by linear theory.

The designer will need to know how to measure stability in a dynamic situation and which particular parameters affect its liability to capsize in the various conditions of loading in service, and in actual sea conditions encountered. This report describes model experiments and a computer program designed to seek tentative answers to the above problem for the case of a fishing vessel in beam seas. The mathematical background is given for the computer program, and a statement of requirements of numerical parameters to specify the ship in the irregular beam seas is given in Appendix I.

2. MODEL EXPERIMENTS

An investigation to help define the conditions of loading which would ensure survival of side trawlers was carried out for the Board of Trade in 1970 by Ship Division of the National Physical Laboratory. The model used in the experiments was an 1/18 scale model of a side trawler with the following full size particulars.

Single-screw side trawler:-

length pp	54.86 m
beam	9.75 m
depth to main deck	5.08 m
trim by stern	1.52 m
draught, mean	4.59 m
displacement, mean	1400 tonnes
GM minimum	0.12 m
GM maximum	0.82 m

The model was of wooden construction, but the interior was filled with plastic foam so that no water would penetrate in the event of a leak developing. Permanent ballast was provided internally under the plastic foam, and variation of the vertical centre of gravity was possible through the movement of a relatively large weight on a vertical lead screw, see Figure 1. The model was provided with a removable deckhouse and superstructure, but in these preliminary experiments no bulwarks were fitted.

The experiments were conducted in No. 4a tank, Ship Division, NPL. The model was ballasted to a certain displacement and positioned near one side of the tank, being held broadside to the waves by two light lines attached to its stern and held by the experimenter. The model tended to drift, with the waves, towards the beach at the end of the tank, and the experimenter moved so that the model remained parallel to the wave crests.

The waves were made by a plunger-type wavemaker, of fixed stroke but of varying frequency; the frequency varied from one cycle to the next according to a preset program. Because of the varying characteristics of the wavemaker at different frequencies, the spectrum of waves produced approximated to an irregular sea, although there was relatively much less variation in wave height than would have been found at sea. The wave-height corresponded to that of a sea of severity Beaufort number 6, with significant wave height 3.9 m. (This is defined as the mean of the one-third highest waves, measured crest-to-trough).

The position of the vertical weight was altered for successive runs and the survivability of the model against capsizing in beam seas was noted for the different metacentric heights thereby produced. If the model was unsafe it usually capsized rather quickly (after about five waves, say): if the model appeared to be safe, it was maintained broadside to the waves while it drifted down some 12 m to the beach. Near the beach the wave steepnesses tended to increase, and survival in these conditions was considered to be indicative of adequate stability. Intermediate between these states were runs in which the model survived capsizing provided it was not touched, but where it would capsize if heeled by hand either into or away from the waves. These runs represented the likely effect of a sudden gust of wind upon the vessel.

The three possible survival states were then plotted with different symbols on the graphs of Figure 4.

The metacentric heights, GM that give specific areas under the GZ curves up to 30 and 40 degrees are also indicated in Figure 4 as e_{30} and e_{40} respectively. The values of these areas have been taken as 0.055 metre-radians for the area up to 30° and 0.09 metre-radians for the area up to 40° and both are in accordance with the current IMCO stability criteria recommended for fishing vessels, Ref. (1). See Appendix II. These values have been calculated from the Division's static stabilizing computer program.

3. DISCUSSION OF MODEL EXPERIMENT RESULTS

The results of these experiments indicated that, as displacement increases and therefore freeboard decreases, an increasing metacentric height is needed in order that the vessel should be safe from capsize in waves equivalent in height to those found in a sea of state Beaufort force 6. This increasing metacentric height can be thought of as providing a larger restoring moment (or sufficient potential energy) to prevent capsize in beam seas.

The scatter of the spots on Figure 2 points to the lack of reproducibility of results. This is partly because of the difficulty of decision on how long a model should survive before it is considered safe. The program for the wavemaker in No. 4a tank lasted 15-20 seconds and then repeated; usually a run lasted about 1½ minutes or 5 complete repeating wave cycles. It is appreciated that the limited sea state in which these tests were conducted will not give adequate definition of the limits of safety for trawlers in extreme conditions, but these results have given an indication of the conditions in which capsize in beam seas occurs. A motion cine film was taken of a number of these experiments.

As already stated it was difficult to decide when a model should be considered safe. If it capsizes at all in the waves it should be considered unsafe. A real ship must survive storm conditions without capsize for perhaps 48 hours at one time, and this period, even at the reduced scale of the model considered, is equivalent of eleven hours of experiment time for just one run. This may be one reason why the GM values

from these experiments are much lower than the current IMCO regulations - the model just did not have to survive for very long in order to be called safe.

The wave-making facilities in most model experiment tanks are inherently unsuitable for long duration experiments because they are based upon repetition of a pseudo-random series or cycles. This is convenient for experiments in which root mean square motions are being measured because the wave conditions should repeat from one run to the next, but extreme waves will not be generated at all, and it is these that will capsize the ship, if any will. At sea, the exceptional wave is composed of several smaller ones of different wavelengths which come together at one point in space and time.

4. EQUATIONS OF MOTION

We consider a ship poised on a train of regular waves moving from port to starboard, and to a sufficient approximation, sinusoidal in form (Figure 2). The ship will be subjected to roll motion because of the variation of the effective slope of the waves with time, and to sway motion because of this slope and the orbital velocities of the water particles. The equation of motion used within the analogue simulation program are as follows:-

Sway motion

The equation of sway motion may be written

$$2 \Delta (\ddot{y} - \dot{v}) + F = 0 \quad (1)$$

where

Δ = ship mass

y = athwartships movement (+ ve to starboard) of ship CG w.r.t. axes fixed in space

v = mean horizontal water particle velocity, taken as equal to that of half-draught

F = resistance to sway relative velocity, see below

ϕ = roll angle, + ve to starboard

$$F = C_D \frac{1}{2} \rho L D (\dot{y} - v) |\dot{y} - v| \text{ at zero speed} \quad (2a)$$

$$F = -Y'_v \frac{1}{2} \rho L^2 U (\dot{y} - v) \text{ at forward speed } U \quad (2b)$$

where

C_D = a drag coefficient

ρ = density of sea water, 1.025 tonnes/ m^3

L = ship length

D = draught (this symbol is used here to avoid confusion with roll period)

Y'_v = sway damping derivative, negative as normally defined (non-dimensional)

It is assumed that the added mass in sway is equal to the ship mass.

Roll motion

$$I^* \ddot{\phi} + k_1 \dot{\phi} + k_2 \phi |\phi| + \Delta GMg (\phi + a_3 \phi^3 + a_5 \phi^5 + a_7 \phi^7) = \Delta GMg \alpha + mg\eta + (F + \Delta (\ddot{y} - \dot{v})) D/3 \quad (3)$$

where

I^* = inertia in roll plus added inertia

$$= \Delta GMg T^2 / 4\pi^2$$

T = roll period

k_1 = linear damping coefficient

$$= C_1 4\pi I^* / T$$

C_1 = critical damping ratio, non-dimensional

k_2 = quartic damping factor = $C_2 I^*$

C_2 = quartic damping ratio, non-dimensional

GM = initial transverse metacentric height

g = acceleration due to gravity, 9.81 m/sec²

a_3, a_5, a_7

= non-dimensional coefficients giving GZ/GM₀ curve up to say $\frac{\pi}{2}$

m = mass of water on deck (initially zero)

η = CG of water on deck

It has been assumed that the force due to sway velocity and the added inertia force due to sway acceleration act D/3 below the ship CG. This assumption can be modified if experimental information is available. Note that there is no movement caused by the inertia force acting on the ship's own mass because of its sway acceleration: the Δ in the last term is the added mass.

Heave motion

The equation of heave motion may be written as given below, where for the purpose of this simulation, the coupled pitch term has been neglected for simplicity. This is considered to have a very small effect on capsize in beam seas. The ship is considered stationary in the forward direction i.e. speed of advance $U = 0$. The heave equation is as follows:

$$a' \ddot{z} + b \dot{z} + cz = z_w \quad (4)$$

where

a' = ship mass + added mass in wave

$$= \Delta + \int A' dx$$

z = heave, + ve upwards

b = vertical damping force coefficient

$$= \int N' dx$$

where

$$N' = \rho g^2 A^2 / \omega^3$$

and

A = ratio of generated wave to heave amplitude for vertical motion induced wave

c = restoring force coefficient

$$= \rho g \int B^* dx$$

B^* = local waterline beam

z_w = the wave excitation

$$= \left[\rho g B^* dx \cdot \zeta + \int N^* dx \cdot \dot{\zeta} + \int A^* dx \cdot \zeta \exp \left(-\left(\frac{\omega^2}{g} h \right) \right) \right] \quad (5)$$

where

ζ = wave height

and

h = is approximated by local sectional draught x local sectional area coefficient

The integral in the above equations is taken over the length of ship. It is noted that the velocity and acceleration terms of the wave height, ζ , are in fact those given by equations (7) and (8) for the mean particle velocities and accelerations.

Wave Motion

For a sinusoidal wave of constant frequency ω the amplitude ζ at a depth $D/2$ is related to the surface amplitude h by

$$\zeta = h \exp \left(-\omega^2 D/2g \right) \cos \left(\omega^2 y/g - \omega t \right) \quad (6)$$

The mean particle velocity (either vertically or horizontally) is given by

$$v = \frac{d\zeta}{dt} = h \exp \left(-\omega^2 D/2g \right) \sin \left(\omega^2 y/g - \omega t \right) \quad (7)$$

and the mean particle acceleration is given by

$$\dot{v} = \omega^2 h \exp \left(-\omega^2 D/2g \right) \cos \left(\omega^2 y/g - t \right) = g a \quad (8)$$

where

a = waveslope at $D/2 = d\zeta/dy$

For the purpose of this simulation, the basic frequency of each of the seven sine waves is slowly increased until it reaches the next higher frequency. This slow increase lasts for say 10 minutes, the period of a statistical sample or simulation run. The basic frequencies are all increased by the same proportional amount, as they are in geometric progression. Since ω is now not constant but given by $\omega = \omega_0 + \omega t$, say, the expressions (7) and (8) for v and \dot{v} will no longer be correct and are modified accordingly.

5. WAVE SPECTRUM

So far the simulation has been in terms of a single sinusoidal wave train only. Although the roll motions of the ship in two superimposed wave trains cannot be added together, the forcing functions due to the wave trains, namely effective wave slope and mean particle orbital velocity, can be superposed. The equations can then be integrated provided that their right hand sides are updated to allow for sway motion y and relative sway velocity $(\dot{y}-v)$.

In the Analogue Simulation program there is just room in the block diagram to represent the seaway by seven sine waves of different frequencies. In order that the whole range of frequencies be covered, these seven central frequencies are 'blurred' so that the final spectrum contains energy from a continuous range of frequencies. The 'blurring' and central frequencies are chosen so that the upper limit of each wave frequency is just equal to the lower limit of the next. The sweep of frequencies should follow a sawtooth pattern in order to provide uniform coverage, but it is difficult to provide this, and it is approximated by a sinewave and third harmonic so that the error is negligible.

The central frequencies and blur ratio (± 0.114) have been chosen so as to cover the frequency range, 0.04 to 0.2 Hz (wave periods 5 to 25 seconds) and thus the program as it exists at present is not suitable for ships with roll period less than about 8 seconds. Moreover, energy

in the wave spectrum outside this frequency range is ignored, and so consequently the significant* wave height of the wave spectrum generated by the program will be less than that of the full spectrum being represented. The comparison of these wave heights is shown below.

<u>Beaufort number</u>	<u>Sign. wave ht., m</u>	
	<u>Actual</u>	<u>Simulated</u>
6	3.90	3.74
7	5.24	5.12
8	7.16	7.08

The amplitudes of each of the seven component waves are chosen so that each has the same energy as the bundle of waves in the continuous spectrum over the same frequency band. The ISSC one-parameter spectral formulation has been taken as a standard, and the semi-amplitudes used are found in Ref. (4).

A comparison of the actual and simulated spectrum for Beaufort 6 conditions appears as Figure 3.

6. SIMULATION OF CAPSIZE IN BEAM SEAS

Although a patch network for the above equations given in Section 4 could be constructed, the problem would require a very large analogue computer and there would be severe problems of scaling and stability. It is possible, however, for an all-digital computer to be programmed as if it were an analogue machine using a system called Continuous System Modeling Program. The user writes the program as if he were drawing out a patch diagram for an analogue computer, but the computer proceeds digitally, integrating the differential equations on a step-by-step basis by brute force. The output is very conveniently arranged to be graphical, just as it would be from an analogue machine, and any points the patch network can be examined. This particular problem has been programmed for the Ship Division Computer to simulate capsizing in beam seas of a side trawler.

The program described above was run for the same side trawler form as used in the model experiments. The

sea state corresponding to Beaufort 6 was used in the program as this was the sea state used in the model experiments. A portion of the print out in graphical form is shown in Figure 5 just as it appears from the analogue simulation program. The results have also been added to the model results given in Figure 4.

The program is designed for full size vessels and the parameters used within the program are therefore ship rather than model values. The roll period and the damping factors k_1 and k_2 have been determined by experiment from a similar ship to the one in question because the model scaling of roll damping is not accurate.² The heaving damping coefficients have been calculated from Ship Division's ship motions computer program which is based on the theory of Korvin-Kroukovsky and the added mass and damping coefficient from the work of Grim.

Arrangements are made within the program to calculate when the deck edge becomes awash. This is done by taking into account the roll angle, wave slope, heave and the freeboard of the vessel with respect of the wave height at every second interval. A diagram showing freeboard of the ship due to roll and heave motions in waves can be seen in Figure 2. When the deck edge does become awash water is assumed to be scooped on deck and the program is arranged to introduce an additional heeling moment due to an assumed amount of collected water. This mass of water is estimated from the prevailing freeboard or more precisely from the height of water above the deck edge when this condition is encountered.

The height of water or freeboard above or below the deck edge due to the motions of the vessel in a seaway are calculated from the following formulae. Now if wave height is given by ζ and the absolute heave is z , then relative heave ξ is given by

$$\xi = \zeta - z$$

*'Significant' motion = $4 \times (\text{area under spectrum})^{\frac{1}{2}}$. It can also be shown to equal the mean of the one-third highest double amplitudes

referring to Figure 2

$$\tan (d_s + \phi) = 2c_o/B$$

where

d_s = wave slope at surface, + ve clockwise

ϕ = roll angle, + ve to starboard

B = ship beam

c_o = change in waterline at ship side with zero relative heave

$$c_o = \frac{B}{2} \tan (d_s + \phi)$$

Now the port freeboard of ship with relative heave is

$$f_p = f_c - c_o - \xi'$$

and the starboard freeboard of ship with relative heave is

$$f_s = f_c + c_o - \xi'$$

where

f_c = calm water freeboard

and

ξ' = the heave term caused by ξ and ϕ

$$\xi' = \xi \cos \phi$$

The program to simulate possible capsize in beam seas was used to investigate two values of GM namely 0.38 and 0.52 m respectively both at a displacement of 1320 tonnes. It was found that the ship condition with the lower value of GM capsized while the condition with the higher value of GM was considered safe. These results can be compared with the model results and it can be seen from Figure 4 that these two sets of results compare favourably with each other. Various assumptions were made for the control parameters of the simulation program; simplified assumptions were made when the deck edge became awash and these were that the mass of water when on deck remained constant and its centre of

gravity remained midway between the deck edge and the deckhouse side. This is clearly an assumption since some of this water would be cleared by freeing ports on one hand and extra water would be taken on board as the vessel took on a list to one side. The vessel was considered to be lost if at any time the vessel heeled to an angle greater than 40° or the flooding angle, whichever was the less.

7. CONCLUSIONS

It may be reasonable to suppose that a vessel will be liable to capsize in a beam sea providing several events occur at the same time. The most important of these events must be the extreme wave forces causing extreme ship motions, the wind forces, including gusting wind forces and water on deck caused by a combination of the former events.

The usefulness of capsizing experiments with ship models and the simulation of this mode of capsize must be seen in perspective as these are only part of the overall picture relating to survival of the ship in its natural environment. Capsizals may occur in following seas due to rolling and due to broaching (Ref. (3)) for example and these too are caused by external forces other than by direct flooding due to damage. However, survival in beam seas should take into account the probability of shipping large amounts of "green seas" on deck causing the vessel to heel. The vessel could be considered safe if the angle of heel produced in this dynamic situation does not exceed 40° or the flooding angle, whichever is less.

A mathematical model of capsize in beam seas is a realistic proposition, providing roll damping coefficients for the ship, rather than the model are used. The question of "adequate safety" is more problematical but the best criterion for survival is considered to be a simplified dynamic approach as outlined above whereby the survivability is measured as the ratio of the restoring moment to the capsizing moment in extreme conditions. The question of good seamanship must not be forgotten as this will always improve the chances of survival of a ship at sea.

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APPENDIX INUMERICAL PARAMETERS FOR ANALOGUE PROGRAM

In order to use the analogue simulator program for a vessel in beam seas, it is necessary to calculate, estimate, or measure by experiment the following numerical quantities.

displacement, tonnes	Δ	1
GM, metres	GM	2
roll period, seconds	T	4
draught, metres	D	5
GZ curve constants	A3,A5,A7	20
Ship roll damping ratios	C1,C2,1	31
Ship heave damping ratios	B1,B2,B3	196
	1,B4,B5	197
C_D (U=0) for lateral motion	$C_D, 0$	36
C_D (U \neq 0) length (m) Speed (m/s) Y'v (n,d)	0,-LUYV	
density (1.025) length/2	RHO.L/2	37
sequence of seven wave	H1	89
semi amplitudes, metres	H2	90
	H3	91
	H4	92
	H5	93
	H6	94
	H7	95
	H1,H2,H3	188
	H4,H5,H6	189
	H7,1,1	190

Details of how many of the parameters not directly calculable may be measured experimentally are contained in Ref. (4). The waveheights corresponding to six different sea states are also tabulated. The above waveheights may be altered because the effective waveslope of shorter waves is reduced if the ship's beam is a significant fraction of the wavelength. The highest frequency wave in the irregular spectrum has a maximum length of 60 metres and a minimum of 28 metres so that some modification to the spectrum on this last count is probably necessary for most ships if their roll period is short.

FIG.1.

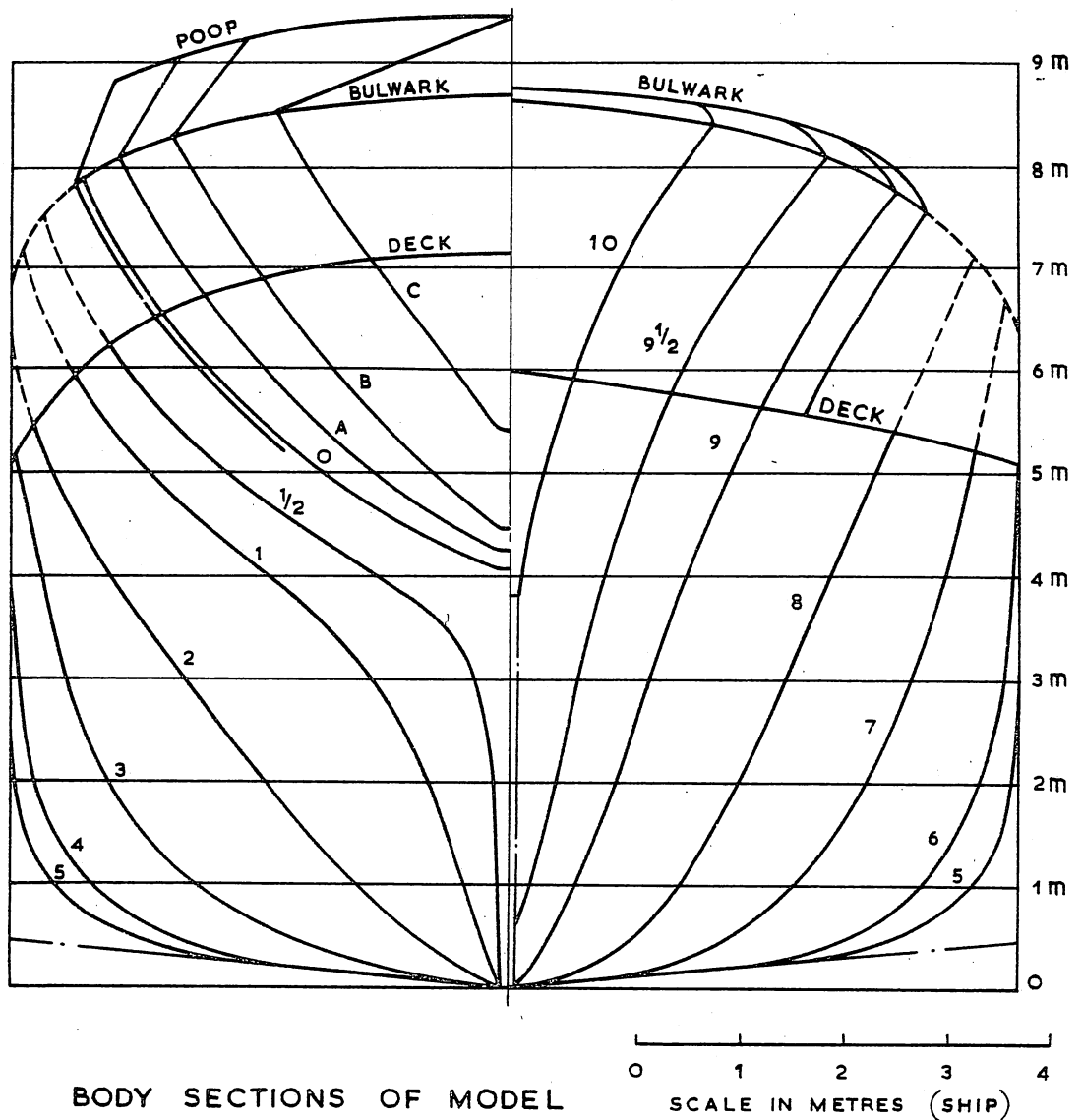
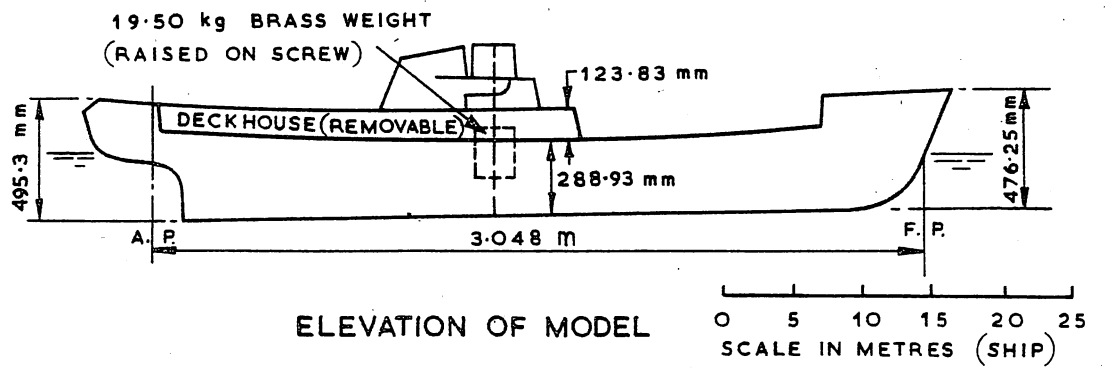


FIG.2.

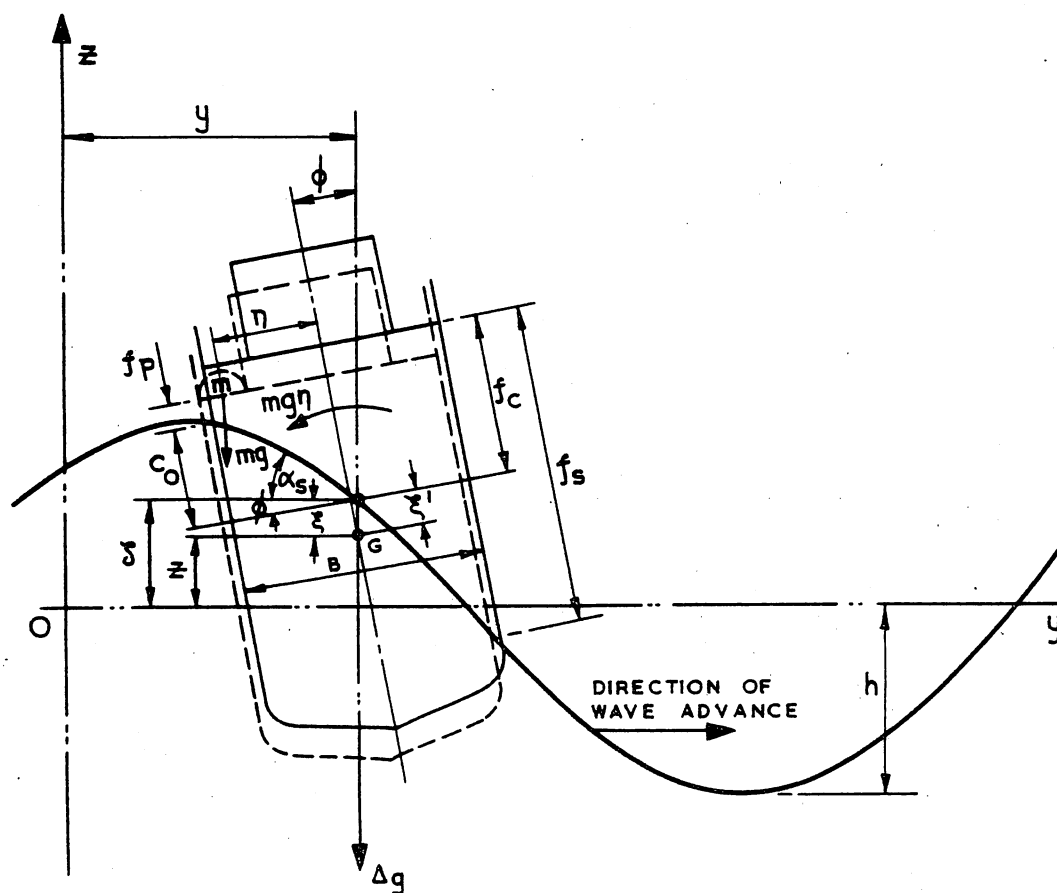
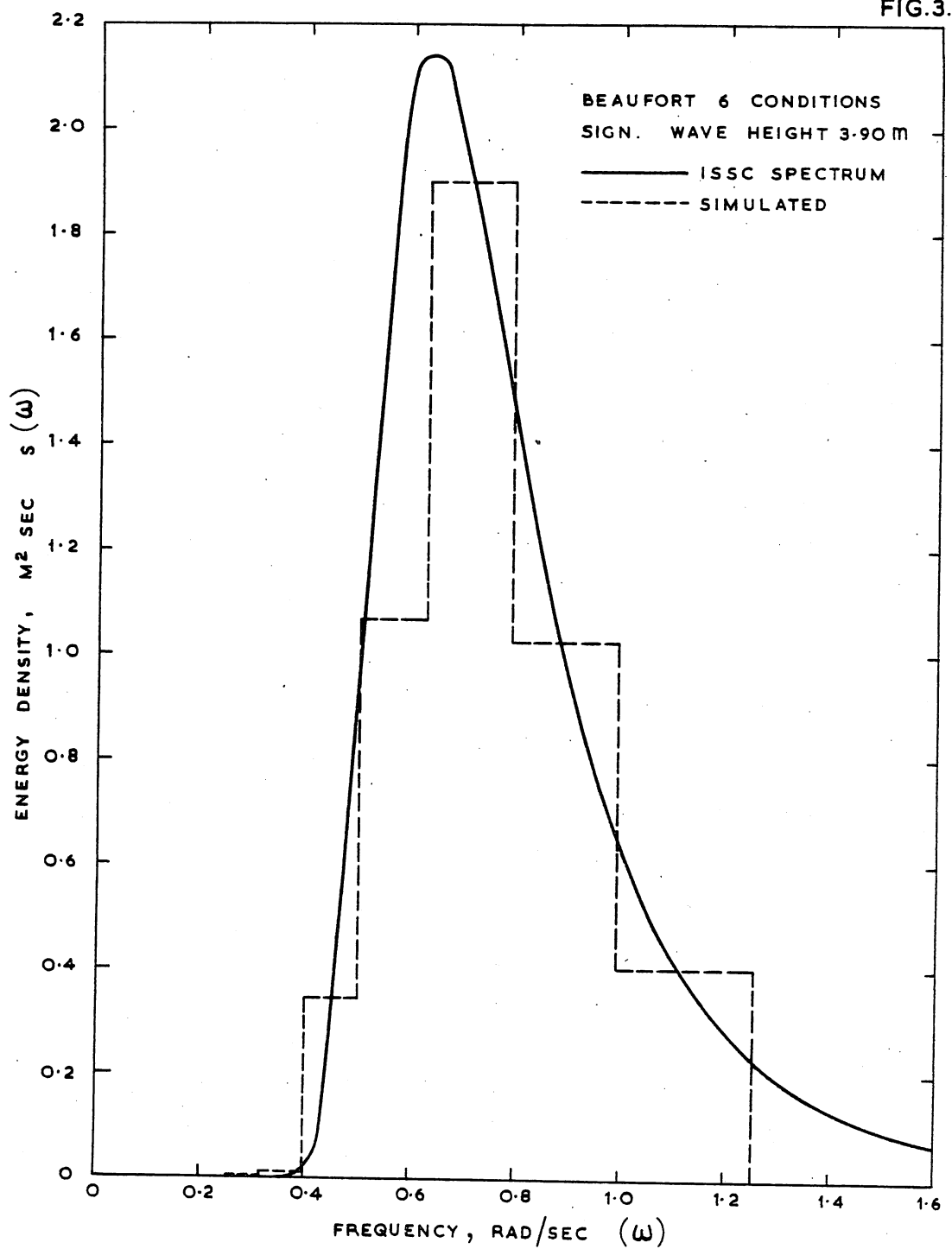
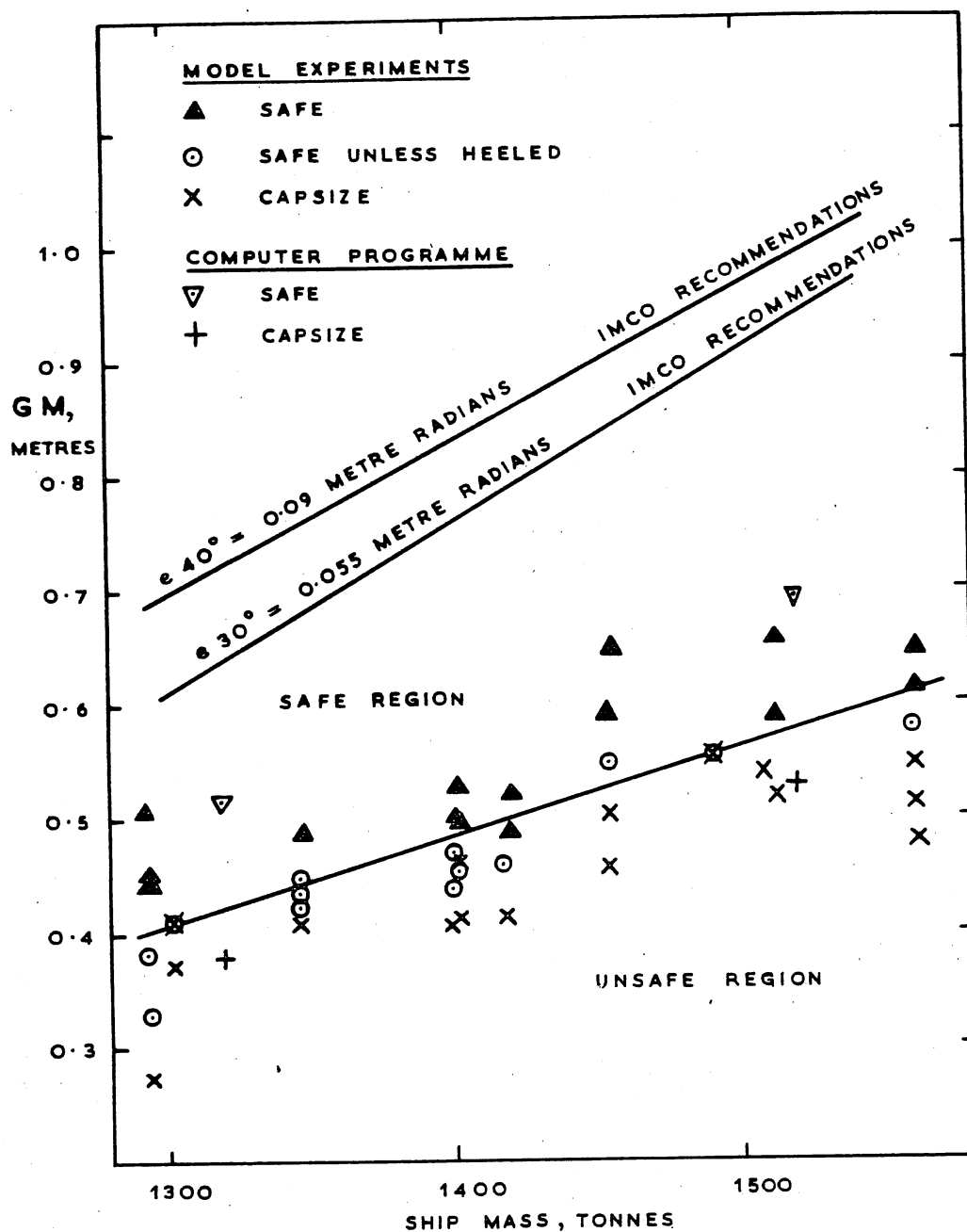


