

EQUILIBRIUM, BALLAST CONTROL AND FREE-SURFACE EFFECT COMPUTATIONS USING THE SSTAB SYSTEM

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Abstract

The methodology presented in this paper is used to calculate the equilibrium and to perform the static stability analysis computing the accurate free-surface effects. The procedure implemented uses a model composed by sets of compartments defined by facets, which allows the analysis of a great variety of shapes. The equilibrium computations are supported by a new algorithm based on the Newton Raphson method that computes, at each step of the search for the equilibrium position, the Hydrostatic Stiffness Matrix using the finite difference approach. This technique allows fast equilibrium computations, and changes in the loading conditions or in the buoyancy are quickly re-evaluated from the current equilibrium position.

The free-surface effect is computed with the change of form of the liquids inside the tanks (ballast, fresh water, diesel, mud, oil, etc.) as the floating system inclines, producing a change in the position of each tank's gravity center. The global effect may be important if the weight of the liquids are significant to the global displacement of the unit and if the subdivisions are being evaluated. In the present work, we present comparative results of stability diagrams built with free-surface effects evaluated both with the change of form in the liquids, and without it, using the traditional KG penalizing approach.

Keywords

Stability Software; Computer Graphics; Rules and Regulations; Numerical Simulation; Geometric Modeling.

1. INTRODUCTION

With the evolution of the geometric modelers, several new challenges in the field of platform and ship stability were given significant importance during the last decade. Most systems for vessel stability are based on transversal-section modelling approach and tend to use this methodology in the

computation of inclinations, free-surface effects, stability diagrams, etc. This technique is very useful for ships and single-hull vessels, even though the internal compartments are usually more complex. For semi-submersible platform geometries the use of this methodology leads to problems and it is difficult to be used in every separated geometry compartment, such as inclined bracings and

irregular intersecting internal trunks.

We here present our approach for a complete static stability system based on a very general surface modeller (MG [1-6]) and on a stability computational algorithm set, implemented in program Sstab, that treats the model with its 3D generality, considering large displacements and angles, and consistently taking into account the free-surface effect.

These modelling concepts allows the fast generation of single-hull floating bodies and even complex semi-submersible platforms, as will be presented in the following sections.

2. MG MODELER

MG has several published references, and its main features are the direct manipulation technique using the interface plane [1], the dual parametric-mesh surface representation [2,6], the use of a topological data structure to automatically identify closed regions [2,6], and the parametric surface intersection algorithm that computes resultant meshes with good geometric quality that can even be used in finite element simulations [3,4,5,6].

MG generates the geometric models to Sstab and also creates the basic parameters, such as: compartment classification (hull, ballast, fresh water, void spaces, etc), fluid specific weights specifications, keel reference plane definition, etc. This task is achieved with the use of an attribute manager library [7] that can be customized independently from the modeller itself.

To exemplify the use of MG in the construction of a ship model, Figure 1 shows a ship drawn with MG.

Figure 2 shows a complete semi-submersible oil platform, including the internal compartments used in stability analysis.

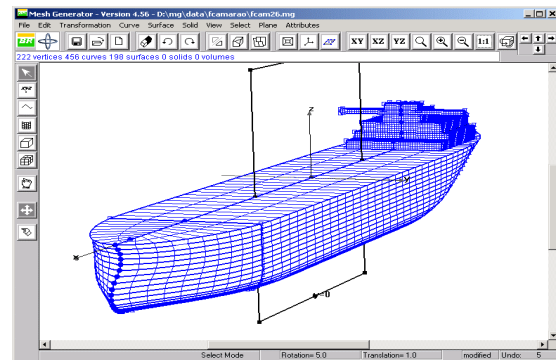


Figure 1 – A ship modelled with MG.

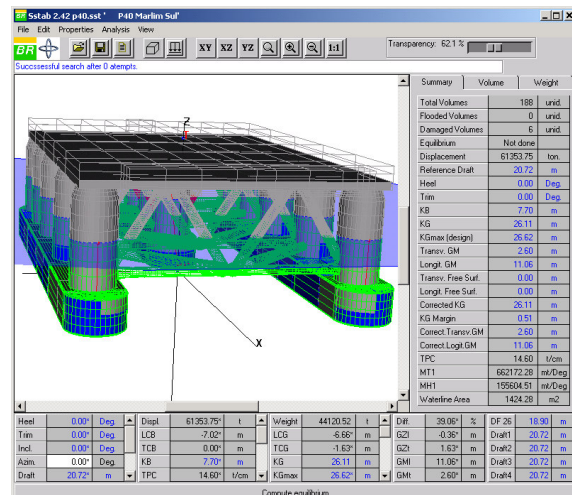


Figure 2 – A semi-submersible platform.