DIAGRAM SHOWING FREEBOARD OF SHIP
DUE TO ROLL AND HEAVE MOTIONS IN WAVES

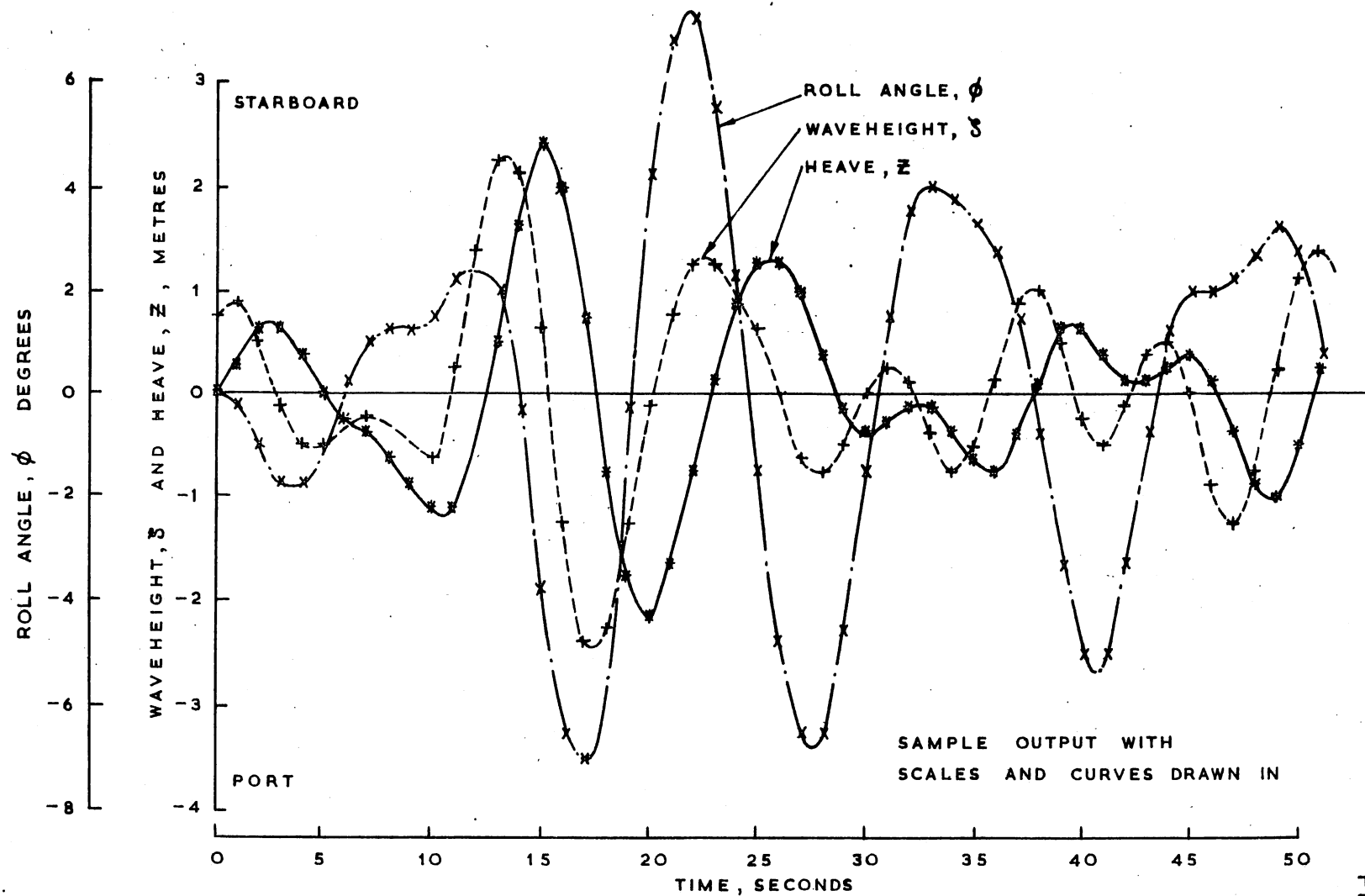


ACTUAL AND SIMULATED SPECTRA FOR
BEAUFORT.6. CONDITIONS

FIG. 4.



MODEL EXPERIMENTS WITH SIDE TRAWLER
SURVIVAL IN BEAM SEAS: BEAUFORT FORCE.6.



SIDE TRAWLER - BEAUFORT 6 WAVES - SIMULATED ROLL AND HEAVE MOTIONS

FIG. 5.

SESSION 5

APPLICATION OF RESEARCH FINDINGS

<u>Paper No.</u>	<u>Author</u>	<u>Title</u>
④ 5.1	Kastner, S. (Germany)	"Long term and Short term Stability Criteria in a Random Seaway"
② 5.2	Kuo, C, Odabasi, Y. (U.K.)	"Application of Dynamic Systems Approach to Ship and Ocean Vehicle Stability"
⑤ 5.3	Tsuchiya, T. (Japan)	"An Approach to Treating Stability of Fishing Vessels"
① 5.4	Nielsen, G. (Denmark)	"A Practical Approach to Ship Stability"
③ 5.5	Dorin, V.S., Nikolaev, E.P., Rakhmanin, N.N. (U.S.S.R.)	"On Dangerous Situations Fraught with Capsizing"

LONG-TERM AND SHORT-TERM STABILITY CRITERIA
IN A RANDOM SEAWAY

by

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1. INTRODUCTION

The best way to judge the safety of a ship from capsizing would certainly be to predict the expected roll motion extremes and their probability of occurrence. However, this is quite a cumbersome task even if performed by computer simulation. The main reasons are:-

- i) The linear theory for motion prediction, based on the superposition principle, cannot be employed because of the nonlinearities at large amplitudes.
- ii) In a random seaway there is no single deterministic capsize, but we are faced with capsizing as a stochastic process. A ship encounters many different waves and wave groups with different initial conditions of her motion. This yields a statistical sample set of different records with roll motion extremes and possibly capsizing events.
- iii) Capsizings are rare events even in some severe stationary seaway, depending on various ship parameters. However, capsizing will occur at an even lower rate within the lifetime of the ship, when she will encounter not only severe, but mild seas too.

But so far motion prediction of extreme roll in the time domain suffers from lack of knowledge on the hydrodynamic parameters or coefficients of the equations of motion at large heel. With recently developed programs for unsymmetrically submerged cross sections of ship hull, this difficulty

might be overcome. Several approximations for the solutions in the time domain have already been developed and this allowed us to generate the types of capsizing found by experiments with ship models in random seas. But further work is needed to refine the computer models and to check results with measurements.

Any numerical time domain analysis of the roll motion consists of two basic parts, see the block diagram in Figure 1. The first one is to determine the time varying forces and moments on the underwater ship hull according to the seaway excitation. The second part consists of integrating the corresponding motion equation in a step-by-step procedure. Both parts of the analysis could be done almost simultaneously at each step of the integration. On the other hand, any required value could be found by interpolation, if computed and stored beforehand for the parameter range in question. With high speed computers certainly the direct computation during the step-by-step integration is more appropriate.

However, the calculation of the righting moment of the ship at different angles of heel alone, already allows some, though not sufficient, judgement on the stability of a ship. Thus it is common practice to calculate the righting arm curve of the ship in still water. Obviously, this curve alone cannot be the basis for stability criteria in random seas for judging the severity of occurring roll motions. There exists quite a large gap between the common practice of using the still water righting arm curve only while applying some

sort of Rahola criterion, and a high level approach based on direct computation of roll motion. Still, the simple calculation of righting arms alone may well be sufficient for most ships, if accounted for the righting arm variations in the seaway and for the dynamic motion excitation. In this paper we will deal with some way to use hydrostatic righting arms in waves, and to account for dynamics involved by simple checking on possible resonance in following and quartering seas instead of solving the equations of motion.

2. LONG-TERM AND SHORT-TERM ANALYSIS

In general, one would like to have a ship that would withstand all possible waves she will encounter in her lifetime. Therefore we would attempt to design for the maximum load predicted. With respect to seaway, a maximum load can only be determined at a certain probability level, which still might be exceeded, though with a very small probability. Natural irregular seaway, as well as seaway induced loads and response, are being described as stochastic processes by statistical methods with spectra and probability distribution functions. Seaway can be considered stationary in a statistical sense within reasonable limits, but this is only valid for relatively short periods of time, say for some minutes up to a couple of hours. At any rate, this is only a short time compared with a ship service lifetime of say 15 or 20 years. A ship encounters a large number of different stationary sea states within her lifetime. Considering the probability distributions of the waves, loads, righting arms, ship motion or any other parameter of interest, only for a single stationary seaway, is called the short-term analysis. The example of stochastic righting arm variations given in Figure 2 is regarded as (stationary) short-term. Looking at the distribution of different stationary seaways and the corresponding ship response, which occur within a particular region of the ocean within years, or which the ship will encounter on her route, we call this the long-term approach.

The long-term analysis has only in recent years become a tool in the study of wave induced loads on the ship structure, in particular on the wave bending moment. Generally, all wave cycles within the lifetime of the ship are taken into account, i.e. even the cycles related to small waves. This

is in order for structural problems when fatigue has to be considered too. However, in ship motion problems, a ship has no memory of different sea states she has encountered at different times as far as the rigid body is concerned. On the other hand, to select severe sea states will impose some arbitrary action on the procedure.

Some authors call any selective approach short-term too as only short periods of time are considered. This would then imply a "non-stationary" short-term approach. But actually, the same computational procedure has to be followed as in the long-term approach, though with a different, reduced sample set of the extreme seaway states only. Therefore, looking at any group of many different short-term distributions might be called long-term too.

3. HYDROSTATIC RIGHTING ARM IN WAVES

Measurements have shown that in a following sea the restoring righting arm for the ship on a wave at large heels may well be calculated hydrostatically, i.e. according to the Froude-Kriloff theory. Consequently, the hydrostatic calculation of righting arms in crest and trough of a regular wave had been suggested. This method has been successfully applied in stability requirements for Navy ships, (2), (3). In setting up these regulations, the required residual righting arm in waves was determined by comparison with results of open water model capsizing tests in irregular seas, thus taking care of any possible dynamic effect involved. More than ten years ago, this was a practical way of combining the advantage of simple hydrostatic calculation with the results of less simple and expensive model tests on the extreme roll motion.

As soon as it is considered reasonable to use hydrostatics, even in a seaway, the question arises how to calculate the righting arm variations in irregular waves. They may be calculated directly for a ship in a long-crested, longitudinal seaway, where the irregular wave profile at the ship hull is determined from a given seaway spectrum with the equation

$$\zeta(x, t) =$$

$$= \sum_{i=1}^N \sqrt{2 S_i \Delta \omega_i} \cdot \cos(\omega_i t + \frac{\omega_i}{g} x + \xi_i) \quad (1)$$

where

- ζ wave elevation ordinate
 S seaway spectrum
 $\Delta \omega_i$ i -th frequency band
 t time
 x travelling direction of wave
 ξ_i equally distributed random phase

For the lines No. 4212 W of the Series 60 the hydrostatic righting arms at different constant heel angles were calculated with a set of 1000 different wave profiles from equation (1), taken at 1 second time intervals [4]. Spectra and distributions of this sample of stochastic righting arms have been calculated, see some results in Figure 2.

This procedure is quite cumbersome, therefore for the design practice hydrostatic calculations in equivalent regular waves are still advisable.

4. WAVE STEEPNESS AND LENGTH

For most ship hull forms, the maximum righting arm reduction results for the ship in a longitudinal wave with the wave crest at the main section. In the wave trough, the righting arm generally increases. The amount of the righting arm variation between wave crest and wave trough depends mainly on the wave steepness, i.e. the wave height-wave length ratio and the wave length. Furthermore, the wave period has to be taken into account for roll resonance. Therefore, choosing only one wave height, even if taken at an extreme value, can only yield a rough estimate of expected righting arm variations, but not cover the whole pattern.

The already mentioned seaway stability criteria from [2] are based on hydrostatic righting arm variations in a trochoidal wave of ship length. This is quite appropriate for comparing different hull forms, in order to ensure a similar stability standard, and to reduce the calculation effort as much as possible. But there is no

general law which states that the maximum righting arm reduction results always at a wave length equal to ship length. As an example, Figure 3 shows the righting arm difference with wave crest - still water versus the wave length - ship length ratio for the American Challenger class with a low freeboard of $D/T = 1.56$. For a constant wave steepness (dashed line) we find the maximum reduction always occurs at a wave length shorter than ship length and is dependent on the heel angle. The same is true for a wave steepness according to the formula given in (2):

$$\frac{H_W}{L_W} = \frac{1}{10 + 0.05 L_W(m)} \quad , \quad L_W = L_S \quad (2)$$

Equation (2) yields a larger wave steepness for shorter wave lengths. Therefore the maximum reduction curve is shifted further towards smaller L_W/L_S values.

5. JOINT PROBABILITY DISTRIBUTION OF WAVE HEIGHT AND WAVE LENGTH

In order to calculate the hydrostatic righting arms for the real seaway conditions we apply the joint probability distribution of wave height and wave length:

$$F_{H_W, L_W}(x, y) = P \{H_W = x, L_W = y\} \quad (3)$$

where

- F probability distribution function
 P probability

Then we call $f(x, y)$ the joint probability density function:

$$f_{H_W, L_W}(x, y) = \frac{\partial^2 F_{H_W, L_W}(x, y)}{\partial x \partial y} \quad (4)$$

We note that

$$P\{x < H_W \leq x + \Delta x, y < L_W \leq y + \Delta y\} \\ = f_{H_W, L_W}(x, y) dx dy \quad (5)$$

The marginal probability densities are

$$f_{H_W}(x) = \int_0^{\infty} f_{H_W, L_W}(x, y) dy \quad (6)$$

$$f_{L_W}(y) = \int_0^{\infty} f_{H_W, L_W}(x, y) dx \quad (7)$$

For any stationary seaway, i.e. where the energy equal to the area under the seaway spectrum is constant:

$$m_0 = \int_0^{\infty} S \cdot d\omega = \text{const} \quad (8)$$

S seaway spectrum

ω wave frequency

we can estimate wave heights and corresponding lengths from the generation of the wave ordinates in time and space according to equation (1). Then the wave height and length can be sampled in order to get the probability density function f_{H_W, L_W} , equation (4).

To extend this procedure to all possible seaways within the lifetime of the ship, we need a long-term distribution of the seaway energies encountered. Then we may accordingly calculate the long-term joint probability density function (4) of wave height and length. To our knowledge, this numerical task has not yet been undertaken. Besides the numerical effort, there is still not enough information on the long-term seaway spectra distribution in different ocean areas available. Projects on prediction, based on fetch, direction, and duration of storms, to cite only the main parameters, with accounting for the nonlinear interaction of waves in the energy transfer, are underway at some oceanographic Institutes, and results will be important for the field of Naval Architecture.

At the moment, long-term observations

from weather ships are in use for ship structures. Here the number of observed wave height-period combinations within certain levels are given, which corresponds to formula (5) for the joint probability density function of the two random variables H_W and T_W .

We can determine the wave length from the period by the relation

$$L_W = k \cdot T_W^2 \quad (9)$$

where

$$1.04 \leq k, \text{ sec}^{-2} \text{radians}^{-1} \text{m}$$

$$\leq \frac{g}{2\pi} = 1.56$$

The choice of the factor k imposes an uncertainty on the wave length estimated from the wave period. It would therefore certainly be better to calculate L_W directly.

The apparent wave heights H_V in the Tables are those which appear to be the most dominant or "significant" in that particular sea state. In many comparative computations oceanographers have tried to relate the observed height with some computational value derived from the full wave ordinate time series. The calculated significant wave height is defined as the mean of all one third highest waves $H_W(1/3)$.

We are going to use

$$H_V = H_W(1/3)$$

even though some correction formula is often suggested and used. However, we feel that this would still be fairly vague and would not improve the accuracy, as it only means a "calibration" of many different trained observers. For our purpose, this relation will be sufficient, unless more accurate data from direct computation are available.

Looking at the $H_V - T_V$ - plane, see Figure 4, any possible combination of $T_{V i} - H_{V j}$ stands for some observed seaway occurring with a certain

probability in that respective area and is found from years of observation. It would certainly be correct to apply any of these combinations in order to get a set of extreme wave heights-wave periods:

$$\{H_{\text{ext},i} \mid H_{V,i}, T_{V,j}\}$$

for all $i = 1, \dots, m, j = 1, \dots, n$

However, we will try to reduce this effort. This could be accomplished by looking at all columns, i.e. $T_{V,i}$

= const, and determine one corresponding significant wave height for each column i :

$$H_{V,i}^{(1/3)} = H_V^{(1/3)} \quad T_{V,i} = \text{const}$$

With a significant energy for each column i , assuming a Rayleigh distribution,

$$m_{0,i}^{(1/3)} = 0,25 H_{V,i}^{(1/3)} \quad (10)$$

we then may determine the extreme wave height $H_{\text{ext},i}$ and the corresponding wave length, resulting in the m -fold pairs:

$$\{H_{\text{ext},i} \mid T_i, L_{W,i} (T_i)\}$$

for $i = 1, \dots, m$

and we can calculate the righting arm variations for all m pairs in regular waves.

The other way would be to look in the $H_V - T_V$ plane at the rows, i.e. for $H_{V,j} = \text{const}$, and to calculate the corresponding extreme wave height, together with the respective period. We will give a numerical example with the latter procedure.

6. LONG-TERM EXTREMES OF WAVE STEEPNESS AND RIGHTING ARMS

We call the largest value to be expected during a certain period of time the "extreme" value.

From the application of order statistics of extremes, Ochi (5) has given a formula for the extreme value \hat{Y}_n , depending on the total spectral energy m_0 , the average

number n of observations considered the spectral bandwidth parameter ξ , and the probability a of exceeding the extreme value:

$$\frac{\hat{Y}_n}{\sqrt{m_0}} = \sqrt{2 \ln \left(\frac{1-\xi}{1+\sqrt{1-\xi}} \cdot \frac{2n}{a} \right)}$$

for $\xi \leq 0.9$, and small a (11)

For dangerous situations in roll excitation, that is in following and quartering seas, we find mainly very narrow frequency bands of the encounter spectra. Thus for easy calculation we might as well employ the limit of equation (11) with the bandwidth parameter ξ tending towards zero:

$$\frac{\hat{Y}_n}{\sqrt{m_0}} = \sqrt{2 \ln \frac{n}{a}}$$

for $\xi = 0$, and small a (12)

The average number of cycles, which during the lifetime of the ship belong to each wave height group j , may be calculated from the relative frequency n/N of the wave height H_V (within the row), and from the average period \bar{T}_V of the corresponding periods, see Figure 4:

$$n_{\text{life}} = \frac{T_{\text{life}}}{\bar{T}_V} \cdot \frac{n}{N} \quad (13)$$

where

$$\bar{T}_V = \frac{1}{n} \sum_j T_{Vij} \cdot n_{ij}$$

The relative frequency n/N is calculated from the number n_j of observations within each wave height group j , and is related to the marginal probability density distribution function from formula (7) by the relation:

$$\frac{n}{N} = f \cdot \Delta H = f_{T_V} (y_j) \cdot \Delta H$$

$$= \Delta H \int_0^{\infty} f_{H_V, T_V} (x, y_j) dx \quad (14)$$

Thus we get the extreme wave height for each wave height group j from the corresponding n_{life} , a , and m_0 .

Now we are faced with the further problem of finding the right corresponding period, as \bar{T}_V is only the average period of the respective wave height class. Sibul gives some data from measurements, compiled in the book by Wiegel [6]. It can be concluded, that the period of the wave with maximum height equals the period of the wave with the mean of the one third highest wave:

$$T_{H_{max}} = T_H(1/3) \quad (15)$$

Thus it seems to be fairly realistic to apply this relationship for the extreme wave. The period of the significant wave again may be determined after Wiegel from the mean period \bar{T} by the relations:

$$T_H(1/3) = 0.89 T^{(1/3)} \quad (16)$$

$$T^{(1/3)} = 1.24 \bar{T} \quad (17)$$

Formula (15), (16), and (17) combined yields

$$T_{H_{max,j}} = 1.10 \bar{T}_{Vj} \quad (18)$$

Figure 5 shows these different periods together with the period probability density at a given wave height H_{Vj} .

Table I shows example calculations based on seaway data after Hogben and Lumb [1]. For comparison two different Marsden squares have been used, area 2 for the North Atlantic, see Figure 4, and area 4 for the North Sea. There are contained 33,146 observations in the North Atlantic and 7,646 observations in the North Sea, both areas at all seasons.

Figure 6 shows the resulting extreme wave steepness versus the wave length.

For the North Sea we obtain shorter but steeper waves than for the North Atlantic, which was to be expected. However, the calculated wave steepness in some points exceeds the wave breaking limit, i.e. waves of this steepness will not be reached, but will break at the crest. This result is due to the minimum possible factor k of 1.04 chosen for the wave length - wave period relationship (7). Although it is believed that the resulting extreme waves with breaking are accurate, in particular for the more shallow water areas in the North Sea, we might better employ some average factor k , say 1.25. However, because of the nonlinear increase of righting arm at extreme wave steepness the resulting righting arms will not change very much, see also Figure 7.

With the now calculated set of extreme wave heights and corresponding wave lengths, respective wave steepness and wave length - ship length ratio

$$\left\{ \frac{H_{ext}}{L_W}, \frac{L}{L_S} \right\}_j \quad j = 1 \dots m \quad (19)$$

We derive the corresponding righting arms by interpolation from hydrostatically computed values for any given hull form, as shown in Figure 7 for the "American Challenger" class.

Figure 8 shows a comparison of the calculated decrease of righting arm at 45 degree of heel in a wave crest for the extreme wave data calculated in Table 1. This graph also includes the mean period and the period of the extreme wave, as well as the marginal probability density function f_j of the observed wave heights H_V .

Looking at Figure 6 again, the wave steepness is also plotted according to the wave height formula

$$H_W = 1.1 \sqrt{L_W} \quad (20)$$

which had been used for longitudinal ship strength calculations before the statistical approach was introduced. Furthermore, the wave steepness according to formula (2) is also shown approximated from the 10^{-3} occurrence rate after data of weather ship observations for the North Atlantic given by Roll.

In comparing all shown wave steepness curves in Figure 6 we find the values of equation (20) closer to the extremes, whereas equation (2) results in the smallest values. However, some designers might consider waves according to equation (2) extremely steep already and too far from reality (which in fact they are not). But Figure 6 shows clearly that equation (2) takes into account only the approximate significant conditions, but will be exceeded considerably by long-term extremes, though very rarely in lifetime.

Which wave steepness should now be recommended for practical use? It certainly depends on the sea area or ship route. For regulations a simple standard formula such as (2) would be advisable. Designing for the extremes might impose too severe limitations on the ship. But it would already show progress to include the seaway righting arm curve in stability criteria at all. Furthermore, for any special ship type or new design, the long-term wave distribution should be considered, in particular the wave height - length - period - combinations, and possible resonance must be checked.

7. MATHIEU RESONANCE IN RANDOM SEA

A capsizing in irregular sea due to resonance has been denoted as capsizing mode 1, low cycle resonance (7). In a following and quartering seaway, the righting arm GZ of the ship or model varies with time, which leads to the Mathieu differential equation for the roll motion, assuming one degree of freedom

$$a\ddot{\alpha} + b\dot{\alpha} + c \text{ GZ}(\alpha, t) = d(\alpha, t) \quad (21)$$

For a linear Mathieu equation resonance occurs if the exciting period is equal to natural multiples of half of the natural period

$$T_{\text{exc}}/0.5T_{\text{nat}} = 1, 2, \dots \quad (22)$$

i.e., for the frequencies

$$\omega_{\text{exc}}/\omega_{\text{nat}} = 2, 1, \dots \quad (23)$$

This is generally true, even if both ω_{exc} and ω_{nat} are not constant in time, but cover some limited range on the real axis, as determined by the probab-

ility density distributions $f(\omega_{\text{exc}})$ and $f(\omega_{\text{nat}})$. Thus we may denote for Mathieu resonance in a random seaway the condition (23) as follows:

Resonance occurs if

$$f(\omega_{\text{exc}}) = f(n \cdot \omega_{\text{nat}})$$

$$n = 2, 1, \dots; \quad \omega \geq 0 \quad (24)$$

The distribution of the natural roll-frequency is determined by the time variations of righting arms, and because of the nonlinearity in the righting arm it also depends on the roll amplitude.

Obviously, equation (24) will rarely hold exactly for all exciting frequencies in a random seaway, but if there is any condition where it holds for some frequency range, particularly around the density peaks (either the peak of the exciting spectrum or the response), in a long run, a partial resonance is likely to occur.

This means that we can expect more Mathieu resonance, the more the existing frequency distribution overlaps the distribution of the natural frequency ($n = 1$) or overlaps a distribution for double the natural frequency ($n = 2$).

If the ship is moving at an angle to the main direction of the waves ($\psi = 0$ for following sea), then the encounter frequency wave - ship is

$$\begin{aligned} \omega_e/(2\pi) &= (c - v \cdot \cos \psi) / L_W \\ &= \frac{\omega}{2\pi} (1 - \frac{v}{g} \cdot \cos \psi) \end{aligned} \quad (25)$$

$$\text{with } c = g/\omega, \quad \omega = \sqrt{2\pi g/L_W}$$

and with

L_W	wave length
c	wave celerity
ω	wave frequency
ω_e	encounter frequency ship - wave
g	gravity acceleration
v	ship speed

From equation (25) follows the maximum of the encounter frequency to be independent of the untransformed wave frequency ω , or the wave length L_w respectively:

$$\text{Max } (\omega_e) = \frac{E}{8\pi \cdot V \cdot \cos \psi} \quad (26)$$

It can be shown from the transformation of the seaway spectrum that at the maximum of the encounter frequency the transformed spectral value goes to infinity, and we get the relations:

$$\omega_v = 2 \omega_{\text{inf}} = 4 \omega_{e, \text{inf}} \quad (27)$$

$$\text{with } \omega_{e, \text{inf}} = \text{Max } (\omega_e)$$

$\omega_{e, \text{inf}}$ singular encounter frequency (where the encounter spectral density goes to infinity)

ω_{inf} corresponding singular wave frequency

ω_v frequency of the wave travelling at ship speed ($\omega_{e, v} = 0$)

In Figure 9, there are plotted the encounter frequencies $\omega_e/2\pi$ versus the ship heading ψ for regular waves of different length L_w according to formula (25), with the dimension Milli-Hertz (mHz).

Naturally, the minimum encounter frequencies are in following seas, the maximum in head seas. The encounter frequencies in beam seas are equal to the wave frequencies.

All curves in the graphs for the encounter frequency for higher speeds show a very remarkable characteristic: they all cross each other at quartering seas, i.e., for a heading ranging from roughly 30 to 60 degrees. This means that the encounter frequency in quartering seas is compressed into a small range on the real axis, no matter what the particular wave length is. In other words, there are speed-heading combinations, which concentrate the total seaway energy into a very narrow frequency band of encounter. In case the natural roll frequency distribution of the ship in the seaway coincides with the narrow banded excitation, the most severe attenuation of roll motion due to Mathieu resonance, equation (23) is likely to occur. Although the maximum

righting arm variations generally appear in following seas, the quartering sea will be even more dangerous from the resonance point of view.

8. CONCLUDING REMARKS

Extreme roll amplitudes incurring the danger of capsizing are rare events within the lifetime of a ship. This is even true for a ship experiencing severe seaway for only a short time, say a couple of hours. But for safety consideration, because of the danger of the total loss of the ship, such rare events as capsizing must be taken into account, and the acceptable risk level must be evaluated.

In determining the extreme conditions which have to be accounted for in a random seaway, the long-term and the short-term approach might be employed. Although computer programs and new model testing techniques in irregular seas in the time domain have been developed, a simple approach such as calculating hydrostatic righting arm variations in wave crest and trough imposes a reasonable effort on the design engineer. The short-term time domain histories may be used in order to gain a basic understanding of the behaviour pattern and to prove requirements.

The dynamics involved can be considered by checking on Mathieu resonance in following or quartering seas. Here ship speed, heading and natural roll frequency are the main parameters to be related to the seaway spectrum. Attention is drawn to the minimum bandwidth of the encounter spectrum in quartering seas at certain ship speeds. This leads to a concentration of the total wave energy at a very narrow frequency band, in other words, practically all wave components in the random seaway of different lengths (say 100' through 1000') add up their energy at almost one single encounter frequency. The encounter spectrum must be compared with a (roughly estimated) probability distribution of the natural roll frequency.

The main advantage in applying long-term wave distributions on to ship righting arms may be seen as follows:

- i) Different ocean areas or ship routes can be taken into account.
- ii) Possible Mathieu resonance can be checked according to the seaway to be encountered on her route.

- iii) It provides a more realistic overall pattern of the extreme roll motions to be expected than with the righting arm reduction in a single hypothetical wave.

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LIST OF SYMBOLS

$y x$	y given x
$\{ \}$	sample space or statistic
C	Righting arm at wave crest
T	Righting arm in wave trough
S	Still water righting arm
$H(1/3)$	Mean of the 1/3 largest wave heights
H_{ext}	Extreme wave height
ΔH	Wave height interval
L_W	Wave length
L_S	Ship length
i	Subscript for wave period group
j	Subscript for wave height group
m_o	Mean total energy of stochastic process

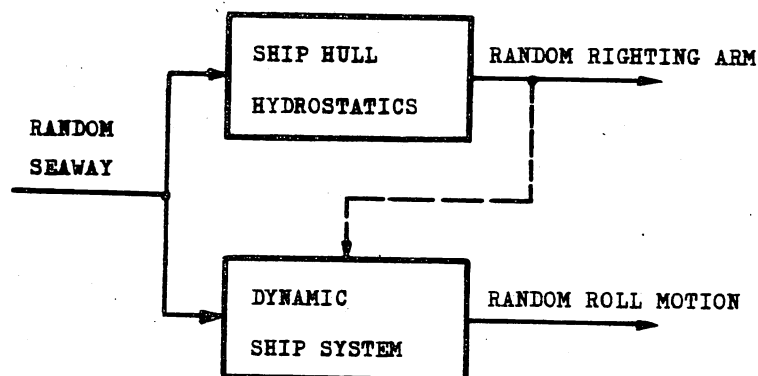


Figure 1: Black Box System for a Rolling Ship in Random Seaway

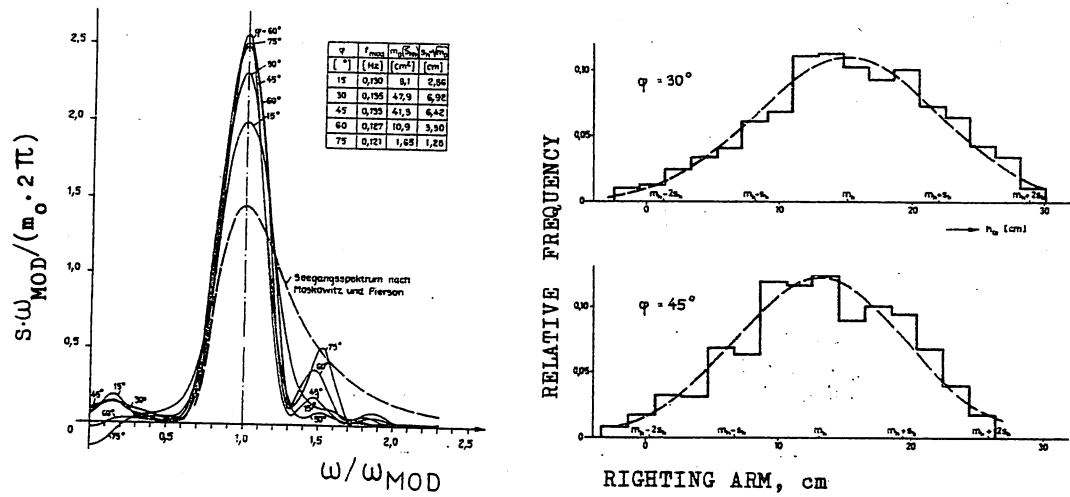


Figure 2: Normalized Spectra and Relative Frequency
of Righting Arms for Ship in a Following Seaway

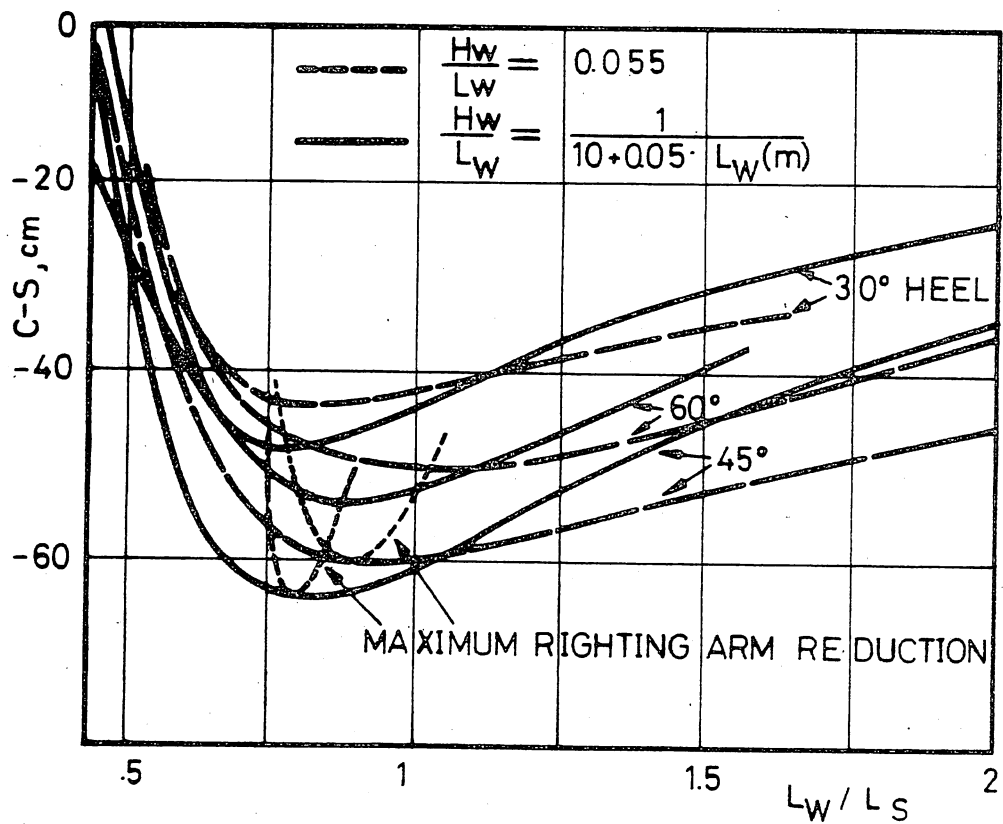
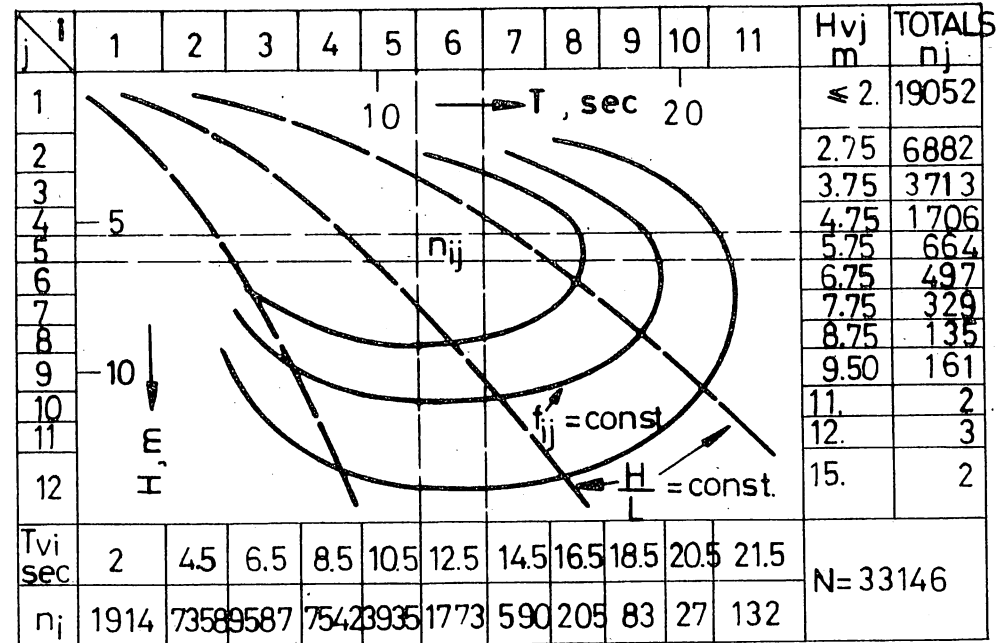


Figure 3 : Maximum Righting Arm Reduction at Wave Crest
versus Wave Length

NORTH ATLANTIC-AREA 2 - ALL SEASONS



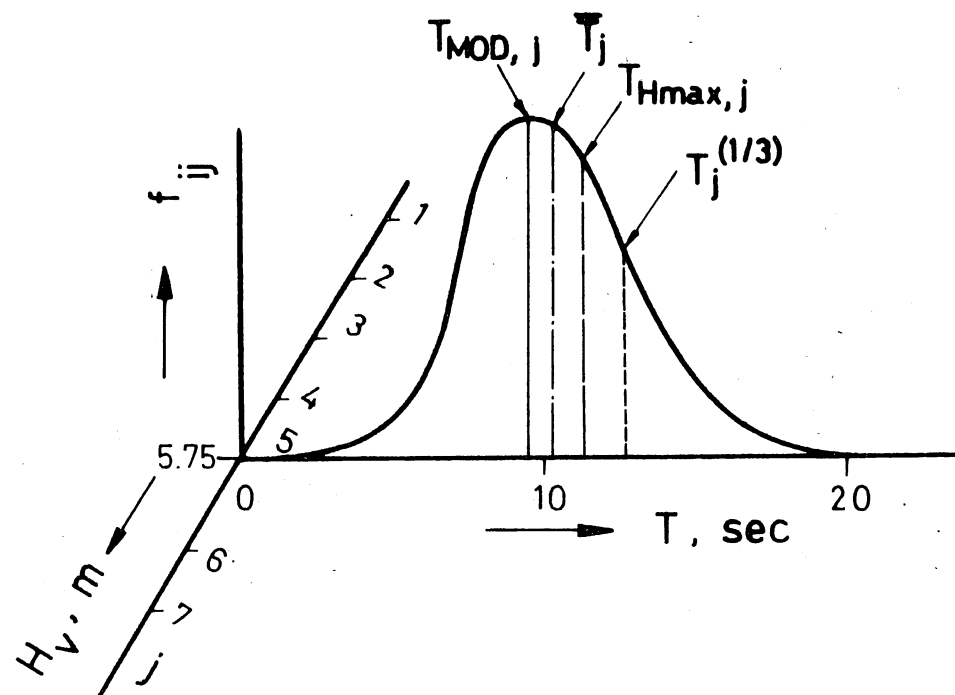


Figure 5 : Density Function of Wave Period at Given Wave Height

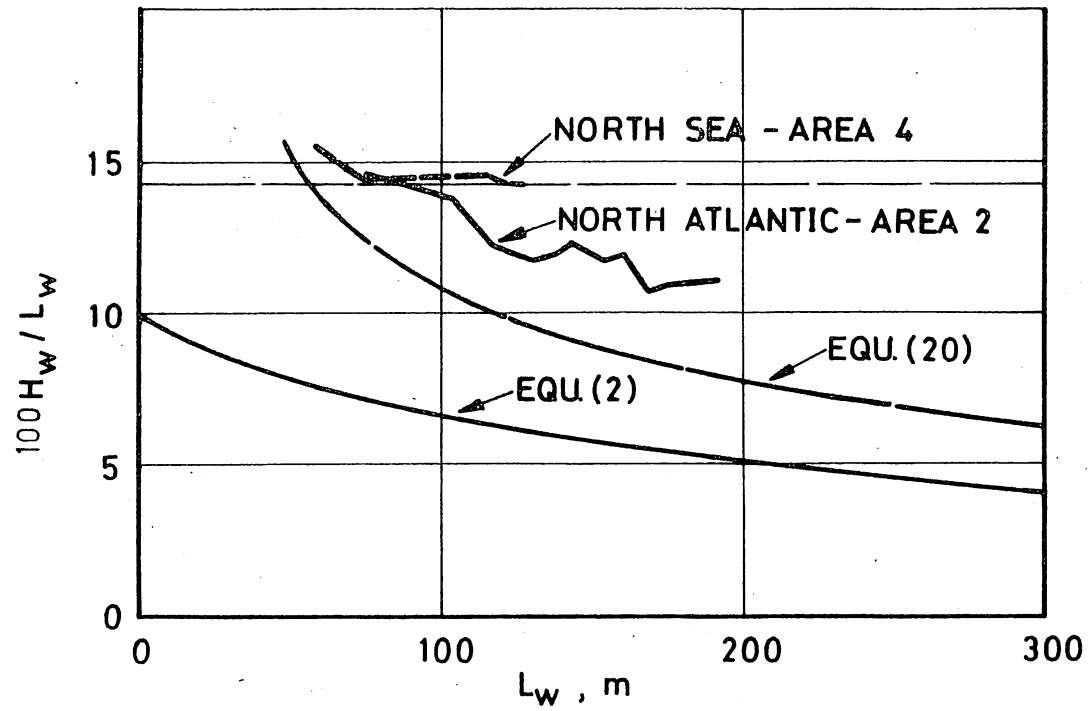


Figure 6 : Wave Steepness Extremes Compared with Standard Formulae

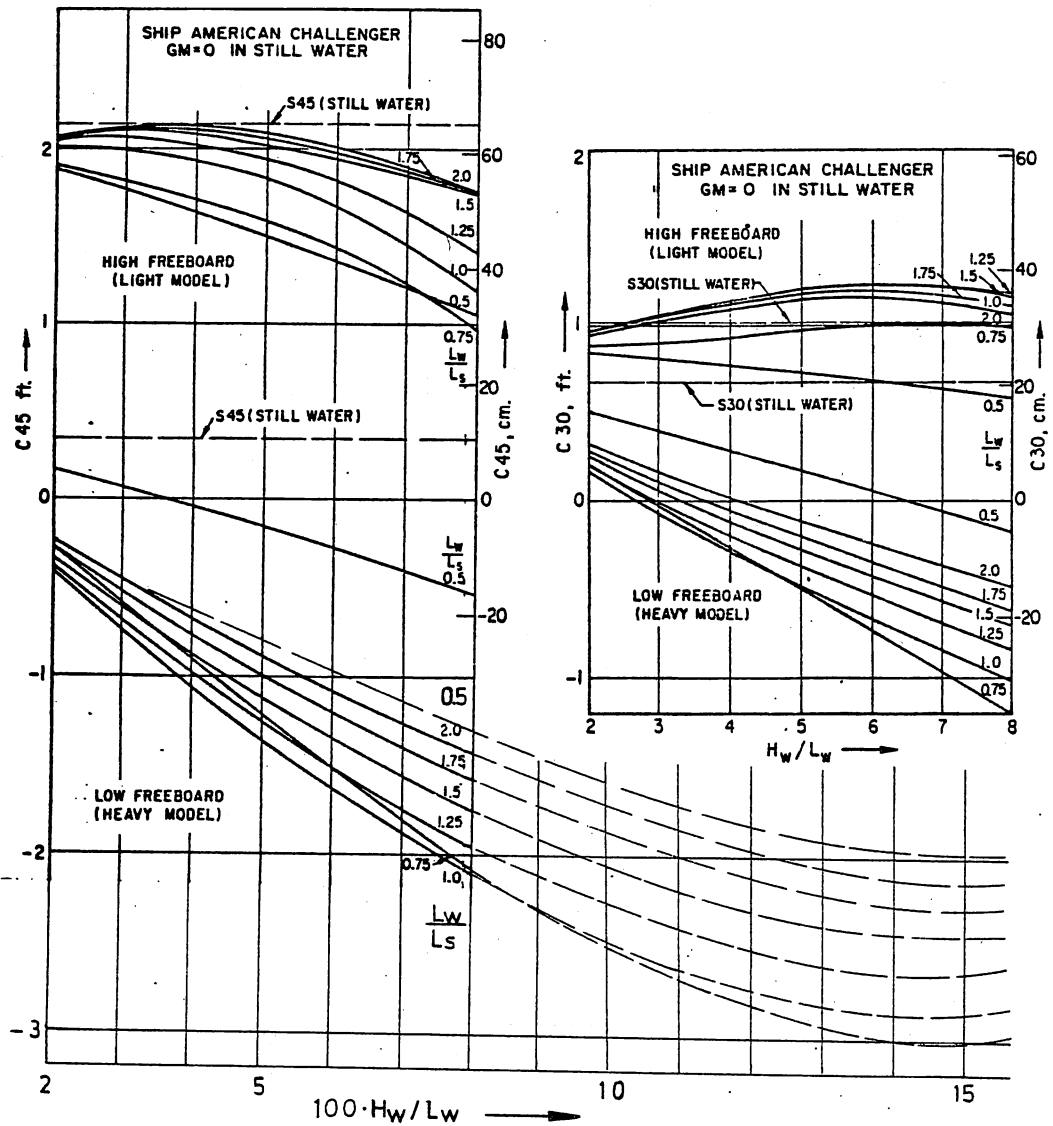


Figure 7 : Righting Arm in Wave Crest for different Wave Length
and Height in Sinusoidal Wave
(dashed curves by rough extrapolation)

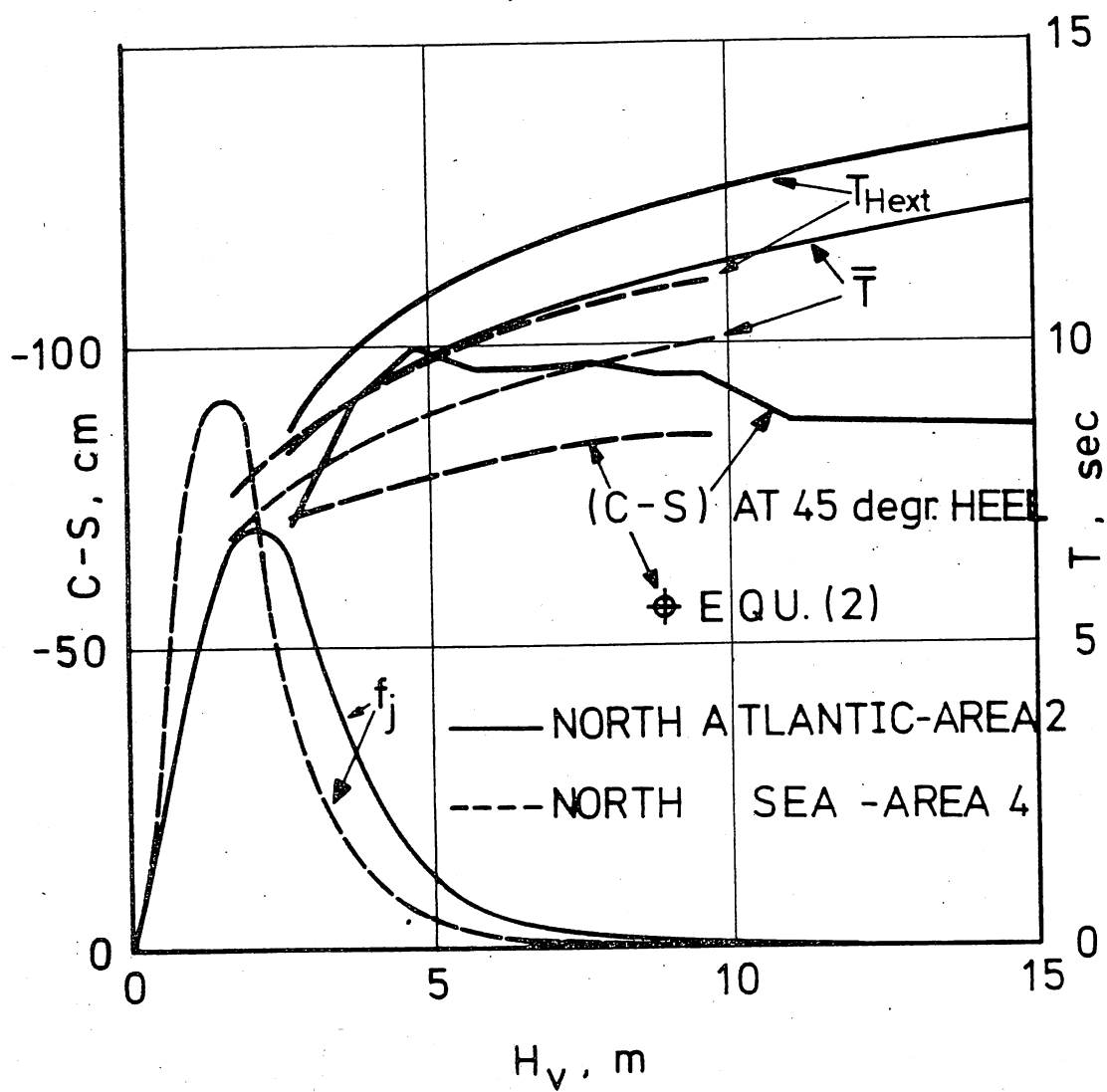


Figure 8 : Extremes of Righting Arm Reduction in Wave Crest at 45 Degr. Heel ("American Challenger", $D/T = 1.56$)

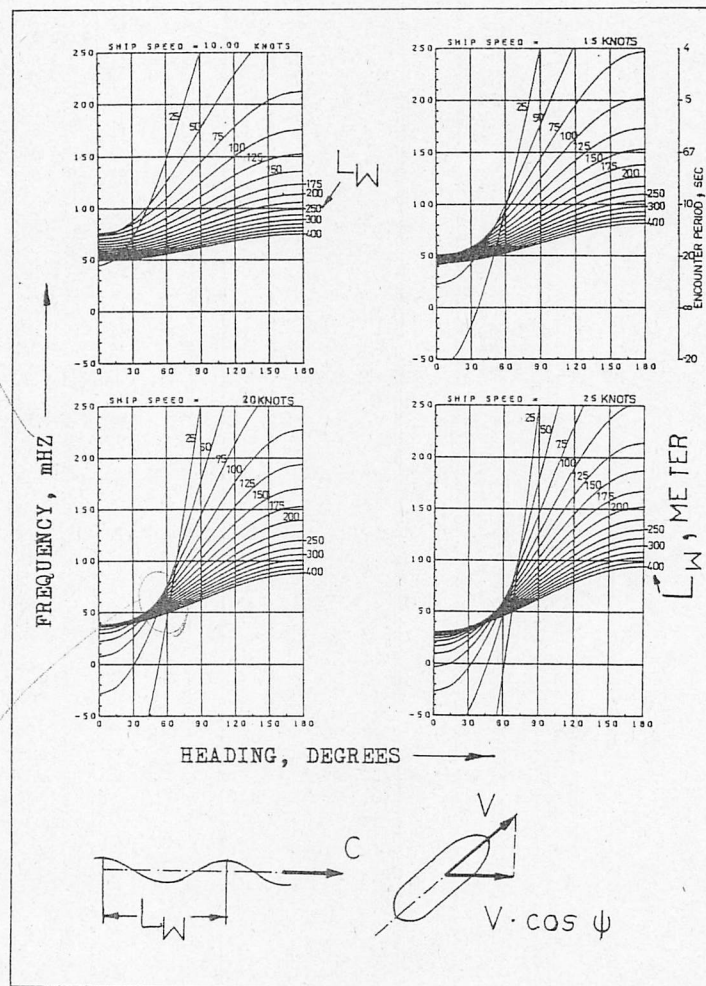


Figure 9: Encounter Frequency Wave - Ship
versus Heading for Different Wave Lengths

Table I : Calculation of Extreme Values of Waves and Righting Arms

$$L_S = 158.80 \text{ m}$$

$$T_{\text{life}} = 5.9 \times 10^8 \text{ secs.} = 20 \text{ years}$$

j	H _V	n	n/N	T	n _{life} · 10 ⁻⁶	H _{ext}	L _W /L _S	$\frac{H_{\text{ext}}}{L_S}$ pct.	(C-S) ₄₅
-	m	-	-	sec	-	m	-	-	cm
(i) North Atlantic, Area 2, all Seasons, N = 33,146 Observations									
2	2.75	6882	.2076	8.4	14.5800	11.4	.50	14.4	72
3	3.75	3713	.1120	9.1	7.2600	13.1	.66	13.8	91
4	4.75	1706	.0515	9.7	3.1300	14.4	.75	12.2	99
5	5.75	664	.0200	10.2	1.1600	15.5	.83	11.8	96
6	6.75	497	.0150	10.5	.8430	16.7	.87	12.0	96
7	7.75	329	.0099	10.7	.5460	17.7	.91	12.3	97
8	8.75	135	.0041	11.1	.2180	18.3	.98	11.8	95
9	9.50	161	.0049	11.3	.2560	19.2	1.01	12.0	95
10	11.00	2	.00006	11.6	.0031	18.1	1.07	10.7	87
11	12.00	3	.00008	11.8	.0040	19.1	1.10	10.9	87
12	15.00	2	.00006	12.3	.0029	21.1	1.20	11.1	86
(ii) North Sea, Area 4, all Seasons, N = 7,646 Observations									
3	2.75	890	.1164	7.7	8.9190	11.2	.47	15.0	72
4	3.75	408	.0534	8.3	3.7959	12.6	.55	14.5	75
5	4.75	135	.0177	8.8	1.1867	14.1	.61	14.5	78
6	5.75	47	.0061	9.1	.3955	15.1	.66	14.5	80
7	6.75	29	.0038	9.4	.2385	16.4	.70	14.8	82
8	7.75	13	.0017	9.6	.1045	16.9	.73	14.6	84
9	8.75	4	.0005	9.8	.0301	17.4	.76	14.4	84
10	9.50	3	.0004	10.0	.0236	18.0	.79	14.3	85

APPLICATION OF DYNAMIC SYSTEMS APPROACH
TO SHIP AND OCEAN VEHICLE STABILITY

by

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1. INTRODUCTION

Intact ship stability is a feature that has concerned the designer since it is closely associated with the safety and efficient operation of the ship. Although the assessment of the ship stability is an essential requirement in design and yet at the same time it is probably the most poorly developed aspect of naval architecture. The main reasons for the existence of this state of affairs are the difficulties in relating stability to the ship's motion characteristics and the general belief that existing methods are sufficient because they can incorporate practical experience.