3. BACKGROUND AND CONCEPTS

All computations are performed over a boundary model represented by a set of faces, then, with the Green Theorem, the areas are computed as line integrals and, with the Stokes Theorem, the volumes are computed as surface integrals. The faces must present normal vectors pointing outside the volumes in order to make the computations consistent. If MG is used, this property is guaranteed during the identification of the closed regions, when surface patches that points to the inside of

volumes have the faces oriented correctly in the exportation process.

3.1. Buoyancy Computations

Figure 3a shows the model of a pyramidal compartment with square basis, composed by outside-oriented triangles. If the compartment in Figure 3a were completely submerged, Sstab would compute the volume by each face's individual contribution with respect to the global Cartesian origin. As the result of the computation, not only the scalar volume is obtained but also the center of buoyancy B , as shown in Figure 3b.

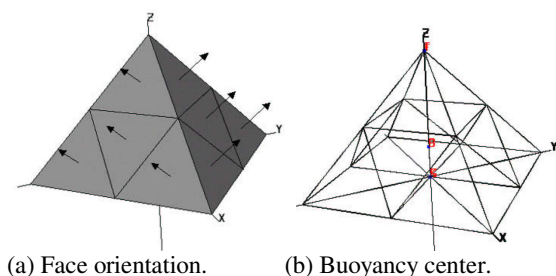


Figure 3 – Volume with surface integration.

If a volume is partially submerged, a cut is performed during the integration process classifying the faces located completely *above* the water surface, the ones that are completely *bellow*, and the *partial* faces that have portions above and bellow the water limit. Faces *above* are not considered in the integration process. Faces *bellow* are considered with all vertex contributions, and faces classified as *partial* are computed by a cutting algorithm shown in Figure 4. Sstab draws the *wet* faces in green and the *dry* faces in gray.

3.2. Hull Compartments and Hydrostatics

After including every compartment in a single model, a classification is applied to each individual compartment. The volumes that are responsible for buoyancy are classified as *hull compartments*.

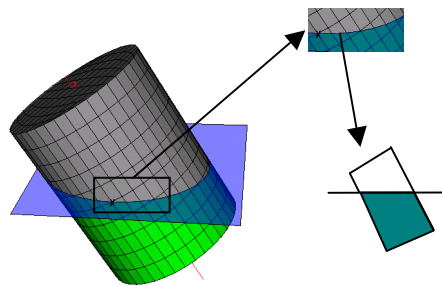


Figure 4: Buoyancy of partially wet volumes.

During the volume computation process, every hull compartment partially submerged cut by the waterplane presents a resultant area, named *cutface*, given by the intersections of each face edge with the plane. The program computes the total cut face area of the water plane, performing the weighted average of each partially submerged volume cut face area. All area parameters are also computed so that, at the end of the integration, it is possible to plot (Figure 5) each cut face area edges (lines on top of the columns), individual centers of flotation (square marks), and the global center of flotation (square mark with 'F' character).

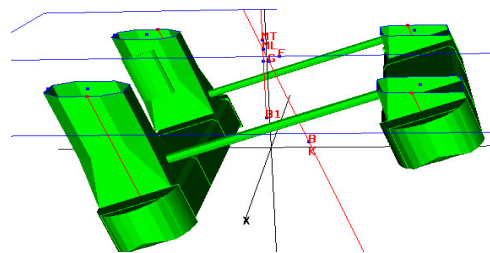


Figure 5 – Waterline area and global properties.

3.2. Load compartments

All compartments having fluid in their interior with the liquid shape changing as the model inclines are classified as *load compartments*. The methodology implemented computes the free surface by iteratively *cutting* the compartment at different plane levels until the computed volume equalizes the user-specified value. This is done for every compartment at

each assumed position of the system and it is not a time-consuming operation due to improvements made on the searching process [4]. At each computed position, the total volume and weight are kept constant but the center of gravity of the modified fluid is updated and affects the global CG. Sstab shows the water inside load compartments in blue and uses gray for dry parts. Figure 6 shows several load compartments (ballast tanks) with their free surface shown (note that the sea surface plane was used to cut hull compartments and each ballast tank has its own fluid level position).

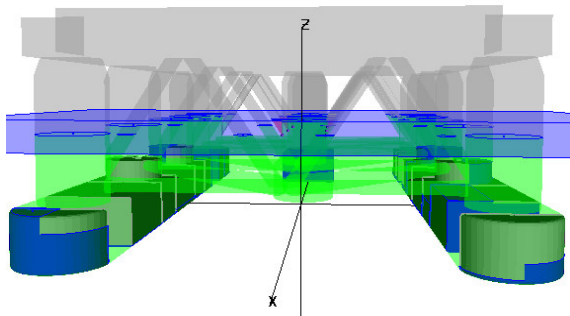


Figure 6- Ballast tanks as load compartments.