In dealing with complicated technological problems such as ship stability there are always two possible ways of achieving a solution. One approach is usually aimed at providing a "simple answer" that will fulfil the immediate needs while the other would attempt to treat the problem more precisely, rationally and logically. Such a philosophy is usually adopted in an engineering context. For the assessment of ship stability the traditional quasi-static method belongs to the former category, while the latter approach demands methods that can effectively take into account the ship's motions in the seaways. However, due to the complex nature of the problem the more rational methods have received only the minimum amount of attention while the solution by the traditional method has dominated the scene over a large number of years. Indeed, the traditional approach is often mistakenly thought to be a sound method of solution.

With increased marine activities being

undertaken by both ships and other ocean structures the traditional method would not be able to cope adequately with the demands for better stability criteria.

During the last decade efforts have been spent in establishing the basic relationships between the stability and the motion characteristics of ships, see for example (1) to (6). However, in spite of the advances made so far, the results are not wholly satisfactory as they have been devoted to either obtaining particular results or have been strongly influenced by the traditional approach. Thus any attempt to make a fresh start should begin by asking ourselves a number of fundamental questions, such as:

- (a) What is stability?
- (b) How do we obtain an accurate mathematical model of the physical phenomena?
- (c) Does the intact stability of a ship depend on the non-linearity of the equations of motion?
- (d) Would the effect of coupling of the other modes greatly influence the rolling motion?

Many more questions may be put but they are unlikely to be answered purely based on the knowledge and techniques developed in classical naval architecture. To achieve success it is necessary to obtain the active support from both the mathematical techniques, as well as the mathematicians and control engineers and carry

out the study in a systematic manner.

In this paper a brief account is given on how the assessment of the stability of a ship and ocean vehicle may be tackled in a logical procedure. Our purpose is not aimed at outlining in detail the theory of the proposed approach or every step in the mathematical formulation as much of the material can be found in Reference (7). Instead, only the basic principles and assumptions will be highlighted with the major effort being devoted to the presentation of the philosophy behind the approach, making a comparison with the traditional approach and illustrating the application with numerical examples.

Within the text the predominate approach is the stability analysis via Lyapunov's direct method which has been introduced by Odabasi to the literature of naval architecture (8). This method provides a comprehensive basis for assessing the stability of motion equation under the most general circumstances and yields the stability regions of interest. Although this treatment is still in its early stages of development as far as ship stability is concerned, there is strong evidence to suggest similar techniques have been successfully applied in the design of control systems for activities such as space flights. In the present application, Lyapunov's method is linked to the equations of motion and the term "Dynamic Systems Approach" is adopted to describe the procedure. In order to ensure that the discussions are not confused by different interpretations of the various items used in stability studies, the traditional and dynamic systems approaches are both defined. It will be noted that the dynamic systems approach is applicable to both ships and floating ocean vehicles and for this reason a term "floating ocean structures" or "FOS" is introduced in the present paper to avoid the need to differentiate between ships and ocean vehicles.

2. THE TRADITIONAL APPROACH

The classical treatment assumes that the stability of a ship can be concluded from its statical properties. This is achieved by calculating the righting moment created by the weight and the buoyancy forces, which form a couple provided the displacement is constant. The lever of this couple is called the "righting arm" and is used to provide a measure of ship stability.

For ship design purposes the usual procedure is to calculate the righting arm at various angles of inclination

while keeping the displacement constant so that a particular statical diagram can be constructed for a given condition such as those for light and loaded drafts. The stability properties of the vessel are then deduced by examining the shape, the magnitudes of righting arms at key locations and the areas under the curve.

Moseley (9) in 1850 presented the idea of "dynamical stability" which was derived from the undamped rolling equation under the action of potential energy, which he called the "dynamic lever", must be more than the work done by external forces, i.e.,

$$\int_{q_4^0}^{q_4^m} W \cdot GZ dq_4 - \int_{q_4^0}^{q_4^m} M_p(q_4) dq_4 > 0 \quad (1)$$

where

q_4^0 is the initial angle of heel,

q_4^m is the maximum angle of roll,

W is the ship's weight,

GZ is the righting lever,

$M_p(q_4)$ is the potential excitation,

q_4 is the angle of roll.

Since both integrals in equation (1) may be considered as the areas under the righting and exciting moments respectively, many naval architects ignored the theoretical basis of the Moseley's study and considered only the areas.

For such a procedure to be effective it is essential to rely on experience in its usage and it is therefore not surprising that use is made of data on capsizing casualties. By carrying out "statistical analysis" on a sufficiently large sample it is possible to arrive at some conclusions or guides provided, of course, the actual data can be regarded as reliable and handled in an appropriate manner. Such studies have been undertaken by Rahola (10) and his important contribution has greatly influenced the selection of ship stability criteria. For example, the IMCO intact stability criteria for ships under one hundred metres in length requires the fulfillment of some conditions regarding the initial metacentric height,

maximum righting arms at certain angles of heel and areas under the righting arm curve up to given locations.

A closer examination of this approach reveals that it involves a number of drastic assumptions and the most important ones are:-

- (a) All types of excitations are of potential nature and stationary,
- (b) There is no contribution due to kinetic energy,
- (c) There is no contribution due to energy dissipation,
- (d) All hydrodynamic forces can be ignored,
- (e) Coupling effects need not be included,
- (f) Buoyancy force remains constant.

Clearly it is important to consider these items if a more realistic representation of the physical phenomenon is desired.

3. BASIC FEATURES OF DYNAMIC SYSTEMS APPROACH

The basic idea behind the dynamic systems approach is to link the stability of FOS directly to their motion characteristics, which in turn must be represented by the equations of motion. The development of such an approach involves the following five steps:-

- (a) Understanding the capabilities and limitations of mathematical simulation,
- (b) Selecting the assumptions to be adopted,
- (c) Formulating the mathematical model of the motion,
- (d) Establishing methods of assessing stability of motion equations,
- (e) Applying the techniques to practical problems.

It is therefore helpful to briefly consider the first three points at this stage while the latter two would be considered in greater detail in Sections 4 and 5.

a) Mathematical Simulation

When the analysis of a physical phenomenon is to be carried out it is usual to define the problems involved first before developing some procedures for

determining their solution. The problem definition may consist of observations and formulation of the physical phenomenon through direct application of some physical laws or hypotheses. In practice, the physical phenomenon is replaced by a mathematical model in such a way that the subsequent studies represent the true occurrences of the important features rather than all the details. In the development of suitable procedures it is necessary to utilise one or more mathematical methods which generally provide approximate solutions that must be coordinated with the experimental results. Thus, by this method it is possible to examine the behaviour of the physical phenomenon through the manipulation of its mathematical model. Clearly the success of the mathematical simulation would depend upon the reliability of the assumptions selected. For example, making unrealistic assumptions or exceeding the limits without physical justification would lead to poor results.

Although the concept of simulation is simple its practical application in the FOS case presents special difficulty since FOS operates in a combined media of air and water. Thus, to avoid being involved in the manipulation of extremely complicated formulations it is necessary to introduce a number of assumptions.

b) Basic Assumptions

Since the motions of FOS are the main point of interest it is most convenient to model the fluid motions as a subdivision of the overall system and let the resultant hydrodynamic contributions be incorporated as parameters in the motions equations. Thus, the key assumptions to be made are as follows:

- 1) The FOS is assumed to oscillate as a rigid body. This assumption is justified since it is stability of motion that is being of prime importance.
- 2) The FOS is assumed to be operating in an ideal, inviscid and incompressible fluid and its motion is treated as irrotational. The equations of fluid motion and associated boundary conditions are discussed in detail by John (11).
- 3) The hydrodynamic forces/moments are expressed in terms of added mass/mass moments of inertia, damping and excitation forces or moments, see Wehausen (12).

The shortcomings of the simulation is due to the extreme difficulties involved in dealing with the hydrodynamic aspect of the problem. Fortunately, five modes of motion fulfil the requirements demanded by the hydrodynamic formulation and for the rolling motion the geometry of FOS makes it possible for us to employ the linear hydrodynamic approximation until the deck is immersed. Apart from these theoretical considerations, it is still necessary to find ways of introducing the effects of energy dissipation due to viscosity of the fluid medium and wave potential energy due to the influence of wave elevation. Since this part of formulation relies heavily on the experimental results and observations it is still necessary to adopt an empirical treatment.

c) Formulation of the Mathematical Model

The equation of motion is formulated in the Lagrangian form using the following equation:

$$\frac{d}{dt} \frac{\partial(T-V)}{\partial \dot{q}_j} - \frac{\partial(T-V)}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = F_j(t) \quad (2)$$

where

T = kinetic energy

V = potential energy

D = dissipation function

q_j = generalised coordinate

\dot{q}_j = generalised velocity

t = time

$F_j(t)$ = general excitation

j = 1, 2, ... 6 representing the modes of motion; surge, sway, heave, roll, pitch and yaw respectively

The kinetic energy is made up to two parts - the more familiar form represented by the product of half mass by velocity squared and a gyroscopic contribution term. The potential energy of the system is expressed in a series form and is obtained by expanding the potential function into a Taylor series about the equilibrium point. The details of this formulation can be found in Reference (13).

Since the present paper is concerned

with non-linear rolling motion under general excitation equation (2) becomes:

$$I_4 \ddot{q}_4 + P(\dot{q}_4) + R(q_4) = F(q_4, t) + f_m \cos(\sigma t - \varepsilon) \quad (3)$$

where

I_4 = mass moment of inertia with respect to the rolling axis

$P(\dot{q}_4)$ = dissipation function which is usually expressed in the form of

$$d_1 \dot{q}_4 + d_2 \dot{q}_4 |\dot{q}_4|$$

d_1 = linear damping coefficient

d_2 = quadratic damping coefficient

$R(q_4)$ = restoring term: $(k_1 q_4 + k_2 q_4 |q_4| + k_3 q_4^3 + k_4 q_4^3 |q_4|) \Delta$
 Δ = displacement

k 's = constant or time varying coefficients of the restoring equation

f_m = amplitude of wave excitation

ε = phase angle

σ = wave excitation frequency

$f(q_4, t)$ = other forms of excitations

Although the equation of motion as represented by (3) is deterministic rather than stochastic, it is our belief and experience that the model provides a great deal of qualitative and quantitative information about the mechanism of capsizing. With this knowledge it is possible to link physical behaviour to the stochastic treatments.

4. METHODS OF ASSESSING STABILITY

Having established the mathematical model to be used for representing the motions of FOS the next task is to develop and apply mathematical methods for assessing the stability of this model. To the naval architect, the concept of stability can have only one

simple physical interpretation - whether the ship or FOS will float up-right or capsize and there is no confusion. Unfortunately, such a fundamental concept cannot so readily be adopted by a simple mathematical formulation. However, the words "stable" and "unstable" in mathematical terms are used to describe characteristics of trajectory which are locations of points in space or plane. It is the presentation of these characteristic features in the various manners that provide information about the behaviour and hence the stability of the mathematical model. For the present study the stability will be defined as:-

"the ultimate boundedness of the motion amplitude within predefined or predetermined limits"

In other words, the motion is regarded as stable provided the amplitude stays within the limits otherwise it is regarded as unstable. By using this definition we can establish a logical procedure to deal with the FOS stability problem.

Two Basic Concepts

Before the mathematical treatment can be applied it is essential to have the necessary "tools" and it is fortunate that the mathematical stability theories provide us with many of the required methods. However, gigantic effort is still required to acquire and to understand the theories before applying the to study motions of FOS. In order to simplify the discussion we define two useful terms.

a) Phase Plane (or space)

Equation (3) which represents the mathematical model can be reduced from a second order system to an equivalent first order system by introducing a new variable \dot{q}_4 as the velocity component of \dot{q}_4

$$\begin{aligned}\dot{q}_4 &= \dot{q}_4 \\ \dot{q}_4 &= -\frac{P(q_4)}{I_4} + \frac{R(q_4)}{I_4} + \frac{F + f_m \cos(\Omega t - \epsilon)}{I_4}\end{aligned}$$

This pair of equations is called the phase plane equations of motion and the graph of the solutions versus is the "phase plane" trajectory of motion, see Figure 1. The motion of a point on this plane caused by varying the time as a parameter will yield prop-

erties concerning the stability of the system.

b) Motion Space

It is also possible to introduce a time axis perpendicular to the phase plane, i.e. taking the time as a state variable instead of a parameter and the space is then defined as a "motion space" which also is useful in dealing with stability of a system, see Figure 2.

Mathematical Treatment

The mathematical treatment of stability is dependent on the magnitudes of various items in the motion equation and can be analysed with the aid of three features which can be grouped under the general headings of

- (a) Characteristics of Equilibrium Positions,
- (b) Domain of attraction, and
- (c) Properties of Steady State Motion.

Thus, the main steps involved in dealing with the motion equation(3) can be illustrated in a flow diagram given in Figure 3. As can be seen, in the case of small amplitudes of motion and excitation the stability assessment will be provided by characteristics of the equilibrium position without the necessity of obtaining a solution to the motion equation. For large amplitudes of motion and excitation use will be made of the domain of attraction. The main advantage of this approach is based on the fact that the solution of the motion equation is again not needed but any purely time dependent excitation amplitude must be small, e.g., f_m should be small. For the general case of large amplitudes of motion and excitation it is necessary to transform the motion equation into its variational form with the aid of an approximate analytical solution. Then we can decide on the stability of motion from the properties of the variational equation.

It would thus be logical to follow the sequence of the three treatments so that the assessment of a system's stability can be properly related to the basic features of the motion equation.

a) Characteristics of the Equilibrium Positions

In classical mechanics a system is

regarded to be in an equilibrium position when the resultant forces and moments acting on that system is zero. This definition implies that the forces and moments applied should either be dissipative or can be derived from a potential. A decision on the stability of the system can be found by examining the stability of the equilibrium position which in turn is reached by studying the behaviour of the trajectories of the motion about this position in the phase plane. Thus, if a trajectory approaches the equilibrium point as time tends to infinity then we regard this trajectory as "stable" while a trajectory that moves away from the equilibrium position is "unstable", see Figure 4. However, if one of the trajectories about the equilibrium position is unstable then the equilibrium position becomes unstable. The theoretical treatment of Routh-Hurwitz can be used to determine stability of equilibrium position, see Reference (14). The coordinates of the equilibrium position can be found by equating the generalised velocities and to zero. It will be noted that for a linear system there is only one equilibrium position and as an illustrative example, Appendix 1 shows how the study of equilibrium point may be carried out. For non-linear systems there are more than one equilibrium point and the study of the behaviour of equilibrium positions becomes correspondingly more involved and can be complicated by the existence of the self-induced steady oscillation effect which gives rise to a "limit cycle", see Reference (14).

Thus, it is possible to determine the stability of equation (3) for small amplitudes of motion and excitation so long as we can ascertain the stability of the equilibrium position. In the case of small ship rolling motions, the conditions required for stable equilibrium position are a positive dissipation term and a positive initial metacentric height.

b) Domain of Attraction

Having established that the equilibrium position is stable it would be logical to attempt to extend this treatment out with the immediate location of the equilibrium position and find a region in the phase plane in which all the trajectories are stable. Such a region is called the "domain of attraction" and to make use of this property it is necessary to define a "stability boundary" for this domain. Thus, any motion starting from an interior point of this region and remain inside the domain of attraction would be stable while unstable trajectory

would go outside the stability boundary, see Figure 5.

The basic technique used to determine the region of stability is the Lyapunov's direct method, (15). The method is dependant on the possibility of formulating a function which possesses certain properties that can be used to test whether or not a system is stable. The major advantage of Lyapunov's method lies in its ability to provide information on the stability of motion without having to solve exactly the equations of motion.

The essential feature of the method comes from energy considerations and the decision about the stability of a conservative or dissipative system is made by examining the rate of change of energy along the trajectory motion. Appendix 2 provides a simple example on the application of the method using small amplitude rolling motions of an FOS. The Lyapunov's direct method generalises the energy argument by introducing energy-like functions which are known as Lyapunov functions. Since the method is actually based on energy considerations it may be thought of as an extension of Lagrange's studies on the stability of conservative systems which in traditional naval architecture is known as "Dynamic Stability".

The Lyapunov's direct method was proposed at the end of last century but its development and application is relatively new because of the difficulties in constructing the Lyapunov functions. However, during the last decade a number of studies have been devoted to developing techniques for generating the various forms of Lyapunov functions and as a result there exists today a number of techniques which are satisfactory and reliable as far as engineering applications are concerned. See for example, (16), (17). Some further information is provided in Appendix 3.

The only limitation of this treatment is that the magnitude of purely time dependent excitation term must be small as, for example, the amplitude of wave excitation should be small but there is no restriction on the magnitude of any other excitation.

c) Properties of Steady State Motion

In many practical problems an equilibrium position may not exist at all. As for example the motion of a ship in regular waves subjected to continuous wave excitation would not have an

equilibrium position. However, under certain assumptions it is possible to reduce the forced motion of a system to that for treating the problem of an equilibrium position. Thus, for a steady state motion we can introduce a moving coordinate system which would perform the same steady state motion as the system under study and by referring the actual motion to this moving coordinate system we can then consider the "perturbed" motion as motion around the equilibrium. Appendix 4 illustrates how this is achieved in practice.

The effect of this approach is that we must now have an approximate analytical solution to the motion equation. Although a very close approximation is desirable, in practice this is not always easy to achieve and from stability point of view it is not necessary to have a very close approximation. The approximate analytical solutions can be obtained chiefly with the aid of methods such as Bubnov-Galerkin (18) and Averaging (19) methods and we note that it is essential to have a range of techniques to serve as cross checks and to give greater credibility to the results. The basic equations are transformed into the variational form and by making use of the determined approximate solution the study of stability of steady state motion is thus reduced to the study of stability of the variational equation. The actual stability analysis can be carried out via the use of Floquet's theory (20) or Lyapunov's direct method.

This approach is not restricted by the magnitudes of motion amplitude or excitation but is much more involved due to the demand for a solution to the motion equation.

5. APPLICATIONS OF THEORY

For a theoretical method to have any chance of serving the needs of the practicing naval architect and marine technologist it is essential to relate the theory to practice. In the present case this entails examining the capsizing statistics, studying the way capsizing actually took place and identifying what mathematical tool is needed to deal with the various aspects of the problem. When an FOS capsizes it is the usual practice to consider first of all the roles played by wind and waves because they are closely associated with the causes of instability. Only these two aspects are considered in this paper but it will be remembered that there many other causes which can play active roles in the capsizing process and these are generally more dependent on the type of vehicle and the operating location. For

example, shifting of cargo, violent motions of liquid in the tanks, accumulation of greenwater on deck and effect of icing can all alter the stability characteristics of FOS or create additional forms of excitation as well as appearing as a combination of both effects. These effects can also be studied with the aid of the dynamic systems approach provided that their influences can correctly be formulated in the equation of motion. To obtain a physical picture of ship stability and gain hints on the mechanism of capsizing it is helpful to study the available time histories of the rolling motion (graph plots of roll amplitude against time). An examination of available records may yield certain patterns such as those shown in Figures 6 to 9 which eventually lead to the vessel capsizing. These capsizings can be grouped under the following headings:-

a) Non-Oscillatory Capsizing

When the restoring moment of a ship is very small in comparison with the excitation moment due to wind, waves and other causes, it is then possible to have this type of capsizing, Figure 6.

b) Oscillatory Sudden Capsizing

This is one of the most interesting types of capsizing because it can occur even when the restoration characteristics of a ship is satisfactory according to traditional understanding of stability, Figure 7. This is usually caused by successive series of waves.

c) Oscillatory Symmetric Build-Up Capsizing

The amplitudes of rolling motion increase rapidly and attain very large values after only a few cycles, Figure 8. The general picture of motion is very similar to that of the linear resonance with possible introduction of beat effect. Capsizing is likely to be due to the action of a series of successive waves.

d) Oscillatory Anti-Symmetric Build-Up Capsizing

In some cases the rolling motion appears to be anti-symmetric with respect to the axis of symmetry about the time axis, Figure 9. This phenomena is again due to a ship traveling through a series of waves and during successive oscillations the amplitudes of motion can attain such large values which make the recovery impossible.

To analyse the stability of these four types of capsizing it is possible to employ the mathematical techniques outlined in Section 4 and put forward the following methods of approach:-

- (a) The non-oscillatory capsizing may be due to an unstable equilibrium position and hence use would be made of the characteristics of equilibrium position.
- (b) A sudden oscillatory capsizing mass comes from exceeding stability boundary and hence use would be made of the domain of attraction.
- (c) A symmetric oscillatory build-up capsizing may be due to the system exceeding the stability boundary or instability due to resonance effects and hence both the domain of attraction and properties of steady state motion would be required to investigate its stability.
- (d) An anti-symmetric oscillatory build-up capsizing may be caused by instability of rolling motion under the influence of some persistent perturbations and hence use would be made of properties of steady state.

6. EXAMPLES OF APPLICATION

In this section we consider some examples to illustrate the application of the developed techniques by studying the forced rolling motion of box-shaped vessels. The form of the equation of motion is as follows:-

$$I_4 \ddot{\theta}_4 + d_1 \dot{\theta}_4 + d_2 \theta_4 |\dot{\theta}_4| + (k_1 \theta_4 + k_2 \theta_4 |\theta_4| + k_3 \theta_4^3 + k_4 \theta_4^2 |\theta_4| + k_5 \theta_4^5) \Delta = WM + f_m \cos t$$

where WM = wind heeling moment

The stability treatment of equation is dependent on the values of mass moment of inertia, damping, restoring characteristics, wind heeling moment and excitation. By choosing three different numerical values of the wind heeling moment while keeping other parameters constant it is possible to show clearly the application of the procedure as outlined in Figure 3. The numerical details and results are given in

Appendix 5, but the basic steps are:-

- (a) Determine the system parameters.
- (b) Calculate the new equilibrium position for the linear case allowing for the wind moment and examine the stability.
- (c) Calculate the new equilibrium position for the non-linear case allowing for the wind moment and examine the stability.
- (d) Determine the Lyapunov function and check whether the trajectory is within the domain of attraction.
- (e) Determine the steady state solution and examine the stability of the solution.

As each of these incidents may equally occur during the operation of a FOS under the varying actions of sea and wind, any application of the dynamic systems approach must go through all the stages given in Figure 3 to serve as cross checks although this may not be necessary in some cases as the stability can explicitly be determined by one of the techniques.

7. COMPARISON OF KEY FEATURES

The success of any method of calculation depends to a large extent on the way in which it can incorporate experience and for this reason the traditional method has been so popular. However, it is useful to make a comparison between the traditional and dynamic systems approaches in the light of the present considerations. Thus, the strong and weak points are highlighted in Table 1. It is perhaps not too difficult to arrive at a conclusion from the seventeen features that the traditional method has a role to play but there is a need to apply and to devote further efforts to the dynamic systems approach because of its overall potential.

8. PROBLEMS IN PRACTICAL APPLICATION

The concept of determining the stability of a system via the motion equation can be readily appreciated by both the designers and operators but the same cannot be said of the theory of stability nor the computational details. Indeed, before the dynamic systems approach can be applied in practice it must overcome a number of serious handicaps and the more important ones are

outlined here together with suggestions regarding how they could be overcome:-

a) Link to Design

The findings of the method must be linked to design so that FOS may include improved stability features. This can be done by:-

- 1) Incorporating the stability assessment as part of the overall computer aided ship design procedure.
- 2) Providing design charts to illustrate the trends from typical geometrical shapes in selected sea states.

b) Practical Experience

The approach must attempt to incorporate practical experience gained by operators, particularly those operating trawlers and small vessels. This task can be done by:-

- 1) Seeking the cooperation of the operators in providing information on circumstances and conditions when the vessels are regarded as having insufficient stability.
- 2) Devising methods for processing the practical information to serve as comparison with theory.

c) Training in Understanding and Usage

The advanced theories used in assessing stability requires the user to possess not only naval architectural knowledge but also adequate mathematical, statistical and hydronamic skills as well as the ability to think dynamically of non-stationary systems. To provide an appreciation of the subject, it is essential:-

- 1) To devise concise courses to introduce the concepts of dynamic systems approach.
- 2) To use small scale physical models in an experiment tank to illustrate the application of the theory.

9. CONCLUDING REMARKS

Based on the work outlined in the paper, the following concluding remarks can be made:-

- 1) Traditional approach has a role in ship stability studies but

there is a need for other methods which can fully incorporate the motion effects.

- 2) Under certain basic assumptions it is possible to reduce the problem of FOS stability to the study of stability of the equations of motion. There are three possible approaches to deal with stability of a system depending on the magnitude of motion amplitude and excitation.
- 3) The basic concepts of dynamic systems approach are given and its great potential has been demonstrated with the aid of examples.
- 4) It is necessary to test the dynamic systems approach in practice in order to gain experience in its application.
- 5) Further work is needed to incorporate the effects coupling and stochastic excitation in the dynamic systems approach.
- 6) Procedures developed are not restricted to a particular type of motion equation.
- 7) An appreciation of the theoretical background is desirable but the practical application may not require specialised knowledge.

10. ACKNOWLEDGEMENTS

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APPENDIX 1.STABILITY OF EQUILIBRIUM POSITION

The main purpose of this appendix is to illustrate how the stability of the equilibrium position can be assessed and in order to signify the treatment a linear system is selected.

For a motion equation of the form

$$\ddot{q}_4 + 2\delta\omega\dot{q}_4 + \omega^2 q_4 = 0$$

where

$$\begin{aligned} \delta &= \text{damping coefficient} \\ \omega &= \text{natural frequency} \end{aligned}$$

the phase plane representation is

$$\begin{aligned} \dot{q}_4 &= q_{10} \\ \dot{q}_{10} &= -\omega^2 q_4 - 2\delta\omega q_{10} \end{aligned} \quad (\text{A1.1})$$

In a general form A1.1 can be written as follows:

$$\begin{aligned} \dot{q}_4 &= l_1 q_4 + l_2 q_{10} \\ \dot{q}_{10} &= l_3 q_4 + l_4 q_{10} \end{aligned} \quad (\text{A1.2})$$

where l_1, l_2, l_3 and l_4 are constants.

The equilibrium positions are found by letting the right hand side of (A1.2) equal to zero. This leads to $q_4 = 0$ and $q_{10} = 0$ provided $l_1 - l_2 l_3 \neq 0$. Using the values of l_1, l_2, l_3 and l_4 we can deduce the stability of the system. The simple way of achieving this is to seek a solution to equation (A1.2) as follows:

Let $q_4 = A e^{\lambda t}$, $q_{10} = B e^{\lambda t}$ and substitute in (A1.2) yields

$$\begin{aligned} (l_1 - \lambda)A + l_2 B &= 0 \\ l_3 A + (l_4 - \lambda)B &= 0 \end{aligned} \quad (\text{A1.3})$$

For a non-trivial solution, we have

$$\begin{vmatrix} (l_1 - \lambda) & l_2 \\ l_3 & (l_4 - \lambda) \end{vmatrix} = 0$$

This leads to λ_1 and λ_2 being evaluated from the relation:

$$\lambda_{1,2} = \frac{(l_1 + l_4)}{2} \mp \sqrt{(l_1 - l_4)^2 + 4l_2 l_3}$$

which yields also the properties:

$$(\lambda_1 + \lambda_2) = l_1 + l_4$$

$$\lambda_1 \lambda_2 = l_1 l_4 - l_2 l_3$$

(A1.4)

Thus depending on the sign of the real parts of λ_1 and λ_2 the stability of the equilibrium positions can be determined. These features are illustrated in the following Table 2.

APPENDIX 2USE OF ENERGY CONSIDERATIONS FOR ASSESSING STABILITY

A simple example is employed to illustrate the application of energy considerations for assessing the stability of a system. Let the equation of motion be

$$I_4 \ddot{q}_4 + P(q_4, \dot{q}_4) \dot{q}_4 + R(q_4) q_4 = 0 \quad (\text{A2.1})$$

where $P(q_4, \dot{q}_4)$ = a general dissipation

$R(q_4)$ = restoring term

Expressing (A2.1) in phase plane, we have

$$\begin{aligned} \dot{q}_4 &= q_{10} \\ \dot{q}_{10} &= -\frac{P(q_4, \dot{q}_4)}{I_4} - \frac{R}{I_4} q_4 \end{aligned} \quad (\text{A2.1})$$

The energy of the system is

$$E = \frac{1}{2} I_4 \dot{q}_4^2 + \frac{1}{2} R(q_4) q_4^2$$

and rate of exchange of energy is

$$\frac{dE}{dt} = I_4 q_{10} \dot{q}_{10} + R q_4 q_{10} \quad (\text{A2.3})$$

On substituting (A2.2) into (A2.3) yield

$$\begin{aligned} \frac{dE}{dt} &= I_4 q_{10} \left[-\frac{P(q_4, q_{10})}{I_4} q_{10} - \frac{R}{I_4} q_4 \right] + R q_4 q_{10} \\ &= -P(q_4, q_{10}) q_{10}^2 \end{aligned}$$

Thus the rate of change of energy is dependent upon the sign of

$$P(q_4, q_{10})$$

since q_{10}^2 will always be positive.

If $P(q_4, q_{10})$

is positive for some region of the phase plane then the energy decreases along all solutions except the zero solution and as time tends to infinity the trajectory tends to the equilibrium position. For different signs of $P(q_4, q_{10})$

the stable and unstable systems are given in Figure (10).

APPENDIX 3

LYAPUNOV'S DIRECT METHOD AND
LYAPUNOV FUNCTIONS

We have already seen, in Appendix 2, how one can obtain some information about the stability of a system from the energy considerations. However, in practice, it is often very difficult to derive an expression for the energy as the motion is generally defined by a nonlinear and nonstationary system of differential equations. To overcome this difficulty Lyapunov's direct method introduces an energy-like function, called, "Lyapunov function", to provide information on the stability of the motion by making systematic use of these functions. The main contribution of the method consists of certain theorems giving sufficient conditions for stability and instability in terms of suitable Lyapunov functions.

For a single-degree-of-freedom system it is possible to attach a geometrical interpretation to the Lyapunov function. Consider the phase plane equations of motion of a system as $\dot{q}_4 = q_{10}$; $\dot{q}_{10} = f(q_4, q_{10})$ and the corresponding Lyapunov function $V(q_4, q_{10})$. When we plot the values of $V(q_4, q_{10})$ for different values of q_4 and q_{10} we obtain a surface, see Fig. . So long as the intersections of this surface and the planes $V = \text{constant}$ are closed and the trajectory of motion remains within the outermost closed boundary, the motion is said to be stable.

The domain enclosed by this outermost boundary is called the domain of attraction. Depending on the relative positions of the domain of attraction and the phase trajectories, we may encounter three different situations, see Fig. (10).

(a) The motion trajectory remains bounded and within the domain of attraction. Then we call this motion stable.

(b) The motion trajectory remains bounded and within the domain of attraction and approaches the equilibrium position as the time progresses. Then we call this motion asymptotically stable.

(c) The motion trajectory leaves the domain of attraction and this motion is called unstable.

Thus the stability of motion can be investigated if we can determine the domain of attraction and examine the behaviour of the phase trajectory with respect to domain of attraction. As the domain of attraction is bounded by a closed curve or surface, we must therefore determine the shape of this envelope.

The concept of closedness in geometry is intimately allied with the concept of sign definiteness in analytical functions. A scalar function $V(q_1, q_2, \dots, q_n)$ is called "positive definite" if $V(0, 0, \dots, 0) = 0$ and $V(q_1, q_2, \dots, q_n) > 0$ for $(q_1, q_2, \dots, q_n) \neq 0$. Similarly negative definiteness implies $V(0, 0, \dots, 0) = 0$ and $V(q_1, q_2, \dots, q_n) < 0$ for $(q_1, q_2, \dots, q_n) \neq 0$. If $V(0, 0, \dots, 0) = 0$ and $V(q_1, q_2, \dots, q_n) \geq 0$ (or ≤ 0) for $(q_1, q_2, \dots, q_n) \neq 0$ the function is termed "positive (or negative) semi-definite".

If $V(q_1, q_2, \dots, q_n)$ is a positive definite function, then $V = K$ defines a closed curve or surface about the origin and by varying the constant K we obtain a set of nested closed curves or surfaces. So long as the function $V(q_1, q_2, \dots, q_n)$ is positive definite these curves or surfaces remain closed.

If we can determine a positive definite function which is associated with the equations of motion, then the condition $V = 0$ yields the stability boundary of the domain of attraction. We call this function a Lyapunov function.

On the other hand if we take the substantial time derivative of the function $V(q_1, q_2, \dots, q_n)$ along the trajectories of motion, i.e.

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + \sum_{i=1}^n \left(\frac{\partial V}{\partial q_i} \right) \dot{q}_i$$

this derivative shows the direction of intersection between the trajectories and $V = \text{constant}$ curves. In order to have a stable trajectory the direction of crossing must be inwards, thus implying DV/Dt should be negative. Consequently the condition of stability is the negative definiteness of the time derivative of the Lyapunov function taken along the motion trajectory. These two conditions are the sufficient conditions for stability.

Results of the Lyapunov's direct method are given in the Lyapunov Stability Theorem which can be stated as follows:

"If the differential equations of perturbed motion are such that it is possible to find a positive definite

function $V(q_1, q_2, \dots, q_n, t)$ for which the substantial derivative DV/Dt taken along the motion trajectory is negative definite, then the unperturbed motion is stable."

It can be shown that in the case of stationary motions, Lyapunov function is equivalent to the dynamical stability of Moseley.

The principles can be illustrated with a simple example. Suppose the equations of rolling motion in the phase plane are given by

$$\dot{q}_4 = q_{10}; \quad \dot{q}_{10} = -\omega_n^2 q_4 - 2\gamma \omega_n q_{10} + k q_4^3$$

Taking Lyapunov function as

$$V(q_4, \dot{q}_4) = \frac{1}{2} (\omega_n^2 q_4^2 + \dot{q}_4^2)$$

we guarantee the positive definiteness condition provided ω_n^2 is positive. The substantial time derivative of the Lyapunov function is

$$\frac{DV}{Dt} = \omega_n^2 q_4 \dot{q}_4 + \dot{q}_4 \ddot{q}_4 = \omega_n^2 q_4 q_{10} + q_{10} (-\omega_n^2 q_4 - 2\gamma \omega_n q_{10} + k q_4^3) = k q_{10} q_4^3 - 2\gamma \omega_n q_{10}^2$$

Thus the second condition requires that

$$k q_{10} q_4^3 - 2\gamma \omega_n q_{10}^2 < 0$$

APPENDIX 4

STABILITY OF STEADY STATE MOTION

In this appendix we shall show how the stability analysis of a steady state motion can be reduced to the stability analysis of an equilibrium position.

Let the equations of motion be

$$\ddot{q}_i = f_i(q_1, q_2, \dots, q_n, t), \quad (i=1, 2, \dots, n) \quad (A4.1)$$

Assume that a steady state solution exists and is the vector $\tilde{\varphi}(t)$ with the components $\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t)$. The real motion will deviate from the steady state motion and we shall denote the components of real motion as $\varphi_1(t) + \xi_1(t), \varphi_2(t) + \xi_2(t), \dots, \varphi_n(t) + \xi_n(t)$. Substitution of $\varphi_i(t) + \xi_i(t)$ into the equations of motion yield the following set of equations:

$$\ddot{\varphi}_i + \ddot{\xi}_i = f_i(\varphi_1, \varphi_2, \dots, \varphi_n, t) + \sum_{k=1}^n \left[\left(\frac{\partial f_i}{\partial q_k} \right)_{q_k = \varphi_k(t)} \xi_k + o\left(\frac{\xi_k}{\varphi_k}\right) \right]$$

and since $\tilde{\varphi}(t)$ is a solution of Eq. (A4.1) we obtain

$$\ddot{\xi}_i = \sum_{k=1}^n \left[\left(\frac{\partial f_i}{\partial q_k} \right)_{q_k = \varphi_k(t)} \xi_k + o\left(\frac{\xi_k}{\varphi_k}\right) \right], \quad (i=1, 2, \dots, n) \quad (A4.2)$$

where $()_{q_k = \varphi_k(t)}$ indicates that the term inside the bracket is evaluated at $q_k = \varphi_k(t)$ and $[o(\xi_k)/\xi_k] \rightarrow 0$ as $\xi_k \rightarrow 0$.

Eqs. (A4.2) are the equations of motion with respect to a moving coordinate system which makes the same steady state motion as the system and are called the perturbed equations of motion. As the steady state solution is known to be bounded so long as the perturbation $\xi(t)$ also remains bounded and stable then the motion is stable. Thus the stability analysis of the steady state motion is reduced to the stability analysis of Eqs. (A4.2). In many cases the linear part of Eqs. (A4.2) gives sufficient information about the stability of motion and the linear differential equation

$$\ddot{\xi}_i = \sum_{k=1}^n \left(\frac{\partial f_i}{\partial q_k} \right)_{q_k = \varphi_k(t)} \xi_k$$

is called the variational equations of motion.

Let us illustrate now the application with simple example. Consider the undamped rolling motion of a ship and the equation of motion is given by

$$\ddot{q}_4 + \omega_n^2 q_4 - k q_4^3 = \int_{-t}^t \cos \sigma t \quad (A4.3)$$

Furthermore we assume that

$$\varphi_4(t) = \alpha \cos \sigma t$$

is an approximate solution of Eq. (A4.3). The variational equation of motion is

$$\ddot{\xi} + [\omega_n^2 - 3k\varphi_4^2] \xi = 0$$

or

$$\ddot{\xi} + [\omega_n^2 - 3k\alpha^2 \cos^2 \sigma t] \xi = 0 \quad (A4.4)$$

As

$$\cos^2 \sigma t = \frac{1 + \cos 2\sigma t}{2}$$

Eq. (A4.4) becomes

$$\ddot{\xi} + \left[\left(\omega_n^2 - \frac{3}{2} k \alpha^2 \right) - \frac{3}{2} k \alpha^2 \cos 2\sigma t \right] \xi = 0$$

This is the well-known Mathieu equation for which the stability analysis is available.

APPENDIX 5

WORKED EXAMPLES

In this appendix three worked examples are given to study the stability of the rolling motion of a box vessel under the influence of wind and wave actions. The equation of motion takes the form of

$$I_4 \ddot{q}_4 + d_1 \dot{q}_4 + d_2 q_4 |\dot{q}_4| + (k_1 q_4 + k_2 q_4 |q_4| + k_3 q_4^3 + k_4 q_4^3 |q_4| + k_5 q_4^5) \Delta = WM + f_m \cos \sigma t$$

The vessel's particulars are:

Length	=	60.00 m
Breadth	=	12.00 m
Depth	=	8.00 m
Draft	=	6.67 m
VCG	=	4.40 m
GM	=	0.73 m
Displacement	=	4920.00 tonnes

Parameters occurring in the equation of motion are found to be:

$$\begin{aligned} I_4 &= 12739.64 \text{ tonne m} \cdot \text{sec}^2 \\ d_1 &= 0.0149 \text{ sec}^{-1} \\ d_2 &= 0.0225 \\ k_1 &= 0.733 \text{ m} \\ k_2 &= 2.795 \text{ m} \\ k_3 &= -14.922 \text{ m} \\ k_4 &= 19.431 \text{ m} \\ k_5 &= -8.242 \text{ m} \\ f_m &= 200.00 \text{ tonne m} \\ \sigma &= 1.00 \text{ sec}^{-1} \end{aligned}$$

Important characteristics of the intact $GZ(q_4)$ curve are:

$$\begin{aligned} GZ_{\max} &= 0.208 \text{ m} \\ (q_4)_{\max} &= 0.332 \text{ radian} \\ (q_4)_{\text{vanish}} &= 0.788 \text{ radian} \\ \int_{(q_4)_{\text{vanish}}}^{(q_4)_{\max}} GZ dq_4 &= 0.097 \text{ metre radian} \end{aligned}$$

The stability of the motion is examined for three different wind heeling moments while other parameters are kept constant.

EXAMPLE 1

$$WM = 316.80 \text{ tonne m}$$

(1) Equilibrium position for the linear case

Coordinates of the equilibrium position are

$$\begin{aligned} q_4 &= 0.09 (5.1^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

$$GM (= 0.73 \text{ m}) > 0$$

and

$$d_1/I_4 (= 0.0149 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

(2) Equilibrium position for the nonlinear case

Coordinates of the equilibrium position are

$$\begin{aligned} q_4 &= 0.07 (4.3^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

$$GM (= 0.92 \text{ m}) > 0$$

and

$$d_1/I_4 (= 0.0149 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

(3) Establishing domain of attractionLyapunov function

$$V(q_4, \dot{q}_4) = h_2 \dot{q}_4^2 + h_3 \dot{q}_4^3 + h_4 \dot{q}_4^4 + h_5 \dot{q}_4^5 + h_6 \dot{q}_4^6 + 0.5 \dot{q}_4^2$$

Values of the coefficients for $q_4 \geq 0$

$$\begin{aligned} h_2 &= 0.4661 \\ h_3 &= 0.0261 \\ h_4 &= -2.4003 \\ h_5 &= 3.2737 \\ h_6 &= -1.3736 \end{aligned}$$

Values of the coefficients for $q_4 \leq 0$

$$\begin{aligned} h_2 &= 0.0187 \\ h_3 &= -2.2667 \\ h_4 &= -5.2886 \\ h_5 &= -4.4987 \\ h_6 &= -1.3736 \end{aligned}$$

Checking the Stability of Motion yields

Maximum possible
half roll = $0.1 (5.7^\circ)$

Lyapunov function
value (=0.000277) > 0

Stability: Motion is stable in the sense of Lyapunov.

(4) Steady State Motion

Approximate solution (Method of Averaging)

$$q_4 = 0.0745 - 0.0245 \cos \tau t + 0.006 \sin \tau t$$

Stability of steady state motion in the sense of Floquet yields Coefficients of Hill's equation in Table 3.

Stability: Steady state motion is stable.

EXAMPLE 2

WM = 668.80 ton m

(1) Equilibrium position for the linear case

Coordinates of the equilibrium position are

$$\begin{aligned} q_4 &= .15 (10.6^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

GM (= 0.73 m) > 0

$$d_1/I_4 (=0.019 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

(2) Equilibrium position for the nonlinear case

Coordinates of the equilibrium position are

$$\begin{aligned} q_4 &= 0.15 (8.90^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

GM (= 0.79 m) > 0

$$d_1/I_4 (=0.0149 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

(3) Establishing domain of attraction

Lyapunov function

$$V(q_4, \dot{q}_4) = h_2 \dot{q}_4^2 + h_3 \dot{q}_4^3 + h_4 \dot{q}_4^4 + h_5 \dot{q}_4^5 + h_6 \dot{q}_4^6 + 0.5 \dot{q}_4^2$$

Values of the coefficients for $q_4 > 0$

$$h_2 = 0.3946$$

$$h_3 = -0.5512$$

$$h_4 = -1.2105$$

$$h_5 = 2.6066$$

$$h_6 = -1.3736$$

Values of the coefficients for $q_4 < 0$

$$h_2 = -0.7643$$

$$h_3 = -4.2882$$

$$h_4 = -7.2441$$

$$h_5 = -5.1658$$

$$h_6 = -1.3736$$

Check the Stability of motion yields

Maximum possible
half roll = $.18 (10.2^\circ)$

Lyapunov function
value (=0.000194) > 0

Stability: Motion is stable in the sense of Lyapunov.

(4) Steady State Motion

Approximate solution (Method of Averaging)

$$q_4 = 0.1556 - 0.0225 \cos \tau t + 0.0005 \sin \tau t$$

Stability of steady state motion in the sense of Floquet yields Coefficients of Hill's equation as in Table 4.

Stability: Steady state motion is stable.

EXAMPLE 3

WM = 844.80 tonne m

(1) Equilibrium position for the linear case

Coordinates of the equilibrium position are

$$\begin{aligned} q_4 &= .24 (13.4^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

GM (= 0.73 m) > 0

$$d_1/I_4 (=0.0149 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

(2) Equilibrium position for the nonlinear case

Coordinates of the equilibrium are

$$\begin{aligned} q_4 &= .21(11.9^\circ) \\ \dot{q}_4 &= 0 \end{aligned}$$

$$GM (=0.59 \text{ m}) > 0$$

$$d_1 / I_{\dot{q}} (= 0.0149 \text{ sec}^{-1}) > 0$$

Stability: Equilibrium position is stable according to Routh-Hurewitz criterion.

Stability of steady state motion in the sense of Floquet yields Coefficients of Hill's equation as in Table 5.

Stability: Steady state motion is stable.

(3) Establishing domain of attraction

Lyapunov function

$$V(q_4, \dot{q}_4) = h_2 \dot{q}_4^2 + h_3 \dot{q}_4^3 + h_4 \dot{q}_4^4 + h_5 \dot{q}_4^5 + h_6 \dot{q}_4^6 + 0.5 \dot{q}_4^2$$

Values of the coefficients for $q_4 \geq 0$

$$h_2 = 0.2932$$

$$h_3 = -0.7355$$

$$h_4 = -0.5922$$

$$h_5 = 2.1809$$

$$h_6 = -1.3736$$

Values of the coefficients for $q_4 \leq 0$

$$h_2 = -1.5521$$

$$h_3 = -5.9266$$

$$h_4 = -8.6333$$

$$h_5 = -5.5915$$

$$h_6 = -1.3736$$

Checking the Stability of motion in the sense of Lyapunov yields

Maximum possible half roll = 13.0 degrees

Lyapunov function value (=0.000114) > 0

Stability: Motion is stable in the sense of Lyapunov.

(4) Steady State Motion

Approximate solution (Method of Averaging)

$$q_4 = 0.2073 - 0.0203 \cos t + 0.00045 \sin t$$

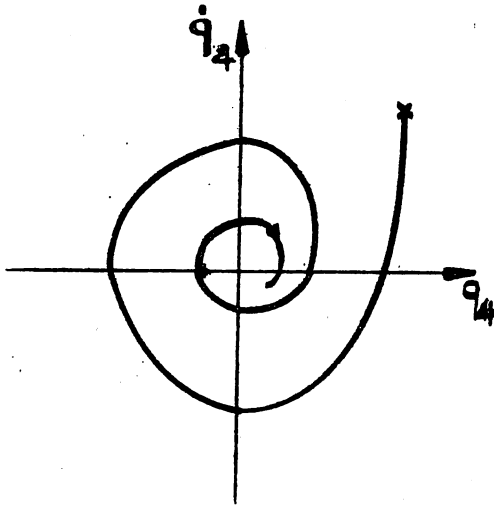


Figure 1

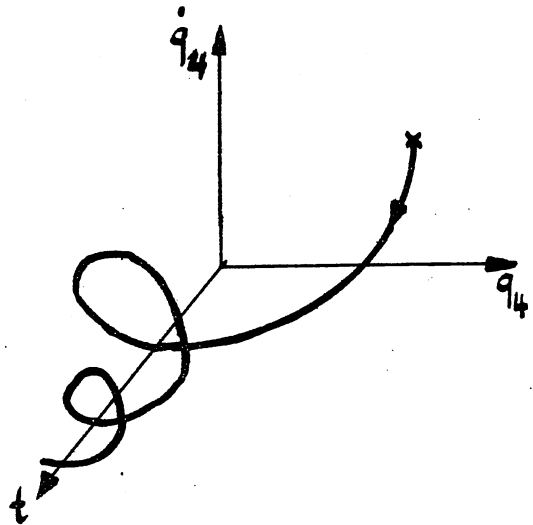


Figure 2

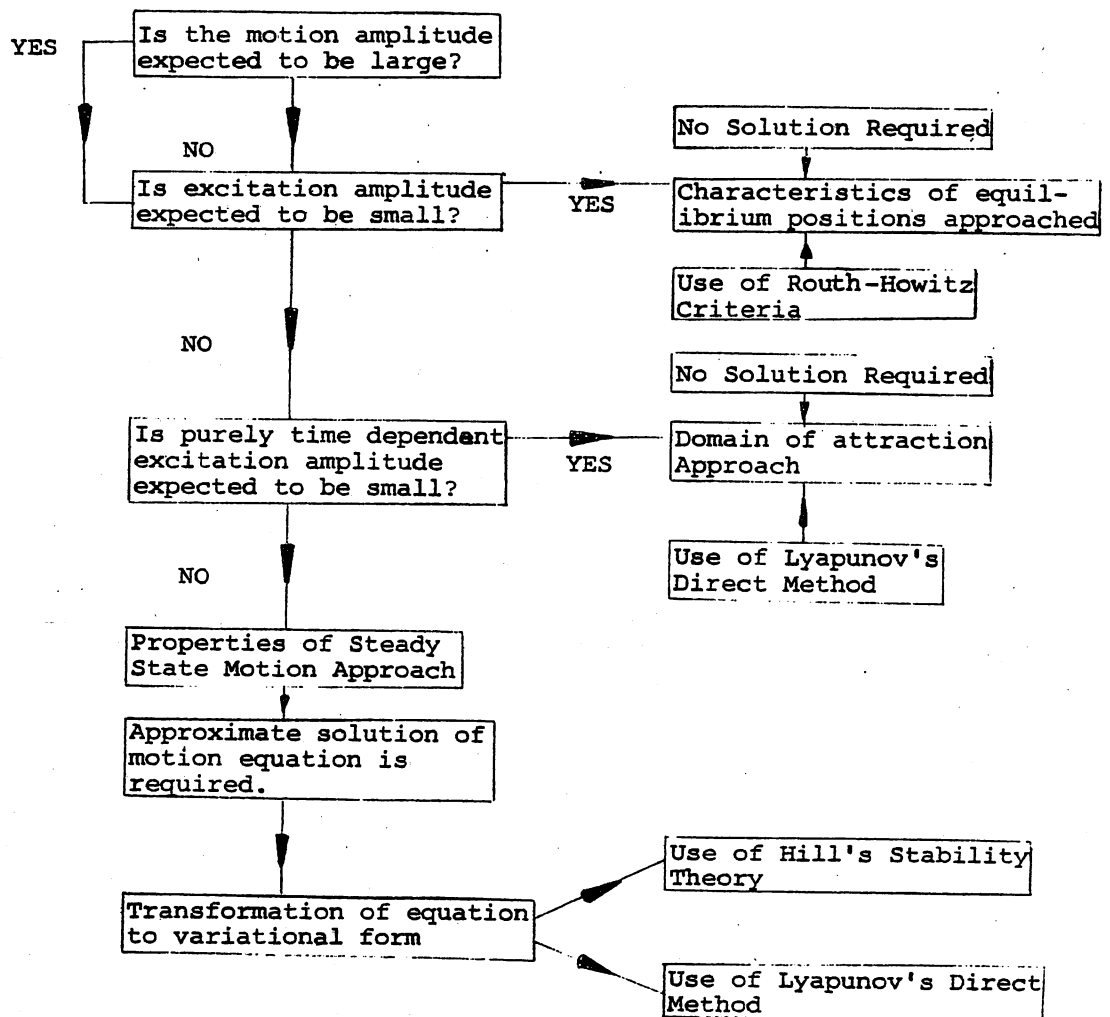


Figure 3

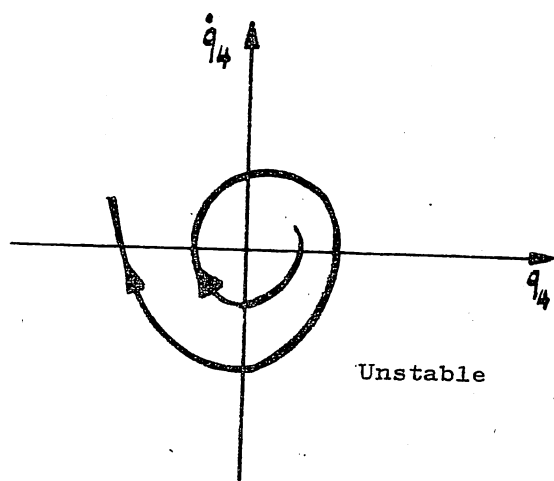
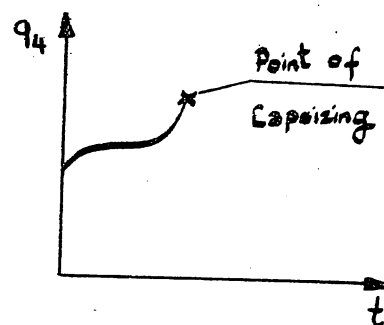
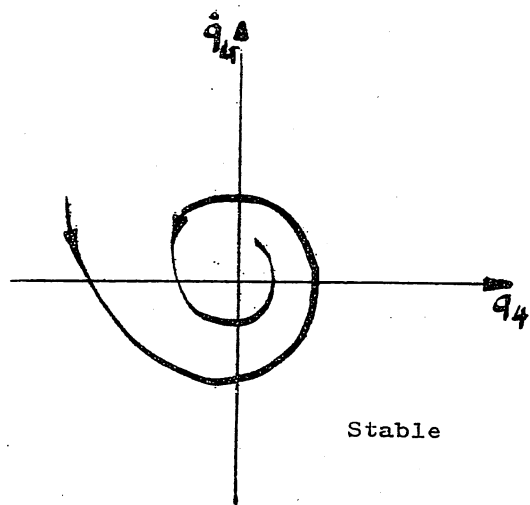


Figure 6

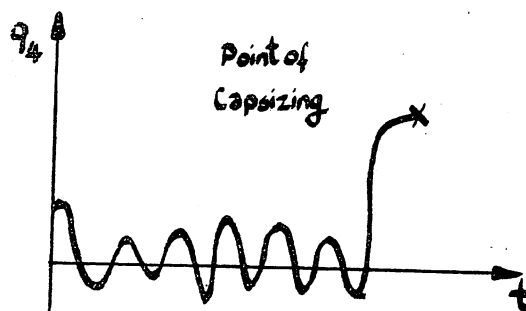


Figure 4

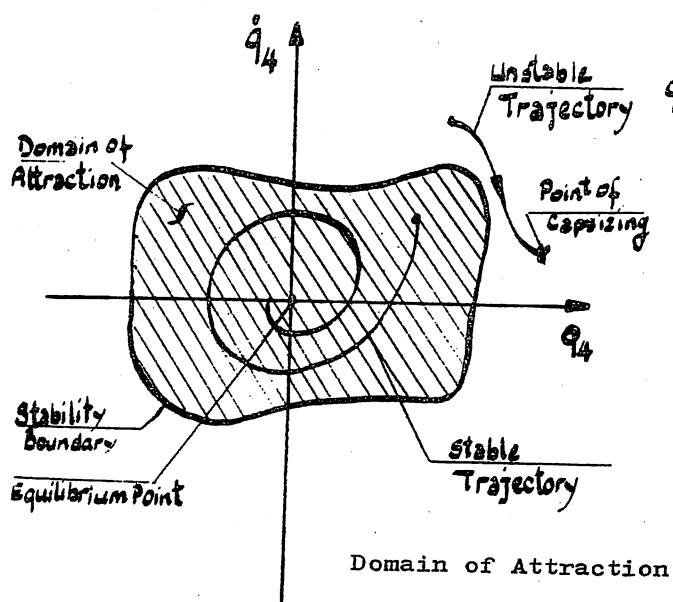


Figure 5

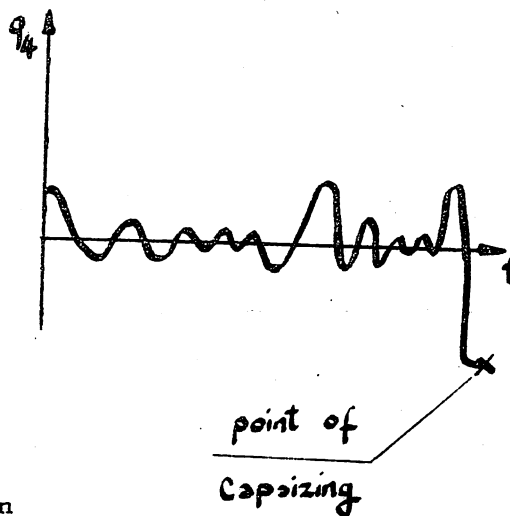


Figure 7

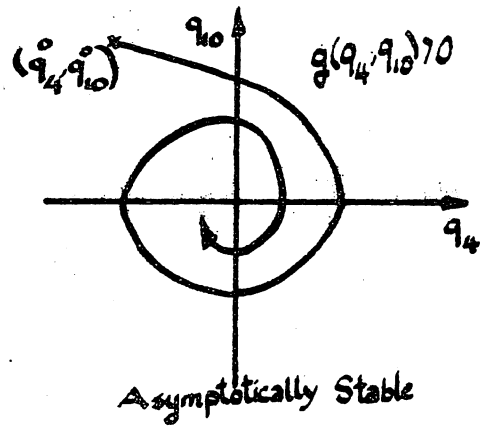
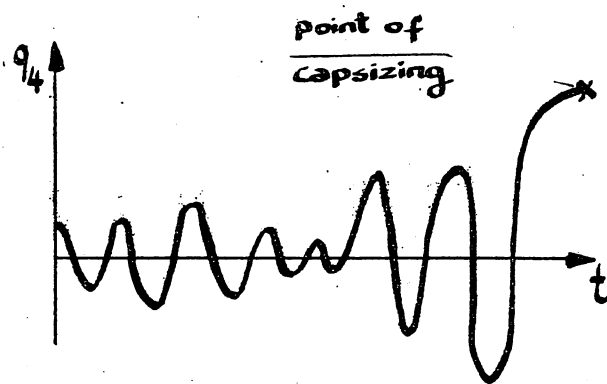