3.3. Equilibrium

Figure 7 shows the relevant degrees of freedom taken into account in equilibrium computations. Azimuth rotations, surge and sway movements are assumed not relevant for static equilibrium. Azimuth is only important in damage analyses where the critical inclination angle is no longer the global X axis.

The following types of equilibrium computations are available:

1. Displacements, heel and trim moments;
2. Displacements and trim moment (given heel);
3. Displacements only (given heel and trim).

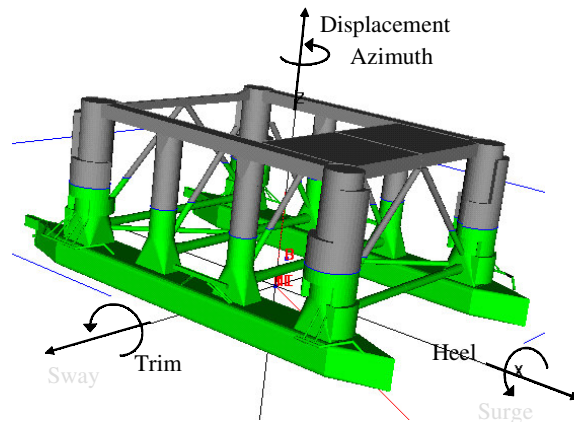


Figure 7 – Equilibrium degrees of freedom.

The first type consists of the generic equilibrium where all forces and moments must be in equilibrium. The initial relative position of the water plane and the floating system defines the unbalanced moments and forces, so the unit is moved until the equilibrium position is obtained. The first type of equilibrium is used to obtain the correct floating status of the unit for every intact and damage situations.

The second type of equilibrium consists, for a given heel angle, in obtaining the displacement and *trim* (composed heel + trim angles) moment equilibrium. This is the most common equilibrium computation used for each inclination angle in the construction of the stability diagram. The second type of equilibrium can be done for different angles of azimuth to obtain the critical-axis diagrams.

The third type consists in performing only displacements equilibrium, accepting heel, trim and azimuth as inputs.

The method employed to calculate the equilibrium uses a technique of computing the next equilibrium position by the evaluation of a hydrostatic stiffness matrix assembled by the finite-difference method. A 3x3 matrix is built at each step performing small movements in each direction of equilibrium. With a small movement in the vertical direction the program

computes the change in displacement, as well as the variation of heel and trim moments, defining the three elements of the first line of the matrix. The other two lines are built with small changes in heel and trim angles, from the current position. The next position vector is a result of the current error vector multiplied by the inverse of the stiffness matrix.

If the equilibrium position is not far from the initial position, and if there are no significant changes in the water plane area, the equilibrium is computed with few iterations.

Figure 8 exemplifies the algorithm's behavior. Only three iterations were necessary to obtain the final equilibrium position with a precision of 0.01 percent of the current displacement and 0.001 meters of arms distance errors. This equilibrium was obtained without the free-surface effect, so the algorithm dealt only with changes in the buoyancy center to eliminate the unbalanced moments and displacements.

Figure 9 shows a comparison of the equilibrium with and without the free-surface effect. In Part (a) the equilibrium at 30 degrees of heel was obtained without the free surface changes. In Part (b), where the free-surface effect was set, the equilibrium position was obtained at 34 degrees of heel.

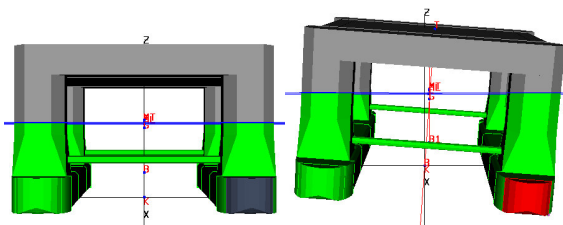
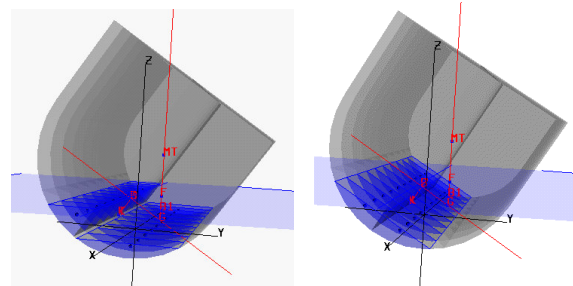


Figure 8- Equilibrium of a damaged platform.



(a) heel = 30°

(b) heel = 34°

Figure 9- Equilibrium without and with the free-surface set.

With the techniques presented so far, Sstab can perform all volume and equilibrium computations.

4. SSTAB FEATURES

4.1 Hydrostatic tables

Sstab presents hydrostatic properties in a single table with several graphs, one for each measurement involved. The user can choose the properties to customize the assembling of the hydrostatic table.

The list of drafts that is used in the construction of the hydrostatic table is also parameterised by the user, who can include the necessary positions and displacements. Inclined hydrostatics are also available and is very useful in emergency situations.

Figure 10 shows a typical hydrostatic dialog, with the graphs referent to the variation in moments to alter one degree of trim and heel selected.

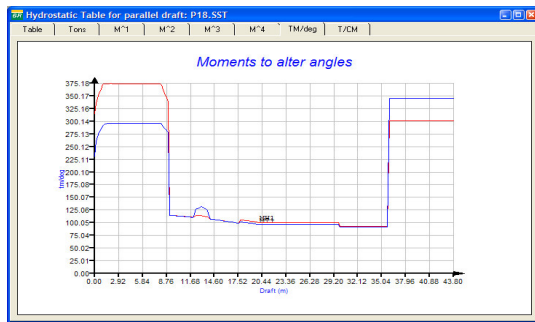


Figure 10- Hydrostatic table with graph tab.

4.2. Stability Diagrams

Given a certain loading condition and a buoyancy configuration, Sstab builds stability diagrams applying inclination angles for a given Wind incidence angle and velocity. At each equilibrium step, Sstab computes the second equilibrium type defining the vertical position and the trim to eliminate the trim moment. Figure 11 shows a typical stability diagram.

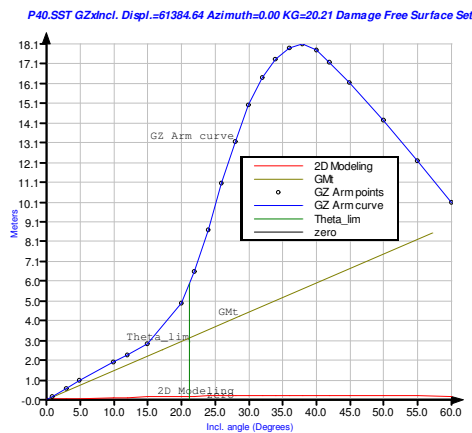


Figure 11- Stability diagram generated by Sstab.

Sstab also presents the table associated with the graphs in a separated window. Rules and regulations are presented in a table where the criteria of each regulation are shown with the parameters computed with the obtained values (Figure 12).

P40.SST GZincl. Displ.=61384.64 Azimuth=0.00 KG=20.21 Damage Free Surface...

Point	Inclin. (deg)	Y Angle (deg)	GZ arm (m)	Draft (m)	Wind Heel (m)	FP Height (m)	Lowest FP
1	0.000	0.000	-0.006	20.750	0.049	18.874	26
2	1.000	0.000	0.195	20.750	0.057	18.212	28
3	3.000	0.000	0.591	20.750	0.073	16.670	28
4	5.000	0.000	0.981	20.749	0.088	15.315	11
5	10.000	0.000	1.933	20.748	0.125	10.892	11
6	12.000	0.000	2.306	20.748	0.140	9.097	11
7	15.000	0.000	2.867	20.748	0.159	6.386	11
8	20.000	0.188	4.922	21.512	0.189	1.227	11
9	22.000	0.167	6.574	21.730	0.200	-0.808	11
10	24.000	0.103	8.689	21.856	0.210	-2.777	11
11	26.000	0.057	11.049	22.024	0.219	-4.765	11
12	28.000	0.016	13.217	22.319	0.227	-6.951	11

Parameter	Value	IMO 1989	DNV 1989	ABS 1991	NMD 1992
Theta_0	0.030		Theta_0:15		
Theta_1	0.287	Theta_1:17		Theta_1:21.2	Theta_1:17
Theta_2	Out of range				
Theta_dif	21.206				
Theta_lim	21.206				
Stab_range	20.919	Stab_range:7		Stab_range:7	Stab_range:7
GZ/vHLL_max	28.139	GZ/vHLL:2		GZ/vHLL:2	GZ/vHLL:1
Area_ratio	17.706	Area_ratio:1.0		Area_ratio:1.0	Area_ratio:1.0
Wind_velocity	40.000	Wind_vel>25 m/s	Wind_vel>25 m/s	Wind_vel>25 m/s	Wind_vel>25 m/s
GM_transv	8.500				

Tunnel Test 2D Modelling 3D Modelling GMt curve GZ curve GZ points Theta_lim

Figure 12- Stability table with classifications.