Figure 8

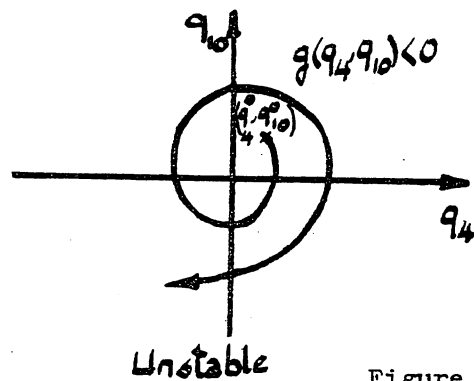
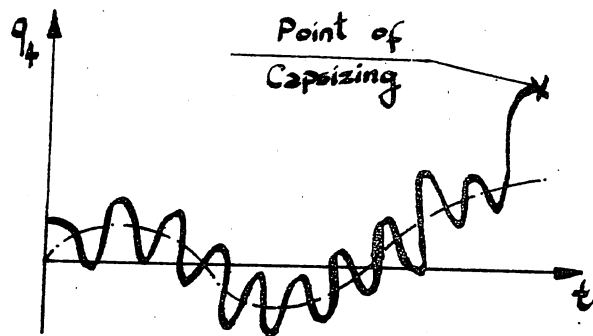
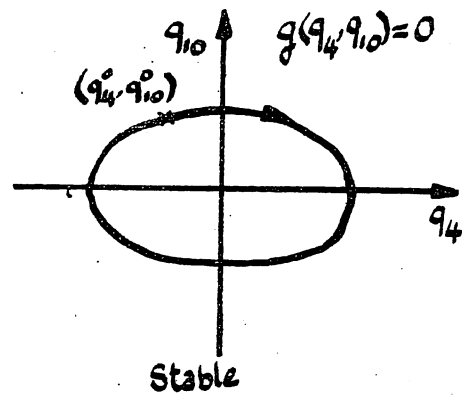
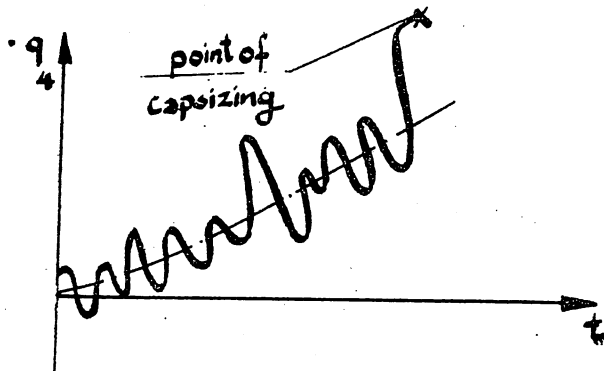


Figure 9

Figure 10

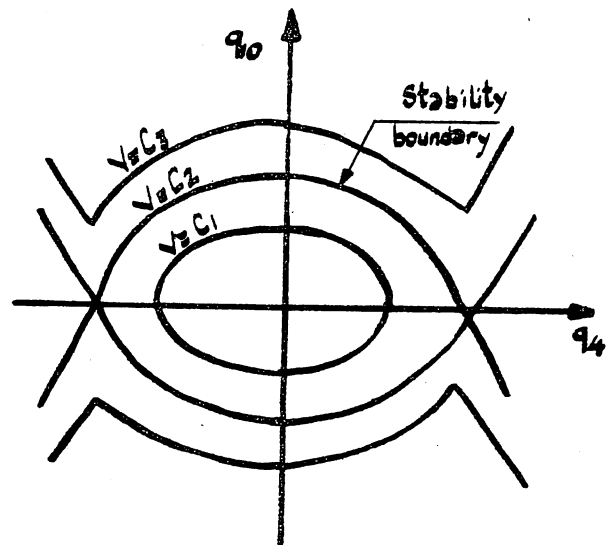
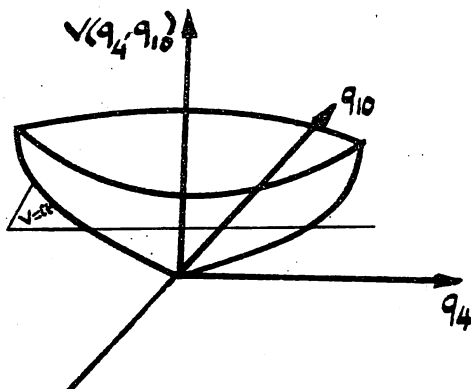


Figure 11

TABLE 1 A COMPARISON OF TRADITIONAL AND DYNAMIC SYSTEMS APPROACHES

NO.	FEATURE	TRADITIONAL APPROACH	DYNAMIC SYSTEMS APPROACH
1.	What is the basis of the method?	Use is made of the theory of statics and dynamics of conservative systems.	Use is made of equations of motion and stability theory and in particular Lyapunov's direct method.
2.	How is the stability being treated?	Examination of the properties of the righting arm against angle of inclination to deduce stability features of the vessel.	Examining the stability of the motion equations so as to deduce whether the system representing the motion of FOS is stable.
3.	Is the modelling realistic?	Insufficiently realistic.	More realistic.
4.	Is there potential for representing more accurately the physical phenomenon by the approach?	Very limited and is done via the use of empirical factors.	Yes. By use of improved mathematical models of the physical phenomenon.
5.	Is the basic concept easy to understand?	Yes. Both the region of validity and limitation concerning applicability to FOS in seaways are also fairly obvious.	The concept is simple and logical but an appreciation of how such an approach can be used in practice to assess stability of FOS is much harder to understand.
6.	How much theoretical knowledge is needed to use the approach?	Classical naval architecture.	By using a computer program to do the calculation it is strictly possible to obtain results without any theoretical knowledge. However, this is highly undesirable and an appreciation of approach is essential if good use is to be made of it. Knowledge of naval architecture, mathematics and stability theory are useful.
7.	What kind of training is needed to understand the theory behind the approach by the designers?	Conventional naval architecture training.	Conventional naval architecture plus mathematical skill in treating the equations of motion as well as understanding the physical nature of the problem.

<u>NO.</u>	<u>FEATURE</u>	<u>TRADITIONAL APPROACH</u>	<u>DYNAMIC SYSTEMS APPROACH</u>
8.	What is the amount of computation involved in its application?	Relatively small. Manual methods are sufficient.	It is essential to carry out the calculations by the computer. But it may be possible to devise simple ways of using the results in practice.
9.	Can the effects of external loads be incorporated in the treatment?	Yes, on their average values, but their time variations can not be introduced.	Yes, external loads can be incorporated in the equation of motion.
10.	Can the effects of non-deterministic excitation be incorporated?	Yes. As in 9.	Yes. The incorporation and solution needing further studies for FOS application.
11.	Can the motion characteristics be incorporated?	No.	Yes.
12.	Can non-linear effects be incorporated?	Yes, only for the restoring term.	Yes. The treatment is more involved.
13.	Can the method cope with more than one degree of freedom with coupling effects incorporated?	No.	Yes. However, the problem becomes more complicated.
14.	Is there any relation between the two approaches?	Yes. (see Dynamic Systems Approach).	Yes. Traditional approach is a very special case of Lyapunov's method of treating stability.
15.	Can the approach incorporate feedback from practice?	Yes. Statistical analysis of casualties plays a major part in refining the criteria.	Not yet being used in practice but there is no reason why feedbacks cannot be incorporated.
16.	Can stability from capsizing be guaranteed if basic conditions are met?	No.	No. Stability from capsizing is assessed from probability of risks under given operating conditions and sea states.
17.	What is the future role of the approach?	This approach will have a role to play but until a better criteria is devised it would be less attractive as a method for use in the stability analysis of a wider class of FOS.	There is still a lot of development work to be done but it offers an essential way of incorporating stability with the motion characteristics of FOS.

Type of equili- brium points	Principal Conditions	Stability Condition	
		Stable	Unstable
Node	$(l_1 - l_4)^2 + 4l_2l_3 > 0$ $l_1l_4 - l_2l_3 > 0$	$l_1 + l_4 < 0$	$l_1 + l_4 > 0$
Saddle Point	$(l_1 - l_4)^2 + 4l_2l_3 > 0$ $l_1l_4 - l_2l_3 < 0$	—	Unstable
Focus	$(l_1 - l_4)^2 + 4l_2l_3 < 0$ $l_1 + l_4 \neq 0$	$l_1 + l_4 < 0$	$l_1 + l_4 > 0$
Centre	$(l_1 - l_4)^2 + 4l_2l_3 < 0$ $l_1 + l_4 = 0$	Stable	—

Table 2

TABLE 3

Coefficients of Hill's Equation

Constant coefficient = 0.356519

<u>n</u>	<u>a_n of (cos 2nt)</u>	<u>b_n of (sin 2nt)</u>	<u>$\sqrt{a_n^2 + b_n^2}$</u>	<u>Inequality</u>
2	-.003256	.000578	.003306	0.640175 > 0
4	.000057	-.000021	.000061	3.643420 > 0
6	.000020	-.000012	.000023	8.643458 > 0
8	.000000	.000000	.000000	15.643481 > 0
10	.000000	.000000	.000000	24.643481 > 0

TABLE 4

Coefficients of Hill's Equation

Constant coefficient = 0.303806

<u>n</u>	<u>a_n of (sin 2nt)</u>	<u>b_n of (sin 2nt)</u>	<u>$\sqrt{a_n^2 + b_n^2}$</u>	<u>Inequality</u>
2	-.001812	.000315	.001839	0.694355 > 0
4	-.000051	.000018	.000054	3.696140 > 0
6	-.000020	.000012	.000023	8.696171 > 0
8	.000000	.000000	.000000	15.696194 > 0
10	.000000	.000000	.000000	24.696194 > 0

TABLE 5

Coefficients of Hill's Equation

Constant coefficient = 0.226527

<u>n</u>	<u>a_n of (sin 2nt)</u>	<u>b_n of (sin 2nt)</u>	<u>$\sqrt{a_n^2 + b_n^2}$</u>	<u>Inequality</u>
2	-.001041	.000176	.001056	0.772417 > 0
4	-.000080	.000028	.000084	3.773389 > 0
6	-.000031	.000017	.000036	8.773438 > 0
8	.000000	.000000	.000000	15.773473 > 0
10	.000000	.000000	.000000	24.773473 > 0

AN APPROACH FOR TREATING THE STABILITY OF FISHING BOATS

by

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(Boat)

1. INTRODUCTION

Thousands of fishing boats ranging from trawlers to coastal fishing boats go out for fishing voyages every day in Japan. Most of the skippers are skilful at controlling their fishing boats, and have their own judgement for the safety of the boats via the use of empirical means although these judgments are sometimes wrong. The boats' hull form, construction, fishing method, and circumstances including the worst stormy weather expected to encounter during the fishing voyage are generally different for each group of fishing fleets. All the boats have wireless equipment to catch meteorological information to escape from the heavy storm. These are the basic circumstances to be considered when the stability criterion of fishing boats is discussed in Japan. A highly simplified criterion would not be suitable as most of the fishermen will not recognize its necessity because of the big gap between their empirical feeling and criterion. This is why a theoretical approach is necessary for assessing the stability of fishing boats in Japan.

2. ACCIDENTS TO BE CONSIDERED

This analysis is based on the concept that the stability of fishing boats should not be judged by a simple geometrical stability standard such as minimum metacentric height GM and freeboard or the shape of righting lever (GZ) curve, instead the following factors should be considered.

- a) The worst weather condition condition expected to be encountered,
- b) The maximum heeling moment caused by the fishing operations,

- c) The critical heeling angle θ -limit or θ -flood beyond which the righting moment will be lost.
- d) The bulwark height,
- e) The profile area on the water line,
- f) The roll-damping characteristics of the boat, etc. should also be considered for this purpose.

In this paper, three kinds of accidents are considered theoretically to be the main causes of capsize or lost accidents of fishing boats, and thus if a fishing boat has enough stability to prevent these accidents then the boat is considered safe. In order to investigate whether these accidents will occur or not, a list of "C" Values are introduced and the terms

are used to denote the three kinds of capsizings, and the boat is judged to be safe by this theory. (Refer Table 2)

The meaning of the "C" values are as follows:

(1) Coefficient C_2-l

If a gust blows from weather side of a boat at a moment when it rolls almost synchronously in the beam wind and irregular waves and heels hardly to the weather side, then the boat heels very hard to the lee side. This condition is thought to be one of the most dangerous conditions for the boat, and the boat is considered safe under this condition when the inequality

$C_2-l \equiv \text{Area } b / \text{Area } a > 1.0$
is satisfied, where a and b are areas relating to the GZ curve See Fig. (1)

(2) Coefficient C_{2-f}

In the above condition, if the critical flooded angle θ -flood (defined as the angle at which see-water begins to immerse into a vessel through its openings) is smaller than the critical angle

θ -limit, then Area b should be replaced by Area b' which is the area up to θ -flood instead of θ -limit, see in Fig. (1). In such case, if

$$C_{2-f} \equiv \text{Area } b' / \text{Area } a > 1.0$$

the boat is considered safe under this condition.

(3) Coefficient C_{1-0}

Some of the fishing boats in Japan were lost by flooded water or shifted cargoes when the boats heeled hard and could not get up easily on the rough sea. Those are thought to be quite different type of accidents from those explained by C_{2-1} or C_{2-f} values mentioned above.

This kind of accident is considered to be caused by the hard rolling motion of the boat in the irregular waves until the bulwark top immerses into the water, which introduces a lot of sea water on the deck. This phenomenon easily occurs before the occurrence of the accidents explained by C_{2-1} or C_{2-f} values especially when the free board of a fishing boat is very low or a boat rolls very hard in the waves. In this report, C_{1-0} is designated to investigate the stability of fishing boats in such case, and it is shown by the following equation:

$$C_{1-0} = \frac{2.(FB_{\min} + H_B)}{B \cdot \tan(\theta_0 + (KAH/W \cdot 6M) \cdot (180/\pi))}$$

where FB_{\min} : Freeboard at the lowest point of upper deck side line under actual loaded condition (m)

H_B : Bulwark height at the lowest point (m)

B : Boat's breadth (m)

W : Displacement (t)

KAH : Heeling moment caused by steady beam wind at the actual loaded condition (t-m)

(4) Coefficient C_{1-m}

If heeling moment M exists due to fishing operation or an unbalanced load, C_{1-m} should be used instead of C_{1-0} , which is designated by following equation:

$$C_{1-m} = \frac{2.(FB_{\min} + H_B)}{B \cdot \tan(\theta_0 + [(KAH+M)/W \cdot 6M] \cdot (180/\pi))}$$

(5) Coefficient C_{3-1}

When a fishing boat is in following sea or oblique sea, it sometimes collects a lot of sea water on the deck. If the fishing boat is not heeled hard at the moment, the water is filled up to the bulwark top of the boat and the deck water sometimes capsizes the boat by the dynamical heeling energy. As it is quite clear that C_{1-0} or C_{2-m} and C_{2-1} or C_{2-f} values do not explain the accidents, C_{3-1} value is used to investigate the possibility of the accidents via the relation

$$C_{3-1} \equiv \text{Area } b'' / \text{Area } a''$$

where a'' and b'' are terms used in Fig. (2) and GZ is the heeling moment lever caused by shipping water on the deck.

In the calculation, the effect of scupper area is neglected, as it is not so effective at the moment to prevent the accident. When C_{3-1} value of a boat is more than 1.0, the boat is considered safe under the condition.

(6) Coefficient C_{3-f}

If θ -flood of a boat is smaller than θ -limit, C_{3-f} should be used instead of C_{3-1} , and the value is given as:

$$C_{3-f} \equiv \text{Area } b''' / \text{Area } a''$$

where b''' is the area up to θ -flood in place of θ -limit in Fig. (2).

Rolling angle θ_0 used in the calculation of C_{2-1} or C_{2-f} value is 70% of synchronous rolling angle θ_c in regular waves which is calculated by a simplified formula based on Sverdrup-Munk's diagram and self rolling period of a boat, (1). θ_0 is also used for the calculation of C_{1-0} or C_{1-m} which is introduced from the concept that rolling motion of a boat in irregular waves is considered a kind of narrow band irregular oscillation and consequently, the occurrence probability of θ_0 in irregular waves can be explained theoretically as shown in Table 1, if θ_c is assumed to be nearly equal to θ 100,000. (Ref.2)

3. PRACTICAL USE

Fishing boats are generally used under many different loaded conditions, and consequently all the C values of such conditions are necessary to be calculated to judge the stability by this theory. But, those calculations are very troublesome work practically, and to simplify them, following diagram method is used.

- (1) When hull form and deck construction of a boat is fixed, contour curves of $C = 1.0$ under several circumstances (named critical curves hereafter) are obtained by digital computer and they can be shown on a cross coordinate of GM and midship freeboard or KG and displacement.
- (2) Each loaded condition of the boat can be plotted on the same diagram.
- (3) By relative position of the plotted points to the critical curves, anyone can easily realize whether C values of a loaded condition under any assumed circumstances is more than 1.0 or not. If trim of the boat changes widely during the voyage, same diagram should be prepared at two or three trimmed conditions.

4. EXAMPLES OF APPLICATION

For many kinds of fishing boats in Japan, such diagrams are calculated, and by the results, it is shown that this theoretical criterion explains the empirical stability standard including skipper's feeling for the fishing boat.

In Fig. 3 - 9, examples of critical curves are shown for several kinds of fishing boats used in Japan and also European trawlers under assumed sea states. In some of the diagrams, critical curves by IMCO's new criterion are shown to compare with the theoretical ones. If the assumed sea state used for the calculation of European trawlers, which is supposedly caused by steady wind speed $U = 26$ m/sec with gust, is insufficiently accurate then, this theoretical criterion almost coincides with traditional stability standard for European fishing boats. From those diagrams, it is possible to estimate roughly how much the sea state, hull form, heeling moment caused by fishing operation, constructions on deck, etc. affect to the stability of fishing boats. Plotted points in the diagrams are actual loaded conditions of the boats.

5. CONCLUDING REMARKS

This theoretical approach is not, of course, perfect and for instance, capsized accidents occurred in oblique seas, or lost accidents occurred after heavy heel in beam sea, are not considered. Therefore, when it is used in practice, reliability of this method should be confirmed carefully by comparing with many actual data of fishing boats' activity, and also with empirical stability standards for different kinds of fishing boats in different areas.

On the other hand, theoretical approach for the accidents mentioned above should be developed as early as possible. It might give us important suggestion for determining more reasonable stability criterion or permissible minimum freeboard of fishing boats.

6. ACKNOWLEDGMENTS

This study has been discussed and consented in "The Research Committee for the Stability of Fishing Boats in the Rough Sea" established in 1968 - 9. In the committee, the necessity of using C_3 value was suggested by Prof. S. Motora, the committee chairman, and the computer program was developed with funds provided by the committee.

The author wishes to acknowledge the suggestion of Prof. Motora, and the assistance of the committee member as well as the association.

REFERENCES

1. M. Yamagata. "Standard of Stability Adopted in Japan", translated RINA, 1959.
2. M.S. Longuet-Higgins, On the Statistical Distribution of the Heights of Sea Waves, Journal of Marine Research, Vol. XI, 1952.

Table 1 Relation between N_1 , a_N/\bar{a} and k

N_1	a_N/\bar{a}	k
1	0.707	0.21
2	1.030	0.30
5	1.366	0.40
10	1.583	0.47
20	1.778	0.52
50	2.010	0.59
100	2.172	0.62
200	2.323	0.68
500	2.509	0.74
1,000	2.642	0.77
2,000	2.769	0.82
5,000	2.929	0.86
10,000	3.044	0.90
20,000	3.155	0.93
50,000	3.296	0.97
100,000	3.400	1.00

Symbols in Table 1 are as follows:

a_{N_1} : Maximum half amplitude expected
in narrow band irregular waves of
 N_1 times

N_1 : Number of irregular waves

\bar{a} : Standard deviation of half amplitude
of narrow band irregular waves
introduced by following equation:

$$\bar{a}^2 = \left(\sum_{i=1}^{N_1} a_i^2 \right) / N_1$$

a_1 : Wave height of irregular waves on
the calm sea level

k : $a_{N_1}/a_{100,000}$

Table 2 Factors being considered for calculation of C values

<u>Factors</u>	C_1	C_2	C_3
Steady wind velocity U	x	x	
Gust		x	
Heeling moment M	x	x	
N value *	x	(if necessary) x	
Trim (fixed value)	x	x	x
Critical angle of heel θ_l **		x	x
Position of flooding opening		x	x
Bulwark Height H_B	x		x
Deck erection		x	x
Deck house			x

* θ_0 is calculated by following equation, and $N = 0.020$ is used for steel boats with bilge keel.

$$\theta_0 = \sqrt{\frac{138 \cdot \gamma \cdot S}{N}} \quad (\text{deg})$$

Where N : Bertin's roll-damping coefficient

S : Steepness of synchronous waves

γ : Effective wave slope coefficient

** The value of θ_l for fishing boats in Japan is assumed as follows;

$\theta_l = 30$ deg. for purse seiner

$\theta_l = 55$ deg. for the other boats

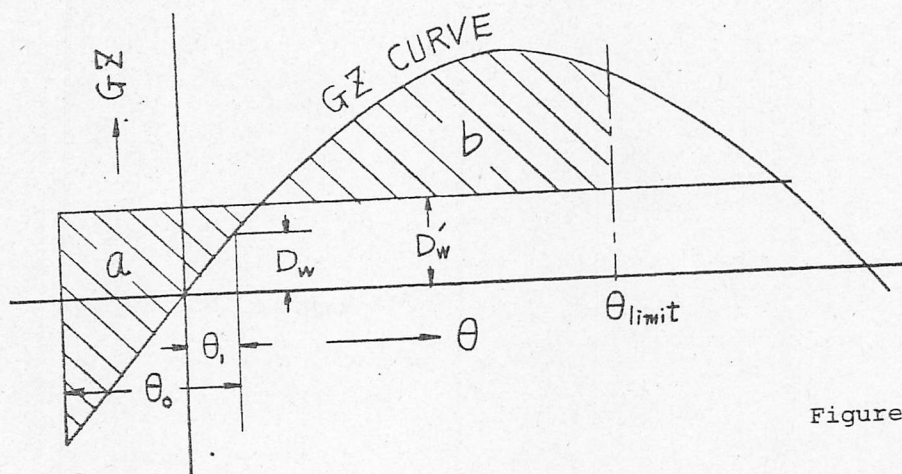


Figure (1)

Fig. 2

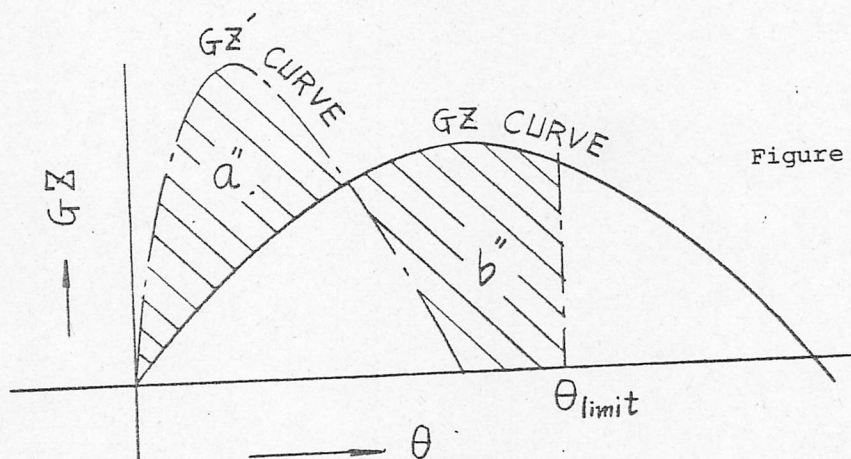


Figure (2)

The symbols in Fig. 1 are as follows:

- θ_0 : Rolling angle at a time of rolling nearly synchronously in irregular waves (half angle) (deg)
- θ_1 : Heeling angle caused by the steady beam wind (deg)
- θ_l : Critical angle of heel (In case the heeling angle exceeds this angle, the position of a vessel's centre of gravity moves by a large margin, because the load on the vessel makes a large movement, therefore, the stability of the vessel is lost.) (deg)

D_w : Heeling moment lever caused by steady beam wind (m)

D'_w : Heeling moment lever caused by a gust (m)

Note:
In the seas adjacent to Japan, a relationship of $D'_w = 1.5 \times D_w$ is known statistically.