In Figure 12, green colours indicate positive attendance to the corresponding criterion.

4.3. Wind Heeling Curves

Two possibilities of wind-curve computations are available in Sstab: *Tunnel Tests* and *3D Modelling*. The Tunnel Tests are simply a database of all data obtained in tunnel tests. Sstab stores the results for every angle of incidence (azimuth) and heeling angle to interpolate in this table in order to obtain all remaining values.

3D Modelling is an implementation that uses the well known Equation 1 (Item 3.2.3 in [8]).

$$F = 0.5 C_s C_h P V^2 A, \quad (1)$$

In this equation F is the wind force (Newton), C_s is the shape coefficient (non dimens.), C_h is the height coefficient (non dimens.), P is the air mass density (1.222 kg/m^3), V is wind velocity (m/s), and A is the projected area of all exposed surfaces (square meters).

The 3D Modelling uses the areas of the compartments assigned to be resistant to wind.

There is a specific dialog for that, where the user can select the faces that are not exposed to wind. At each position, all assigned areas are projected in the wind direction, and Sstab computes the areas and centers above and below the sea surface. The final wind moment computed is then divided by the displacement to obtain the wind-heeling arms. Figure 13 shows a model with the wind elements above the main deck represented to be used as wind resistant.

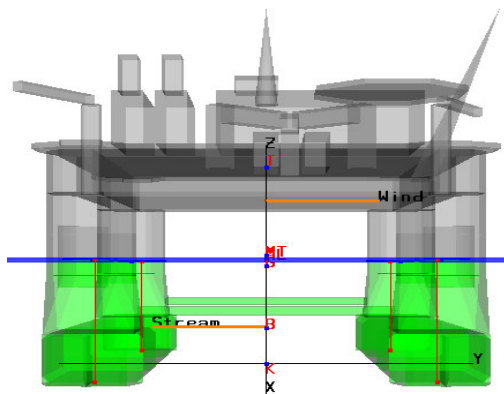


Figure 13- 3D Modelling wind heeling lever.

4.4. Damage Stability

Sstab computes the weight of the unit with all weight items defined plus the weight of the load compartments, with the center of gravity computed for each individual tank. Sstab also computes the buoyancy with the active and intact volumes, keeping the buoyancy and weight information separated. For each damage condition the user can specify the precise state of each tank, not having to make simplifications with the model, such as using negative coefficients. The volumes can be intact, flooded or damaged. The difference between damage and flooding is simple: the first is filled with water to sea surface, and the second is not related with the sea surface, so tanks can be flooded to their top or maximum volumes (external volume reduced by permeability values).

After defining the damage condition, the user can compute the precise equilibrium position

and observe the critical inclination angle. After that, the assemblage of the stability diagram and the computation of the corresponding wind curve is done automatically by the program. Figure 14 shows a damage position equilibrium computation, given by heel, trim and displacement positions, and also shows the critical axis with a blue line.

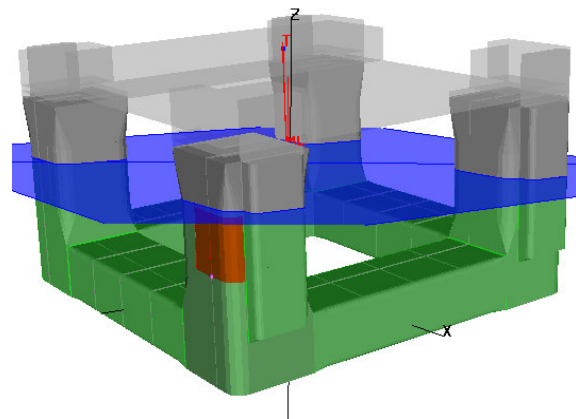


Figure 14- Damage position equilibrium computation.

Figure 15 shows the damage stability diagram computed with the critical equilibrium position shown in Figure 14. In Figure 15, the sudden reduction in the GZ graph is related with the flooding of some compartments, exactly nearby the down flooding angle region.

GZincl. Displ.=79986.00 Azimuth=34.28 KG=25.74 Damage Free Surface Set Void_Spaces_AP_33_AP_34_D.

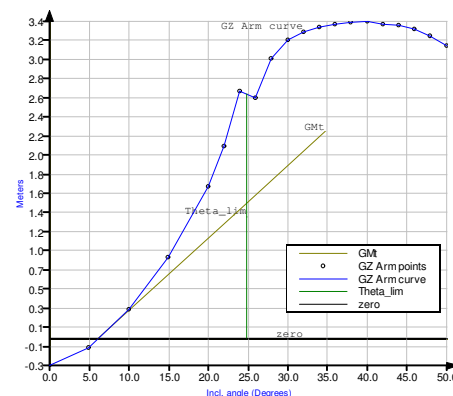


Figure 15- Damage stability diagram evaluated for critical configuration shown in Figure 14.