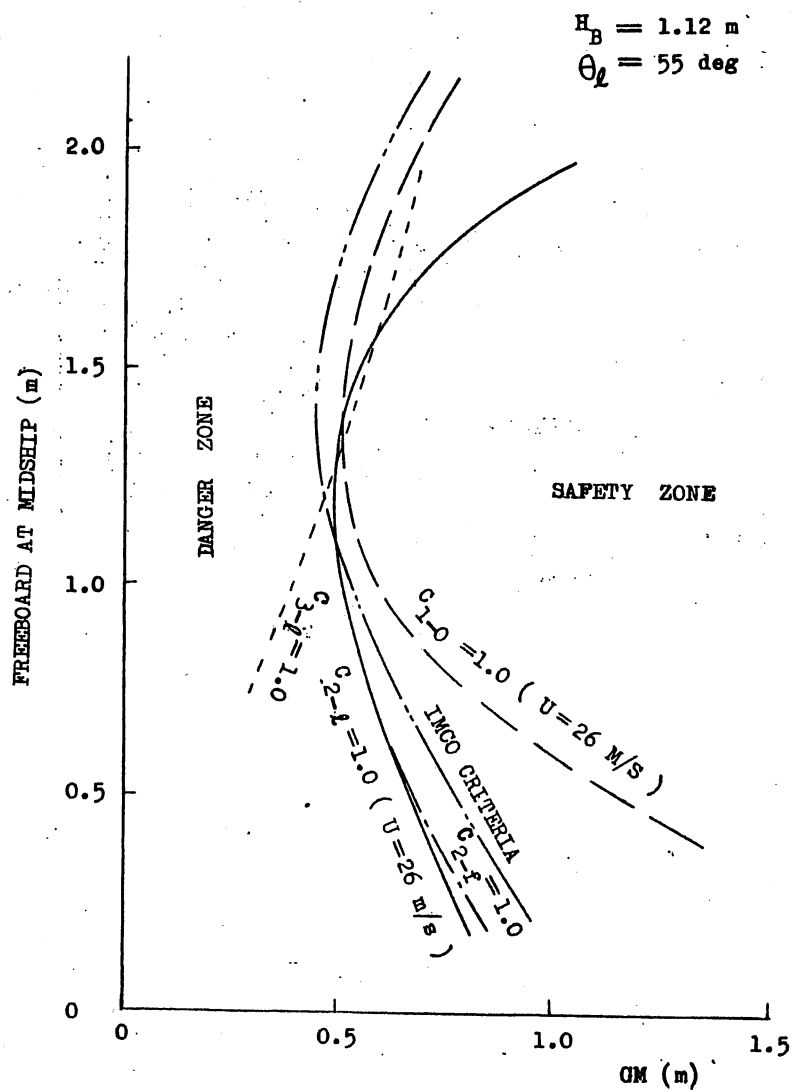


Fig. 3 For 127 ft European Type Steel Side Trawler

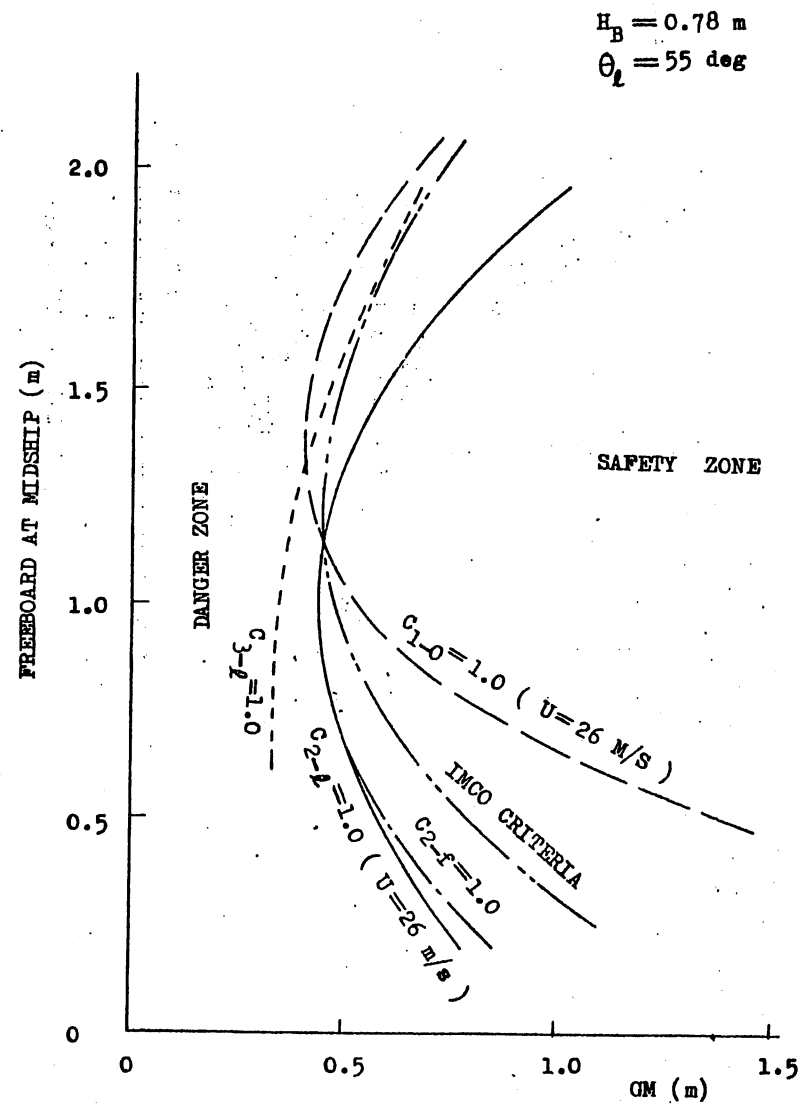


Fig. 4 For 72 ft European Type Steel Side Trawler

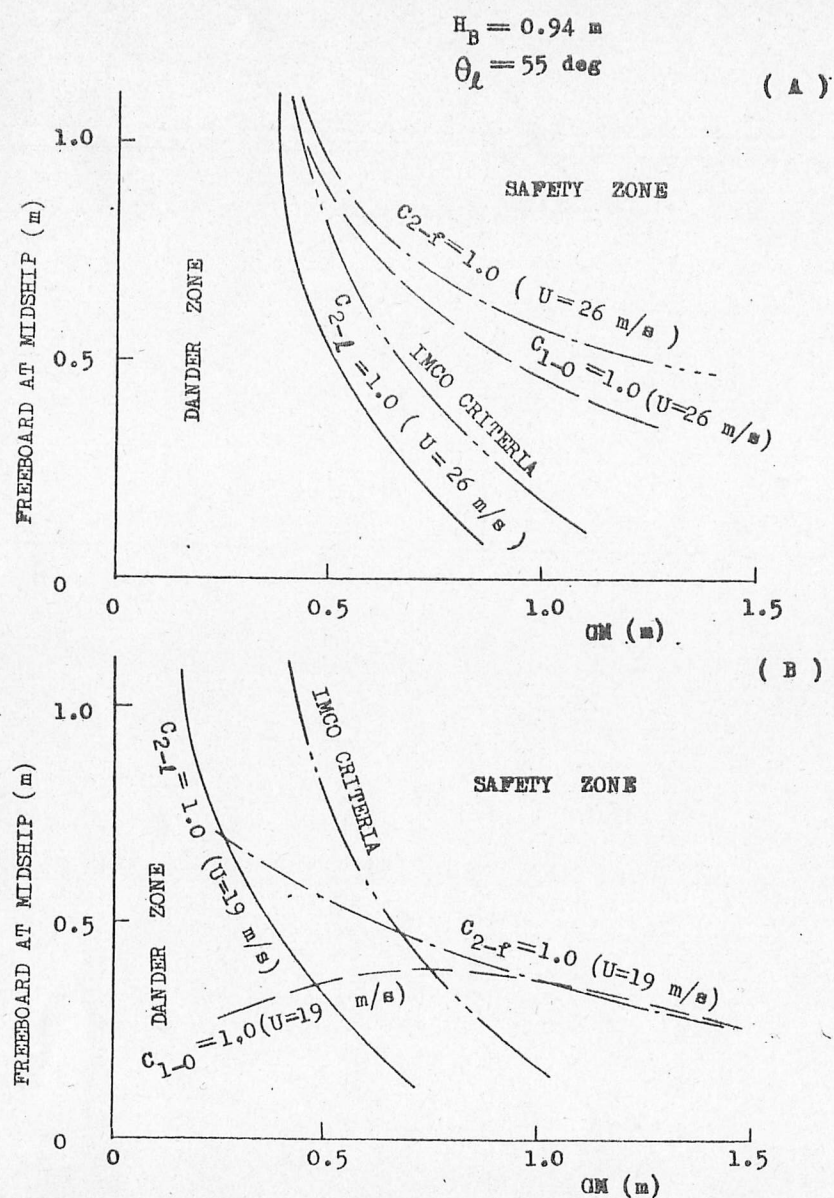


Fig. 5 For 25 Meter European Type Wooden Side Trawler

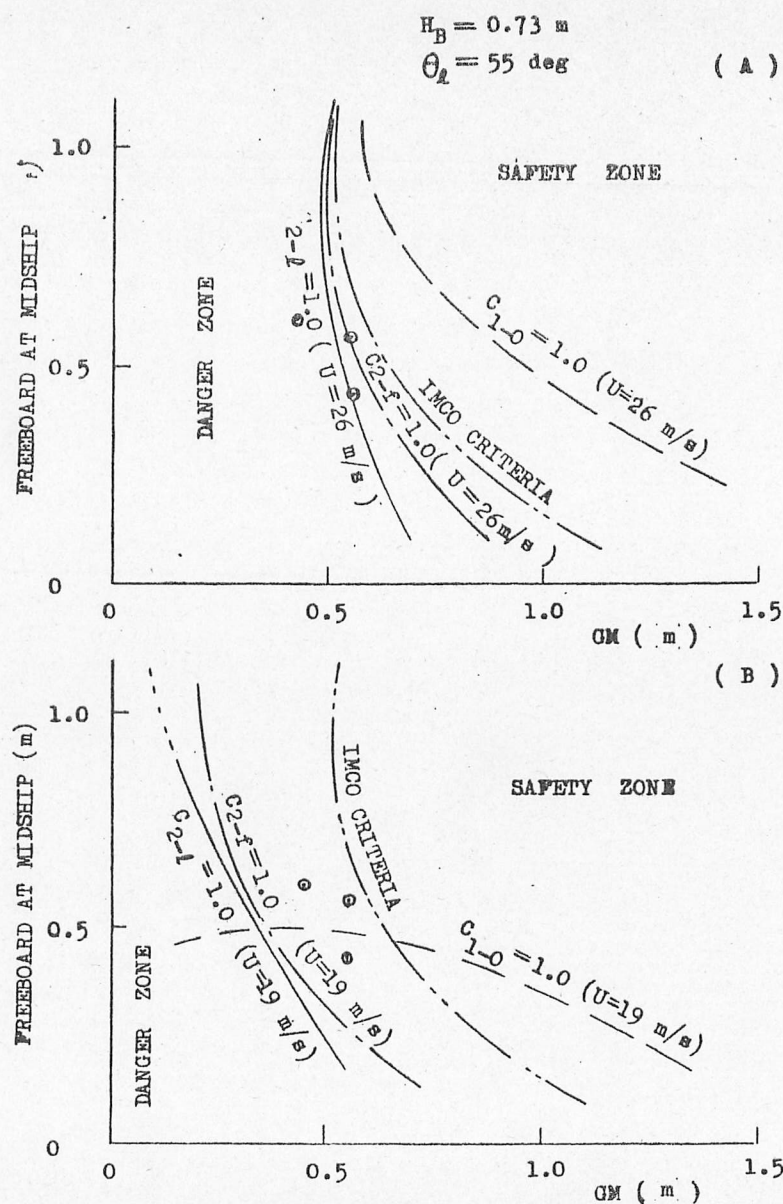
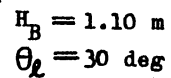
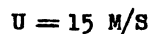
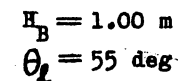


Fig. 6 For 29 Meter Pair Trawler in Japan

$$\begin{aligned} H_B &= 1.10 \text{ m} \\ \theta_L &= 30 \text{ deg} \end{aligned}$$

$$U = 15 \text{ m/s}$$

$$\begin{aligned} H_B &= 1.00 \text{ m} \\ \theta_1 &= 55 \text{ deg} \end{aligned}$$


Tsuchiya

PRACTICAL APPROACH TO SHIP STABILITY

by

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1. INTRODUCTION

The paper now to be presented is made up of a few scattered points of view which are meant to represent those who employ and train the men whose duty will one day be to bring the ships with their passengers, crews and cargoes from port to port.

Previously, in most cases ships were built and arranged according to the principle that the cargo had to be adapted to the ship, but today the situation is that ships are built to satisfy existing transport requirements for certain definite units and consequently we are to an ever increasing extent, designing ships to suit the cargoes. This fact, in connection with the increased size of ships, has resulted in a number of problems to which solutions are sought by national authorities, classification societies, universities and independent research institutions in cooperation with the shipowners and the shipyards.

The stability problem has been attacked from widely different quarters, and the setting up of IMCO and the activity of that organization has led to cooperation and exchange of research results, the effect of which is that a vast amount of information and knowledge is available to us today. So vast indeed that few persons, if any, are capable of grasping a sufficiently comprehensive all round picture of the situation. Introduction of the electronic computer together with the use of complex mathematical models has enabled the description and treatment of the problems to be carried out in a manner not dreamt of before the Second World War. If conventional

aids were to have been used in the process, this would have taken up so much time as to be quite impracticable.

This, however, has the effect that many results are presented and registered in an elegant language of formulas or are prepared for programming for use in the computer. In the documentation of shipyards or research institutes intended for national or foreign authorities, who of course have technical experts at their disposal, a similar form of presentation is used.

Together with the growing understanding and knowledge of the factors which are crucial as far as sufficient stability is concerned, the data supplied to ships by shipyards have become more comprehensive and detailed. In many cases the information is the same as that which is required by the supervising authority in order to decide whether or not the ship can be accepted.

The ship will also frequently be furnished with particulars for the purpose of showing the authorities of foreign countries that the ship in a certain condition complies with a given criterion.

It should be borne in mind, however, that the information which a specialized expert is capable of extracting directly from the documentation need not emerge quite so easily for a ship's officer, master or mate.

Although the period of training for the mates and masters gives the

navigator a sound basis for mastering stability problems, it should be remembered that the problems surrounding the stability of ships only represent a small - though important - part of the navigator's duties.

It is important that the information which is to be used in practical everyday work is presented in the simplest possible manner, and if it is necessary for the purpose of documentation required by foreign authorities to supply tables with information which is of no use to the navigator then such tables should be placed in a separate booklet.

During the last generation, where those on board wanted to judge the sea-keeping capabilities of their ship, it has been the custom in Danish ships to note the metacentric height and plot the isocarene GZ curve and consider the flow of the curve without really having on board any particulars on which to base such evaluation.

The appearance of the IMCO criteria has changed the situation. To introduce these criteria a number of new concepts were used about which the navigator could not be assumed to have any previous knowledge. Although, out of interest, the members of the profession have made an effort of their own accord, or through instruction under the auspices of shipping companies, to increase their knowledge, it must be admitted that as far as stability matters are concerned there is a communication gap between the technical person and the sea-going navigator.

It is possible, indeed probable, that the continued work of scientists and technicians in theory, on model and full scale experiments and also on statistical evaluation of experience gained from marine casualties will result in the establishment of stability criteria for different ships, cargoes and weather conditions. If complied with, such criteria will provide the necessary condition which will enable the ship to survive virtually any situation, but if the men responsible for the loading and handling of the ship fall short in this respect casualties will continue to occur.

An attempt is made in the following sections to outline guidelines to assist in the formulation of these instructions which are furnished as an aid to navigators on board ships.

2. PRESENTATION OF STABILITY CRITERIA FOR SMALLER SHIPS

During the period 1965 to 1967 a number of losses and casualties happened to smaller Danish ships which led to the appointment of a Commission under the Ministry of Maritime Affairs. Having investigated the casualties the Commission produced a report (1) which shows that out of 22 casualties investigated capsizing had certainly occurred in 10 cases. Further, it appeared that in none of the 10 cases, in which the ships were intact, had capsizing taken place without shifting of cargo; for instance, timber deck cargo, broken stones, salt, soda, ammonium sulphate, china clay and wet sand, which are not considered liable to shift. The cargo had, generally speaking, in all cases been dealt with in accordance with ordinary practice.

The Commission made the remark that lack of adequate stability criteria for intact ship loaded with non-shifting cargo, with shifting cargo and with cargo not liable to shift, and to some extent lack of experience of the officers in handling small ships as well as inability to recognize the shifting character of the cargo, were the reasons why firstly the cargo was not secured either by trimming or shifting boards and secondly that in bad weather the ship was not handled in a way to reduce the influence of rough sea by changing course and speed.

Among the conclusions deserving a mention, the Commission stated that the stability criteria contained in IMCO recommendation No. 167 ought to be put into force as soon as possible. Likewise the Commission proposed that IMCO's Code of Safe Practice for Bulk Cargoes (1965) should enter into force after certain supplementary investigations, including investigations in case of a list.

Caused by the loss of a small government ship in 1970, another commission was appointed with the object of bringing forward proposals for securing of ships' stability.

The Commission recommended that IMCO's Recommendation on Intact Stability for Passenger and Cargo Ships under 100 m in length and the corresponding recommendation for fishing vessels with a few modific-

ations, should, until further notice, form the basis of approval of new ships in Denmark and also for ships above 100 m in length.

In connection with preparation of its report (2) the Commission transformed the IMCO criteria so that they all appear as GM-minimum requirements.

The transformation is considered in the next section and, as can be seen from Appendix I, this basic principle leads to a simple but clear manner for presentation of calculation results.

3. TRANSFORMATION OF IMCO CRITERIA FOR DIRECT DETERMINATION OF MINIMUM GM

The minimum requirement to stability mentioned in Reg. 5.1 of the IMCO Recommendation can, with reference to Figure 1, be stated as follows:

- 1) The hatched area e_1 should be greater than or equal to 0.005 metre-radians.
- 2) The hatched areas e_1 and e_2 should in all be greater than or equal to 0.090 metre-radians.
- 3) The hatched area e_2 should be greater than or equal to 0.030 metre-radians.
- 4) The righting lever GZ should not be less than 0.20 m at an angle of heel equal to or greater than 30° .
- 5) The maximum righting lever GZ should occur at an angle of heel θ_M preferably exceeding 30° , but not less than 25° .
- 6) The metacentric height GM should be greater than or equal to 0.15 m for cargo ships and greater than or equal to 0.35 m for fishing vessels.

It appears from Figure 2 that $GZ = MS + GM \sin \theta$, and it is now in common use in Denmark that by calculation of stability, MS is determined for a certain amount of displacements and angles of heel. By means of these MS curves a distinct coherence between GZ and GM for any displacement and angle of heel is established.

In Figure 1 the designation 'e' is used for the area under the curve of stability, i.e.

$$e_1 = \int_0^{30} GZ d\theta = \int_0^{30} MS d\theta + \int_0^{30} GM \sin \theta d\theta$$

$$= q_{30} + GM \cdot (1 - \cos 30^\circ)$$

$$\text{or } e_1 + e_2 = q_{\theta_f} + GM(1 - \cos \theta_f) \text{ for } \theta_f < 40^\circ$$

(when $\theta_f \Rightarrow 40^\circ$, θ_f is in this and the following formula substituted by 40°)

The IMCO criteria can therefore be written as follows:

- 1) Reg. 5.1 (a) 1: $e_1 \geq 0.055 \text{ m}$
or: $GM \geq \frac{0.090 - q_{30}}{0.134}$
- 2) Reg. 5.1 (a) 2: $e_1 + e_2 \geq 0.090 \text{ m}$
or: $GM \geq \frac{0.090 - q_{\theta_f}}{1 - \cos \theta_f}$
- 3) Reg. 5.1 (a) 3: $e_2 \geq 0.030 \text{ m}$
or: $GM \geq \frac{0.030 + q_{30} - q_{\theta_f}}{0.066 - \cos \theta_f}$
- 4) Reg. 5.1 (b): $GZ \geq 0.20 \text{ m}$

for at least one value of θ in the interval 30° to θ_f° .

$$\text{or: } GM \geq \left(\frac{0.20 - MS\theta}{\sin \theta} \right)_{\min}$$

The minimum value of $\frac{0.20 - MS\theta}{\sin \theta}$ is found by successive insertion of a series of θ values in the interval 30° to $\theta_f(40^\circ)$.

- 5) Reg. 5.1 (c): $\theta_M \geq 25^\circ$,

$$\text{where } \left(\frac{dGZ}{d\theta} \right)_{\theta_M} = 0$$

$$\text{or: } GM \geq \left\{ \frac{-\left(\frac{dMS}{d\theta} \right)_{\theta_M}}{\cos \theta_M} \right\}_{\min}$$

The minimum value is found as above, however, with the interval starting

at 25° and without other limit upwards than θ_f , as a maximum is acceptable also for $\theta_M > 40^\circ$.

- 6) Reg. 5.1 (d): $GM > 0.15$ for cargo and passenger ships
- $GM = 0.35$ for fishing vessels

For passenger ships, further requirements are:

$$\text{Reg. 5.2 (a)} \quad GM \geq \frac{M_P}{0.172 \Delta}$$

$$\text{Reg. 5.2 (b)} \quad GM \geq \frac{KM - 0.5d}{1 + 0.6 \frac{L}{V^2}}$$

where

- M_P is the moment in metric tons due to crowding of passengers in one side
- L length of ship at waterline in m
- V service speed in m/sec
- d mean draught in m

When determining M_P it is supposed that all persons on board are placed on all open decks with a density of four persons per sq m as close as possible to one side of the ship, assuming the weight to be 75 kg per person or 300 kg per sq m.

Immediately following this work of the committee, a working group prepared a calculation form with the purpose of securing the stability in smaller ships and a co-operation with owners of coastal ships and the Danish Ship Research Institute (DSRI) has been initiated.

Appendix II is an example of the mentioned calculation form and includes instructions. The information thus given is intended specially for use by skippers and is attached to demonstrate its simple and exact form.

4. PRESENTATION OF GRAIN LOADING REGULATIONS

On 9th December, 1971, a Danish vessel of considerable size carrying maize capsized in the Pacific during a voyage from Mexico to Japan. The Ministry of Commerce brought about an investigation, and on 15th May, 1972, a

statement was issued (3). An impediment to the investigation was that the master as well as the chief officer died in the accident. However, based on the official enquiry and details of the loading condition on departure the following description of the capsizing, which is a short extract from the report, could be given.

A small list to port side was compensated for by using fuel from the port side fuel tanks during the first part of the voyage. As the list increased up to about $10^\circ - 12^\circ$, filling of No. 5 starboard wing tank with water ballast was started, and the list was brought down to about 1° . During an alteration of course caused by damage to No. 1 hatch the wind now came in on the port side and the ship fell towards starboard with a list of $27^\circ - 30^\circ$. By pumping water ballast from No. 5 starboard wing tank to No. 5 port side wing tank an attempt was made to bring the ship to an upright position, and at the same time filling of port bottom tank No. 3 began. The list now increased to 40° and the ship had to be abandoned.

Calculations based on the principles of the equivalence rules to SOLAS Chapter VI showed that the heeling angle in the loading condition for the voyage would be greater than the assumed angle of 10° with the horizontal plane of the maize, which had caused it and thus would have a self-increasing tendency.

There is no doubt that the accident was started by shifting of the maize and the list having had a selfincreasing tendency. It must be considered a fault to attempt balancing the list by fuel consumption from one side only and one-sided filling of No. 5 wing tank. It is obviously a fault to create two free surfaces by pumping from No. 5 starboard wing tank to No. 5 port side wing tank. Producing another free surface by filling bottom tank No. 3 must be assumed to be catastrophic.

A possible correct procedure must be to try to improve the stability from the very first by filling the bottom tanks one by one paying no regard to the vessel being brought to a draught deeper than the load lines.

Calculations showed that the ship, even though complying with SOLAS 1960 Chapter VI, did not comply with the rules of equivalence.

The capsizing caused precipitation of the work by a committee on implementation of IMCO recommendation No. 184 for Danish ships concerning equivalent rules for SOLAS 1960 Chapter VI.

Doubt was expressed from various sides in Denmark as to the application in practice of the equivalent rules for navigators at sea. The problem was solved in a co-operation between research organisations, industry and educational institutes as production of a layout for the Grain Loading Stability Booklet was started based on DSRI's preceding work on the equivalent rules. The explanations to the individual tables of this booklet are expected to be ready during 1975.

In a booklet prepared for use in Denmark, see Reference [4], the various suggested alterations to the equivalent rules have been taken into account. However, the booklet has not been brought quite up to date with the final formulation of SOLAS 1974 Chapter VI, but it appears that the final result will be a booklet suitable for practical use on board ships loading grain.

5. UNCONVENTIONAL SHIPS

As mentioned in the introduction, developments have brought about special purpose ships - gastankers, container ships, roll-on/roll-off ships, to mention a few. These developments have led to the preparation of various regulations with requirements for construction of the ships as well as for their operation.

By taking into account that environmental problems also require attention, everyone will surely appreciate the amount of literature which must be studied by those carrying responsibility on board.

As the crew, often with short notice, may happen to have responsibility for unconventional types of ships, it is essential that exact descriptions and instructions are ready at hand in cases where it is to be expected that the operating of the ship would differ from what the crew has previously experienced.

Thus, an instruction of this nature is very useful, although it should be in a suitable form and there is a need to use very clear language.

6. CONCLUSION

The marine casualties referred to in

the foregoing which, alas, are but few among many, seem to indicate that in many cases there is a lack of understanding as to where the limit of acceptable stability is and also insufficient understanding of the principles which make it possible to take the right precautions to prevent or reduce the consequences of a casualty which has occurred or which is developing.

While previous stability criteria in nearly all cases relied on elements derived from the varied positions of the ship in an immovable mass of water with plane, horizontal surface, a number of the papers read at this conference would seem to indicate that other parameters derived from dynamic conditions in the ship and its surroundings will be introduced.

A great number of new research results should be capable of being described without recourse to higher mathematics, and in order to motivate sea-going personnel to follow developments it would be a blessing if someone could and would take upon himself the task of popularizing the results so that people other than those with highly specialized training, by keeping up with these matters in their own technical magazines, may acquire some background for understanding the principles behind new criteria, methods of calculation or directions for action in situations involving risk of accidents.

It is important that the information supplied to the ship is simple and precise and written in a language which can be understood by the navigator. The problem must be dealt with in two stages:

- 1) An attempt should be made during training of the navigator to provide him with sound basic knowledge and, when joining ships where special knowledge is important, to provide the necessary follow-up training.
- 2) The expert, when formulating technical information for use on board ship, should always endeavour to meet the level at which those on board ship may be expected to work and to formulate it in such a manner that only those particulars which are relevant to the navigator are given without involving technical data or documentation which may only prove useful to national or

foreign authorities.

Model for Grain Loading Stability Booklet, 1973

Since the results of the conference no doubt will have much impact on the future work at IMCO, it is my view that this gathering is the right forum to emphasize the wish of industry that the obligations imposed on those responsible on board by additional provisions must be reduced to the necessary minimum, and finally I would like once more to underline the necessity of providing for exchange of experience between technicians and ships' officers. There is no doubt at all that seamen have a store of knowledge based on experience only waiting to be formulated so that it may be included in the field of activity of scientists.

I wish to wind up this paper by venturing the postulate that technicians may very well be capable of handing over to the shipmaster such a ship as will possess the necessary conditions for survival in almost any conceivable situation, but for the time being these conditions will not in practice amount to more than 75% of the sufficient conditions. This paper should, therefore, also be considered in the light of the desirability of bringing the remaining 25% into the fold.

7. ACKNOWLEDGEMENTS

This paper was prepared in co-operation with the Danish Shipowners Association.

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- 2) Danish Ministry of Commerce
Betaenkning vedrørende SKIBES
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- 3) Danish Ministry of Commerce
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1972
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BC XIV/INF.3)
- 4) Danish Shipowners' Association
Directorate of Maritime Education
Danish Ship Research Institute

APPENDIX IPRESENTATION OF CALCULATION RESULTS

The transformed form set up above allows a direct calculation by computer. By means of ordinary available programs hydrostatic data are calculated and drawn on a curve sheet. The MS values are also calculated, and these calculations should be carried out for constant moment of trim. The results are presented as isocarene MS-curves and, if requested, also as isocline MS-curves.

For a number of displacements, the GM minimum requirement corresponding to each of the GM formula stated is then calculated, and the results are drawn graphically, the GM-values being plotted against the displacement and with an additional scale for the corresponding values of mean draughts (see Figure 3).

The upper envelope represents the minimum requirements to GM corresponding to each displacement.

Finally, a diagram showing the maximum allowable KG is plotted against displacement, Δ , or showing $\Delta \times \text{KG}$ max against Δ . Whether the first or the second way of presentation is preferred, plots corresponding to each individual loading condition are inserted in the diagram. A loading condition can only be approved if the corresponding plot lies below the curve.

When using the curve $\Delta \times \text{KG}$, the advantage is obtained that for a loading condition which does not meet the requirements - the plot lies above the curve - it is easy to find how much more ballast or less cargo is needed to place the plot on the right side of the curve.

Figure 4 shows such a graph, where $\Delta \times \text{KG}$ is plotted against Δ . Different loading conditions are shown. Two of these, A and B are unacceptable, the plots lying above the curve. Point A can be brought on to the curve, if ballast is added to the loading condition. Assuming that ballast can be placed 1 m above keel a line is drawn from point A with a slope taken from the additional graph in the right hand side. Where this line intersects the KG-curve an acceptable condition is obtained. The horizontal move of the point A to A_1 gives the increase of the displacement, i.e. the necessary amount of ballast. Likewise point B being a deck loading condition

can be changed to the acceptable point B_1 by removal of some of the deck cargo.

The graph is particularly useful on board for examination of conditions, which are not calculated beforehand, $\Delta \times \text{KG}$ representing the total vertical moment. Calculation of KG or GM is therefore superfluous, and the total control work only involves insertion of $\Delta \times \text{KG}$ against Δ and making sure that the point lies below the curve.

APPENDIX II

GUIDE FOR CONTROL OF STABILITY FOR CARGO SHIPS BELOW 100m IN LENGTH

The present guide describes a simple method enabling the master to ensure before a voyage that his ship has enough stability.

The method is devised for cargo ships below 100 m length which are fully or nearly fully loaded, and which are on a nearly even keel. However, the method can not be applied to shifting cargoes such as grain or similar cargo.

The idea behind this is, by means of the form on page 13, to calculate the ship's displacement and vertical moment. These two values are plotted into the diagram which is titled the Capsizing Safety Diagram.

If the intersection point lies below the curve in the diagram, the ship complies with the stability criteria given by IMCO as a measure for safety against capsizing under normal conditions.

The calculations must be made before departure from loading port for the departure conditions as well as for the expected condition of arrival, taking into account consumption of fuel, freshwater and provisions during the voyage.

Calculation Form (Figure 5)

The attached form for calculation of displacement and vertical moment comprises four columns, (A), (B), (C) and (D). In column (A) weights in metric tons are inserted, the sum of which, Sum (A), represents the ship's total displacement. With regard to weights of LIGHT SHIP, STORES, CREW and PROVISIONS these are considered to be constant and may, therefore, be inserted in the form beforehand.

The weight of the cargo in metrical tons is inserted as stowed, abreast HOLDS, TWEEN DECK and DECK CARGO with the option of subdividing each room into, for instance, forepart and afterpart.

The weights of fuel, ballast and freshwater are inserted abreast the tanks in question.

To be inserted in column (B) is the height of the centre of gravity above the keel measured in metres for each of the weight values in column (A).

Also in this connection the values for LIGHT SHIP, STORES and PROVISIONS are considered constant and, therefore, inserted as fixed figures.

Capsizing Safety Diagram

In the capsizing safety diagram, a curve is drawn indicating the values for the vertical moments corresponding to varying displacements. This curve is established by IMCO expressing that the ship can be considered safe against capsizing under normal conditions.

The left terminus of the curve corresponds to the ship being half-loaded.

The curve from this point to the point corresponding to an approximately two-thirds loaded ship is dotted and the remaining curve is fully drawn.

The explanation hereof is the following:

The IMCO-criteria have been set up on the assumption that the ship is fully or nearly fully loaded corresponding to the fully drawn part of the curve. The dotted part of the curve therefore indicates that the result of the calculations of these loading conditions is only a guidance for judging the stability of the ship.

On the horizontal scale of the diagram SUM (A) obtained from column (A) of the calculation form is plotted and a vertical line drawn through this point.

On the vertical scale of the diagram SUM (C) + (D) obtained from column (C) of the calculation form is plotted and a horizontal line drawn through this point.

If the point of intersection of the two lines is placed below the curve the ship has sufficient stability to be safe against capsizing under normal conditions.

If the point of intersection of the two lines is above the curve the ship has insufficient stability and it will be unwarrantable to leave harbour. In the latter case, precautions must be taken - filling the ballast tanks, removal or unloading of cargo or the like - to improve the stability and another calculation must be made for the altered condition to ensure that the point of intersection plots below the curve.

By such alterations the ship should continue to be on a fairly even trim.

The centre of gravity in metres above keel for different parts of the cargo is taken from the ship's capacity plan. Having inserted the cargo on the plan, the centre of gravity above keel is estimated and afterwards the distance from the keel is measured.

For tanks which are filled or partly filled, centres of gravity are likewise estimated.

In column (C) are inserted vertical moments for the different weights obtained by multiplying the values in column (A) by the corresponding ones in column (B).

The sum of the vertical moments results in SUM (C). In column (D) are inserted corrections for free surfaces which must be added to the total vertical moment for that or those tanks which are either filled or empty.

If a tank is empty or filled the figure abreast that tank is struck out and thus is not included in the total correction for free surfaces SUM (D). At departure the correction for free surfaces should, however, be made for fresh water tank and fuel oil tank, from which consumption takes place, whether such tanks are filled or not.

SUM (D) is passed to column (C) and is added with SUM (C) to SUM (C) + (D) which gives the vertical moment.