4.5. Cross Curves

Cross Curves can be built with Sstab by specifying only the sequence of inclination angles. This assemblage uses the list of drafts defined for the construction of the Hydrostatic Table to compute a list of displacements associated to these drafts, when the platform is in even keel. After that, Sstab will heel the model for each specified value and then compute the correct displacement, storing the heeling arm. At the final stage, it is possible to obtain every Stability Diagram for a given buoyancy state and azimuth.

Figure 16 shows the Cross Curves Dialog, with the first tab selected. The user can specify the displacement and observe the corresponding Stability Diagram and Cross Curve Table in the third and second tabs, respectively.

Sstab can also plot the *surface of stability*, what is done with the insertion of a third axis related to the heeling angles to the 2D plot of Figure 16. This plot is shown in Figure 17. The red curve corresponds to the current displacement's Stability Diagram.

All graphs and tables that Sstab shows can be directly exported to other applications.

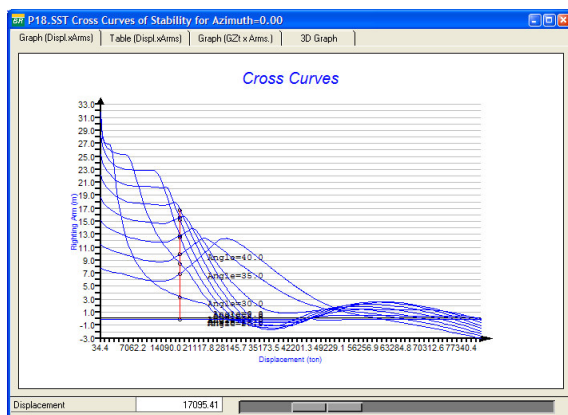


Figure 16- Sstab Cross Curves Dialog.

The 3D plots can be exported as images or PostScript files. The 2D graphs can be exported

in several different image formats and several different vector formats. The tables can be exported as ASCII files with tab separations, in Html format, or in LaTeX format.

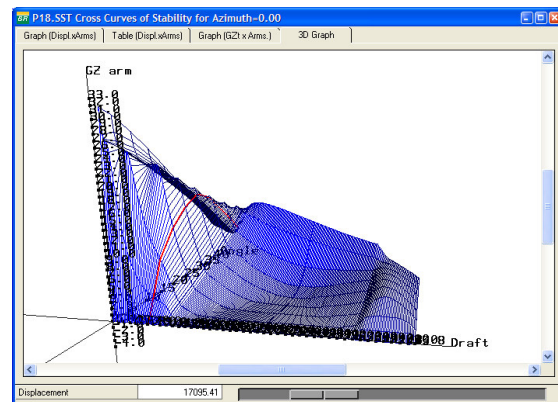


Figure 17- Sstab *Surface of Stability* built with cross-curves.

4.6. Longitudinal Strength

Sstab computes the Longitudinal Strength of ship hulls using the volume algorithm to evaluate the buoyancy and the distribution of the light weight and other loads to compute the bending moments and the shear at each specified section. The main issue here is that the Longitudinal Strength evaluation is consistent with the equilibrium of the floating system, so the user can evaluate the longitudinal strength at any equilibrium position, either in even keel or not.

To perform the buoyancy at each desired interval, between two adjacent sections, an algorithm that cuts the floating compartments at the sea-surface, at the first longitudinal section, and at the final longitudinal section, was implemented. Figure 18 shows an instance of this computation.

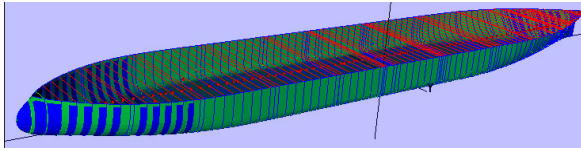


Figure 18- Interval buoyancy computation for longitudinal strength.

Figure 19 shows the Dialog with the elements related to the longitudinal strength. The user informs the envelopes of bending and shear, and the system computes the safety factor.

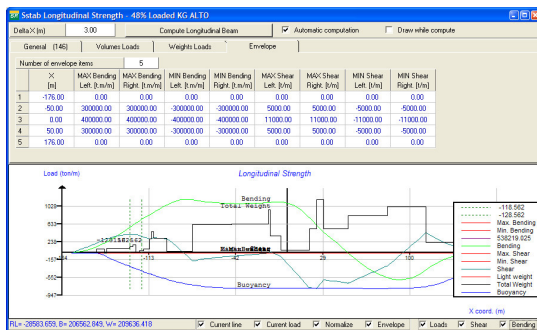


Figure 19- Sstab Longitudinal Strength Dialog.

4.7. Mass Matrix

Sstab uses its capability of computing volumes to obtain with good accuracy the mass matrix of the platform. For that, we use the result of the inertia of a tetrahedral formed with a triangle connected to the origin [9]. To obtain the moment of inertia with respect to the Cartesian axes we use the following formula:

$$I_0 = 0.8 (\mathbf{b} \cdot \mathbf{b}) + 0.05 (\mathbf{p}_1 \cdot \mathbf{p}_1 + \mathbf{p}_2 \cdot \mathbf{p}_2 + \mathbf{p}_3 \cdot \mathbf{p}_3), \quad (2)$$

Where I_0 is the moment of inertia with respect to origin, \mathbf{p}_1 , \mathbf{p}_2 , and \mathbf{p}_3 are the triangle points. With this equation, it is easy to compute the moment of inertia around the 3D Cartesian axes, and also the products of inertia.

Weight items can also have their moments of inertia, and the program performs the transference to the evaluation point using the parallel-axis theorem. Hull elements are treated in a different way, using the inertia of a shell with a thin width, assuming that all the mass is

concentrated in the boundary of the compartments. Figure 21 shows the Mass Matrix Dialog of a FPSO, shown in Figure 20. The elements above the main deck are considered mass compartments that have a specific weight compatible with the total weight of each module. Non homogeneous modules can be set with the position of the center of gravity of each module. Figure 18 shows the inertias of each hull volume, cargo tanks, ballast tanks and mass compartments.

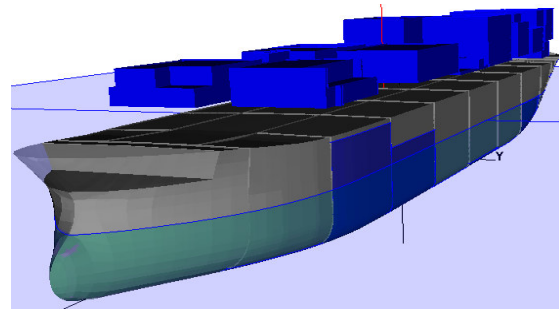


Figure 20 Deck modules represented as mass compartments.

Vol	Name	Active	Spec/W	Weight	Xcg	Ycg	Zcg	Ixx	Iyy	Izz
1	PS4	Yes	24.341	53638.0	-1.7	-0.7	22.3	2.72e+007	4.15e+008	4.27e+008
2	TQ OIL 1	Yes	0.920	11621.9	109.4	-0.0	5.6	5.49e+005	3.24e+006	3.54e+006
3	TQ OIL 2	Yes	0.920	16099.2	65.9	-0.0	13.3	1.52e+006	2.29e+006	1.93e+006
4	TQ OIL 3	Yes	0.920	16098.7	34.0	-0.0	13.3	1.52e+006	2.30e+006	1.93e+006
5	TQ OIL 4	Yes	0.920	13556.6	-45.5	-0.0	5.6	6.26e+005	4.74e+006	5.09e+006
6	TQ OIL 5	Yes	0.920	493.4	-90.5	-0.0	0.5	1.74e+004	-7.28e+003	1.01e+004
7	TQ OIL 1 BB	Yes	0.920	243.7	123.3	15.1	0.7	2.57e+003	2.22e+004	2.47e+004
8	TQ OIL 1 BE	Yes	0.920	243.7	123.3	-15.1	0.7	2.60e+003	2.42e+004	2.67e+004
9	TQ OIL 2 PS	Yes	0.920	12863.2	97.5	18.7	13.4	1.04e+006	1.79e+006	1.35e+006
10	TQ OIL 2 SB	Yes	0.920	12727.8	97.5	-18.7	13.2	1.01e+006	1.76e+006	1.33e+006
11	TQ OIL 3 PS	Yes	0.920	9500.0	65.8	18.7	9.7	5.15e+005	1.08e+006	1.01e+006
12	TQ OIL 3 SB	Yes	0.920	410.7	65.8	-17.9	0.5	7.67e+003	3.04e+004	3.80e+004
13	TQ OIL 4 PS	Yes	0.920	410.7	34.0	17.9	0.5	7.59e+003	3.46e+004	4.26e+004
14	TQ OIL 4 SB	Yes	0.920	410.7	34.0	-17.9	0.5	7.60e+003	3.32e+004	4.08e+004
15	TQ OIL 5 PS	Yes	0.920	10000.0	2.2	18.3	1.1	2.07e+004	8.44e+004	1.04e+005
16	WHS 100 KCB	Yes	0.920	8000.0	9.9	-10.9	-0.0	1.40e+004	6.79e+004	6.76e+004

Figure 21- Mass Matrix Dialog showing the individual properties of each compartment.