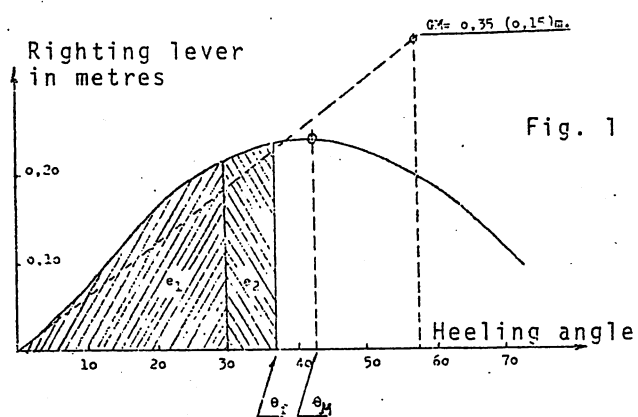


Figure 1

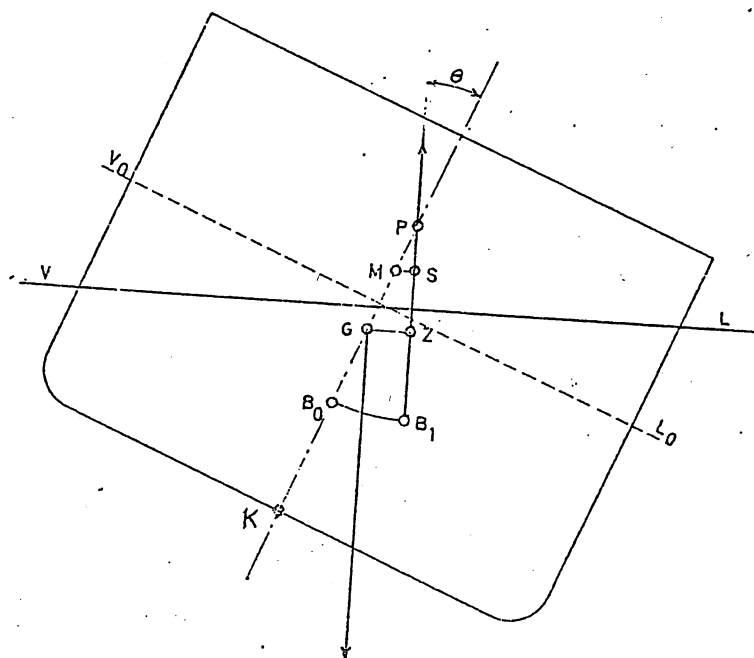


Figure 2

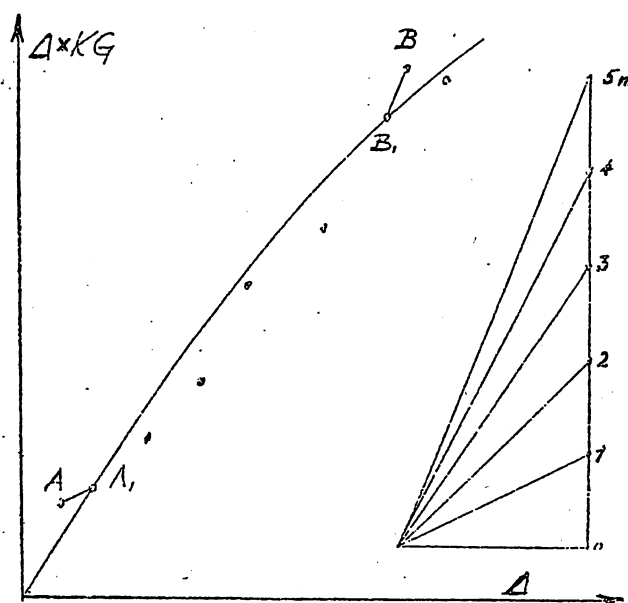


Figure 3

GM - Requirements

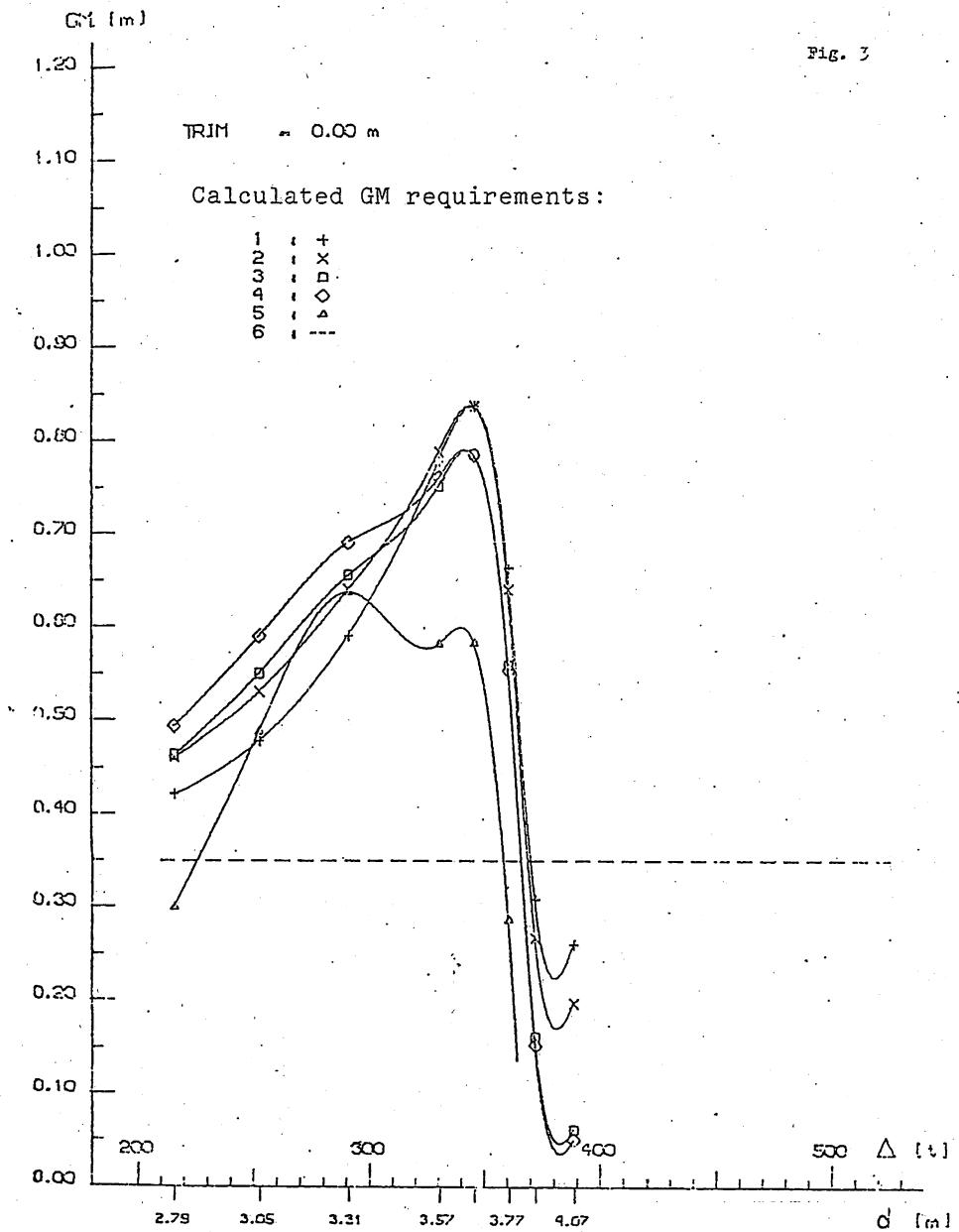


Figure 4

BEFORE DEPARTURE BOTH DEPARTURE AND ARRIVAL CONDITIONS SHOULD BE CALCULATED

M/V
DEPARTURE CONDITION
ARRIVAL CONDITION

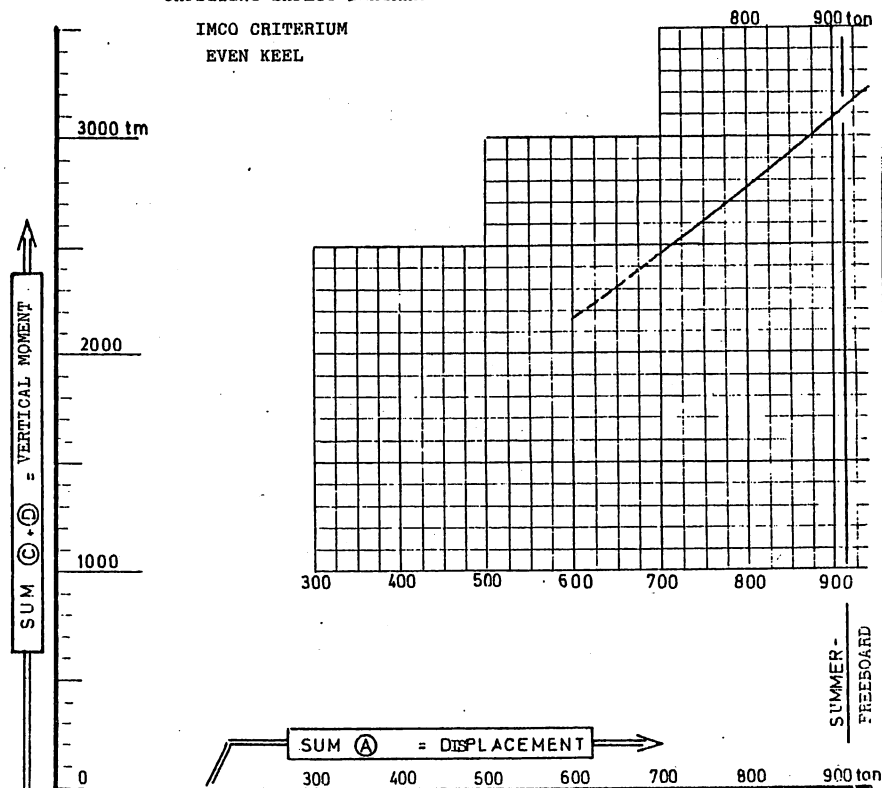
CALCULATION OF DISPL. AND KG

VOYAGE NO. FROM TO		COLUMN (A) WEIGHT	COLUMN (B) C.O.G. ABOVE KEEL	COLUMN (C) VERTICAL MOMENT
DATE OF DEPARTURE		tons (t)	meter (m)	tonsmeter(tm)
ITEM				
LIGHT SHIP		213.0	3.73	1093
STORES, CREW, PROVISIONS		12.0	5.50	66
HOLD				
TWEEN DECK				
DECK CARGO				
COLUMN D FREE SURFACE CORR.				
DBT. NO. 3 CENTER 14/31		43.0		
DBT. NO. 2 CENTER 31/45		37.0		
FRESH WATER TANK 3/5		26.1		
FOREPEAK		6.0		
DBT. NO. 1 S/P 45/65	S/P	36.0		
DBT. NO. 2 S/P 35/45	S/P	7.4		
DBT. NO. 3 S/P 23/35	S/P	8.8		
DBT. NO. 4 S/P 14/23	S/P	3.5		
AFTERPEAK		10.5		
FREE SURFACE CORR. (tm)				
SUM (D)				
SUM (A)				
SUM (C)				
SUM (D)				
SUM (C) + (D)				

IN COLUMN (D) STRIKE OUT THE FIGURES FOR TANKS WHICH ARE EITHER FULL OR EMPTY. WHEN COUNTING OMIT THE FIGURES STRUCK OUT.

CAPSIZING SAFETY DIAGRAM

IMCO CRITERIUM
EVEN KEEL



PLOT DISPLACEMENT AND VERTICAL MOMENT
IN THE DIAGRAM
POINT OF INTERSECTION MUST LIE BELOW THE CURVE.

FOR FURTHER DETAILS CONSULT THE EXPLANATION.

DATE

SIGNATURE

Figure 5

ON DANGEROUS SITUATIONS FRAUGHT WITH CAPSIZING

by

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1. INTRODUCTION

While formulating the problem of ship's intact stability regulations, it is advisable to accept that

- (i) The diagram of stability (on condition of considering the free trim when calculating the righting arms) completely defines the ability of the ship to return to the initial position after ceasing of the heeling moment, which caused the inclination.
 - (ii) It is impossible to ensure the immunity of a ship against capsizing in any situation regardless of environmental circumstances, prudence and good seamanship exercised by the Master. Some specific ocean vehicles are the only exception.
 - (iii) The improvements in stability usually brings forth the deterioration of other qualities of a ship, ensuing from changing the relations between the main dimensions, the hull shape, the stowage of cargo, the order of consumption of stores and ballasting.
- (i) a static or a dynamic effect of some heeling moments,
 - (ii) a reduction in stability caused by various phenomena,
 - (iii) circumstances bringing about a simultaneous reduction of stability and the arising of heeling moments.

The negative factors pertaining to these three categories are manifold and may be regarded as random. Because of the random character of these factors which must be considered while developing stability regulations, the requirements for the stability of ships may have but a comparative character.

In other words, we may speak only about the indirect requirements for the probability of non-capsizing and non-appearance of the heeling angles exceeding some given angles, under certain conditions (in certain situations).

Considering the above propositions the problem of intact stability regulations becomes that of development of the requirements for diagrams of stability which would provide a rational minimum of the danger from capsizing and of excessive heeling during the ship's operation.

The capsizing of a ship or her inadmissible heeling may result from:

Such approach to the intact stability regulations brings forth three main questions to be solved:

- (i) Under what conditions (in what situations) must the stability regulations be carried out?
- (ii) What must be the standard of requirements? Equal for all ships or different for ships of different purposes operating

in different conditions, the probability of non-capsizing and the probability of non-appearance of inadmissible heeling?

- (iii) What must or can be numerical values of the criteria defining the adequate intact stability?

The views of the authors on these questions are given in the following sections.

2. DANGEROUS SITUATIONS

The probability of simultaneous acting of all the unfavourably negative factors affecting the stability or causing a dangerous heeling is apparently very small. At the same time certain factors may be decisive for ships with some specific peculiarities of design and conditions of operation. Therefore it seems reasonable to identify a certain combination of negative factors (a certain situation) which must be carried out for testing of the stability of any ship. It is also reasonable to identify the additional factors which need to be regarded while testing the stability of ships with some peculiarities of design and operation.

The identified situation is to be tested and must satisfy three conditions:

- (i) The situation must be realistic enough on the probability of simultaneous acting on the ship of a combination of negative factors corresponding to this particular situation must be large,
- (ii) The consequences of the influence on a ship of the given combination of factors must be determined by as large a number of ship's parameters as possible such as for principal dimensions, shape of hull, architectural type, etc.
- (iii) The procedure of the calculation should be sufficiently simple.

Conditions (i) and (iii) are evident. As to condition (ii), it arises from the wish to generalize the main stability criterion, so that it could be applied to non-traditional types of ships of unusual architectural forms.

The main unfavourable factors which affects the ship's safety with regard to capsizing or the appearing of a dangerous

heel may be classified in the following way:

(a) Factors generating the heeling moments

- a1. The static pressure of wind on the side surface of a ship.
- a2. The dynamic effect of a gust on a ship.
- a3. The rolling effect of the waves.
- a4. Forces affecting a ship at circulation.
- a5. Crowding of passengers on one side of a ship.
- a6. Non-symmetrical position of cargo and stores in regard to the centre plane.
- a7. Shifting of the cargo.

(b) Factors reducing stability

- b1. Non-rational location of cargo which elevates the ship's centre of gravity.
- b2. The lowering of the centre of buoyancy when the ship's draft changes as a result of partial loading of cargo and consumption of the stores.
- b3. The appearance of free surfaces in tanks as a result of consumption of fuel, water, or oil.
- b4. Non-rational order of consumption of the stores which elevates the ship's centre of gravity or brings about larger than the necessary minimum free surfaces.
- b5. Icing of topsides.
- b6. Wetting of the deck cargo.
- b7. Reduction of ship's stability in following seas.

(c) Factors reducing stability and generating the heeling moments simultaneously

- c1. Non-symmetrical icing.
- c2. Non-symmetrical wetting of the deck cargo.
- c3. Crowding of passengers on a higher deck.
- c4. Water trapped on deck during the motion of a ship in heavy seas.

From these factors a6, b1, b4 may occur only in case of breaking by the Master the instructions on the loading of cargo and the succession of consuming the stores. Factors c1 and c2 may arise only when the ship takes a certain course relative to wind and waves. The heel which appears as a result of their effect can be corrected by changing the course. In practice it is advisable to eliminate the danger of factor a7 by means of some designing

measures and corresponding stowage. Therefore it seems right not to consider these factors in stability regulations. We must consider the necessity to eliminate the very possibility of their appearance during the ship's operation.

Factors b3, b5, b6, may under certain circumstances exercise continuous effect on the ship and they must be taken into account while testing the stability for any situation. Naturally, for certain ships they can physically appear for example, the wetting of the deck cargo on the timber-carriers, the icing of the ships going in areas where icing is possible.

We can hardly imagine the situation when all the adverse factors would affect the ship at the same time. For that we must accept the possibility that the Master would permit the ship to perform circulation at full speed in rough seas having a heel caused by the static effect of wind on her windage area, when all the passengers would crowd on one side, and at the most unfavourable moment, with the ship being on the crest of the following wave, she would suffer the dynamic effect of the squall. It is evident that, exercising prudence and good seamanship, the Master will avoid such a situation. The possibility of simultaneous action on the ship of the unfavourable combination of wind (factors a1 and a2) and rolling (factor a3) seems much more real. As to the other factors (a4, a5, b7, c3, c4,) it is more advisable to regard their influence separately or in combination with factor a3 when it seems relevant. Some of these factors may prove decisive for ships of different types with different peculiarities of design or other special features.

Thus, it is expedient to demand that ships suffering rolling in heavy seas should not capsize as a result of the squall of a certain force. Irrespective of this, the stability of a ship must be sufficient for her to avoid a dangerous heel or loss of stability when influenced by each of the following factors:

- heeling moment caused by the forces arising during circulation;
- heeling moment caused by the crowding of passengers on one side;
- heeling moment caused by water trapped on the deck during the motion of a ship in rough seas;
- reduction of stability while going in following seas;
- heeling moment and reduction of stability due to crowding of

passengers on a higher deck. heeling moments caused by specific distribution of hydrodynamical and aerodynamical forces affecting the hull and the appendages of novelty crafts such as aircushion vehicles, hydrofoil boats, etc.

3. CONDITIONS OF NAVIGATION

It is obvious that the dangerous situation may occur at any moment of the trip. Strictly speaking, while developing the stability regulations, one must consider a different probability (for ships with different character of service) that the dangerous situation will occur at the worst load condition. But, since it is rather difficult to consider this circumstance, we may accept the equal (for all ships) probability of meeting with the dangerous situation while a ship is in the worst load condition with regard to stability.

Since the probability of a certain combination of weather conditions depends on areas of navigation, it is reasonable not to put forward equal requirements for the stability of ships navigating in different geographical areas. The research, specially carried out in (4), proved that the probability of capsizing of the ship in different regions of the ocean differs by several orders due to different exceedance of the dangerous combination of wind and waves.

In the modern state of short-term weather forecasting and the existence of means of communication, ships which are a certain distance off the port, may avoid a dangerous situation. With regard to this, it is advisable to vary the rigorousness of stability requirements, easing calculated weather conditions for the ships, which due to their service are used on limited distances from their ports of shelter.

4. RIGOROUSNESS OF STABILITY REQUIREMENTS

For the assumed dangerous situation and chosen conditions of navigation for which stability calculation will be made the rigorousness of the relevant requirements is determined by the combination of the calculated wind speeds and amplitudes of rolling, as well as by the values of the maximum permissible angle of heel during circulation of the ship and crowding of passengers on one side, by the values of the minimum permissible stability in rough seas

with the water trapped on the deck. All these values may be identified on the basis of:

- generalization of data on stability of successfully navigated ships,
- analysis of stability of the capsized ships and those which had inadmissibly large angles of heel,
- carrying out special experiments.

But using only one of the above three ways does not seem right. If we take into account the stability of successfully navigated ships only, the requirements may be far above the margin of safety. If we take into account the data of capsized ships only, the requirements are inadmissibly under the margin of safety, since we do not know how much the stability of a capsized ship is under the margin of safety

5. CALCULATION SCHEMES FOR THE BASIC DANGEROUS SITUATION AND FOR ADDITIONAL FACTORS

For estimating the ship's ability to withstand the simultaneous action of wind and rolling, it is possible to apply the scheme used in (1). Here the ship is considered to have sufficient stability in regard to the main criterion if under the most unfavourable condition of loading, the dynamically applied heeling moment produced by wind pressure is equal to, or less than minimum capsizing moment. The main criterion is adopted on the assumption that the ship is rolling in resonance conditions and is at the same time exposed to dynamic effect of wind pressure under squall conditions. With the above assumptions in mind, it is expected that the most unfavourable conditions for stability would arise when, at the beginning of the external heeling moment caused by a squall, the ship has the greatest list from resonance roll in the direction opposite to that of heeling moment and performs a swing under the action of both the heeling and the disturbing moments of waves.

The compilation of an energetic balance accounting for the moments produced by the forces affecting a rolling ship, as well as for the heeling moment from the squall with regard to the swing performed by the ship, enables us to obtain an approximate equation of works for determining the ship's heeling. If we then assume that the disturbing moment of waves is fully compensated by the damping moment and, furthermore, that the external moment applies instantly and keeps constant during the whole period of swing, it becomes possible to fix a simple procedure for the determination

of minimum moment capable of capsizing a ship.

The minimum capsizing moment is to be determined from the diagram of statical stability assuming the work performed by the capsizing and righting moments to be equal and taking into account the effect of rolling. For this purpose the curve of statical stability is continued into the region of negative abscissa for a length equal to the amplitude of rolling θ_m see Fig. (1). Then a straight line \overline{mk} parallel to the abscissa is chosen so that the stripped areas S_1 and S_2 are equal. The ordinate \overline{Om} corresponds to the minimum capsizing moment: $M_{caps} = \overline{Om}$. In accordance with the proposed concept, the main stability criterion may be written as follows: $M_{heel} \leq M_{caps}$. Here M_{heel} is a dynamically applied heeling moment produced by wind pressure.

The heeling moment produced by the centrifugal force at the helm circle is proposed to be estimated from G.A. Firsoff's formula:

$$M_{heel} = 0.233 \frac{D}{g} \cdot \frac{v^2}{L} \left(z_g - \frac{T}{2} \right)$$

here D - displacement of the ship, t;
 L - length of the ship measured at the load waterline, m;
 T - draught of the ship, m;
 z_g - height of the ship's centre of gravity above the base line, m;
 g - acceleration of the gravity force, m/sec²;
 v - speed of the ship, at the beginning of the helm circle assumed as equal to 80 per cent of the full speed, m/sec.

In this formula the radius of circulation is so accepted as to provide the maximum value of the heeling moment caused by the centrifugal force.

It is suggested that the heeling moment due to passengers crowding on the ship should be estimated with regard to the peculiarities of space and deck planning for the ship concerned.

As to crowding of passengers on the ship's side and the helm circle performed with sidescuttles, doors, etc. being open, they are not prohibited if the weather is calm. These factors usually take place when the ship is approaching the port, and their effects may form a sum. Therefore, they are proposed to be considered both apart and summed up.

The influence on the ship's stability of the crowding of passengers on the higher deck may be estimated from the known formulas of statics given for a load shifted along the vertical line.

Water trapped on the deck may be dangerous for comparatively small low-freeboard ships ($L < 100m$) where the size of the deck-well is relatively large. The danger is connected with the appearance of a specific heeling moment, whose character of influence depends on the ship's course relatively to waves.

When going in countering or following seas the ship has inevitably an angle of heel, and after burying bow or stern into the wave performs a dynamic inclination under the action of the heeling moment M_{w1} , created by the water trapped on the deck and acting like a liquid load (5), (6), (7). The moment M_{w1} is shown in the fig.2 as function of heeling angle θ_L . The dynamic effect of this moment serves the reason of inclination of a ship by angle θ_L , which is easy to determine processing from the equality of areas S_1 and S_2 , limited by the curve M_{w1} and the curve of restoring moments M_R . Fig (2). Normalizing the angle value θ_L one may avoid the danger of capsizing in the given situation.

When the ship is going in beam seas the danger of capsizing may arise due to the systematic flooding of the deck over bulwark. The main feature of the systematic flooding is that the water from the previous wave has not flown overboard by the moment the next wave covers the deck. The periodic disturbing action of the waves is aggravated by the influence of the static moment M_{w2} , which as different from the moment M_{w1} does not disappear when the gunnel is submerged under the water ($\theta = \theta_0$), but preserves some finite value (6). Under its influence the ship acquires a static heel θ_s Fig. (3), and her rolling becomes unsymmetrical (8). To avoid the danger of capsizing under the influence of moment M_{w2} , it is necessary to fulfill the requirement given in (9)

$$\theta_v - (\theta_s + \varphi_0) \geq C_\theta$$

where θ_v - angle of the static stability diagram vanishing point;

φ_0 - amplitude of relative rolling;

C_θ - some positive value which

can be the object of the regulation.

The reduction of ship's stability in following seas can take place for a more or less considerable period of time according to the relation between the ship's speed and the mean speed of waves. As a rule, this reduction proves the biggest when a ship's middle gets on the crest. The reduction, for a given shape of wave profile, may be determined from the hydrostatic calculation (10) and be considered in stability regulations. It is necessary to point out that due to the irregularity of the seaway, the estimation of this factor in the determined presentation can be regarded arbitrary to a great extent. Therefore, direct application of this estimation in the form of restoring moment reduced by the action of following seas may lead to superfluous rigorousness of intact stability regulations. At the moment it is more reasonable to recognize the approach, based on the indirect consideration of the taken factor as a requirement to some elements of the stability diagram for calm weather condition, as it is done, for example, in (1). The alternative to such an approach is the correct analysis of the factor under study from the probability standpoint. But such analysis will inevitably lead to the estimation of a ship's capsizing probability from the standpoint of random processes (11), (12), and consequently, to probabilistic scheme of stability regulations.

6. CONCLUSION

The authors consider that on the basis of all this, taking into consideration the national experience of intact stability regulations, it is possible to evolve a satisfactory scheme of regulations, which would allow for physical peculiarities of interaction of a ship with the factors reviewed in this paper. Although the major factors dangerous in relation to ship capsizing are studied well, it is thought expedient to undertake a further study of the process of a ship's capsizing in a relatively seldom situations connected with such factors as the loss of manoeuvrability, broaching, parametrical rolling in following seas, etc.

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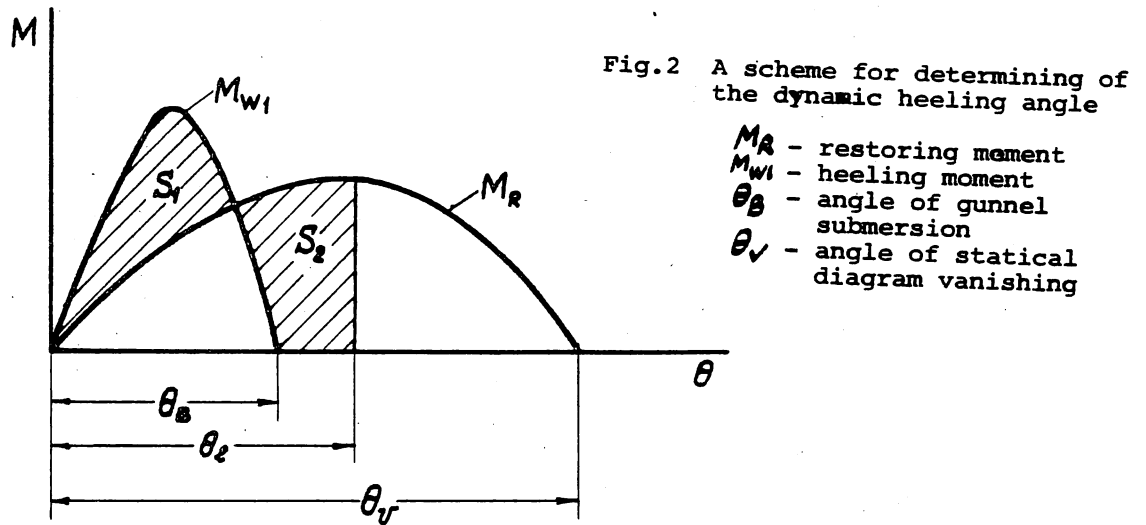
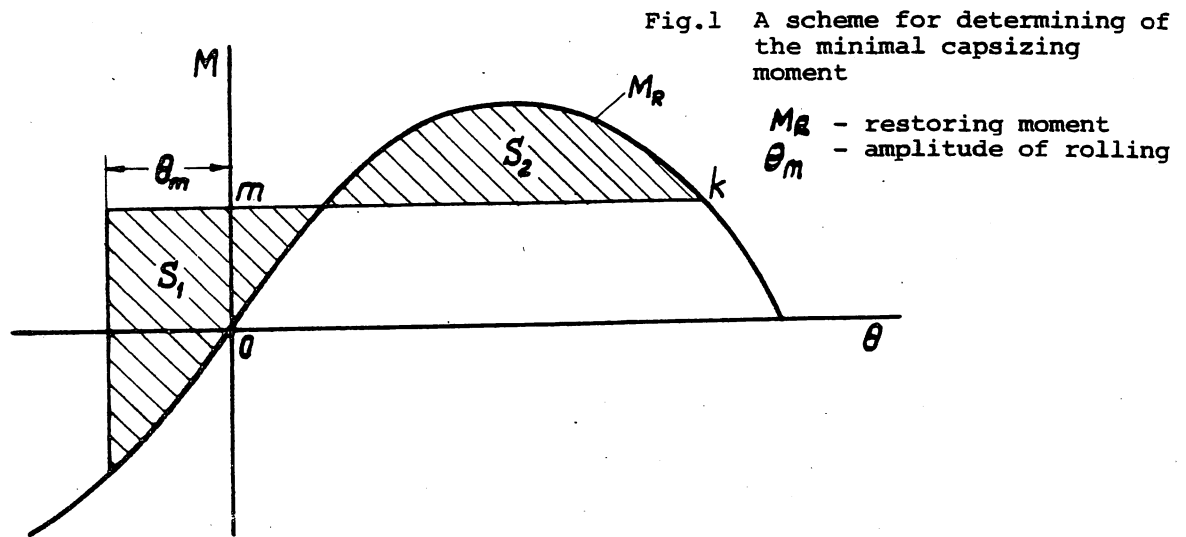


Fig.2.