Figure 22 shows the final Mass Matrix that can be exported to programs that compute vessel movements. Note that the evaluation point can be informed by the user, and there is an easy way to put it on the CG position.

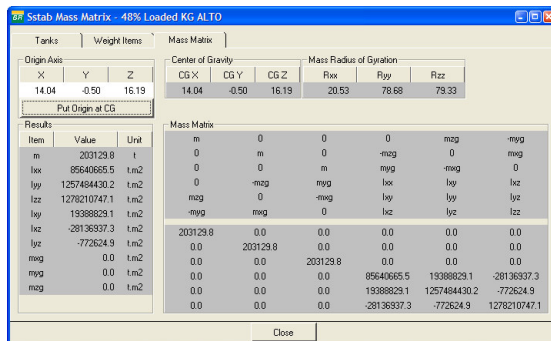


Figure 22 Dialog showing the mass matrix.

5. DESIGN PROCEDURES

Sstab stores the conditions of each separate loading condition and flotation status to keep all verifications made with the program available at any moment. All necessary parameters to restore a certain condition are kept in the data structure, including the critical axis azimuth and the equilibrium conditions, allowing Sstab to plot the model in the equilibrium position by just selecting the desired condition.

Figure 23 shows the dialog that controls the system conditions. Every intact, damage or flooded situation can be stored in the correspondent category, all parameterised by the user.

6. USE ON BOARD

Sstab is now being used on board of offshore platforms in Brazil helping ballast operators to perform their daily calculations and reports. Figure 24 shows the Report Dialog that the operators use in daily work, with the environment and ballast data that they have to deal with. The use of the program enables the operators to perform their tasks with the help of the graphical interface.

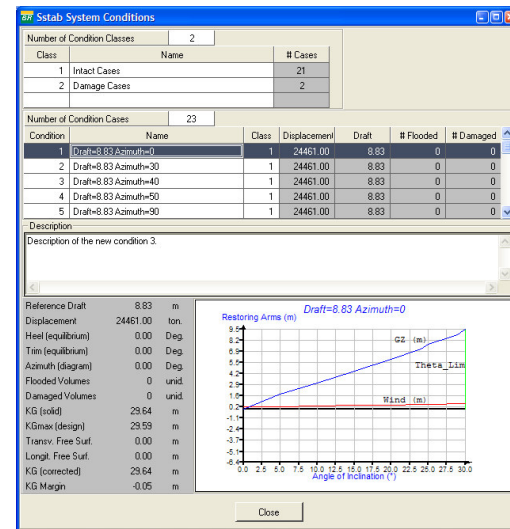


Figure 23- Conditions Dialog showing the current condition.

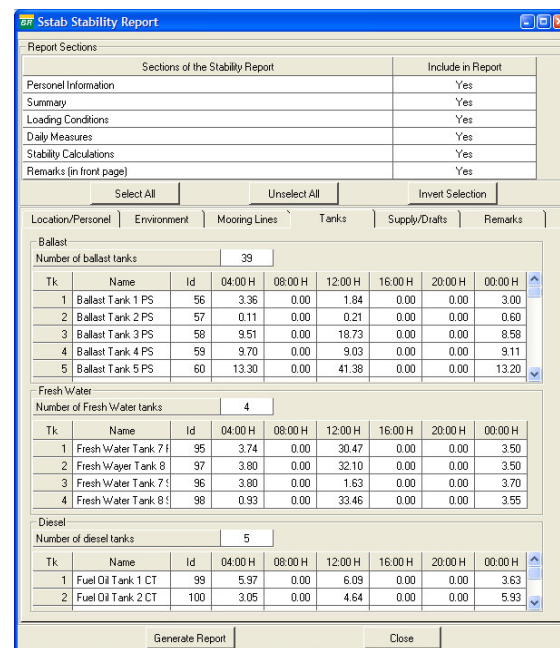


Figure 24- Sstab Daily Stability Report Dialog.

7. EMERGENCY SITUATIONS

A very useful functionality of Sstab is the possibility to force a platform to assume a position given by measured drafts. In normal

equilibrium computations, the drafts are a result of the final position of the floating system, but the user can also inform the drafts at the measured positions to obtain a computation of the buoyancy and the center of gravity. With this result, Sstab users can provide additional information to the salvage offshore team in accidents where the unit inclines. Figure 25 shows a picture of an accident where the information of some drafts was used to evaluate the extension of the flooding. Figure 26 shows the Sstab forced position for the model of Figure 25.



Figure 25- An accident with flooding and inclination.

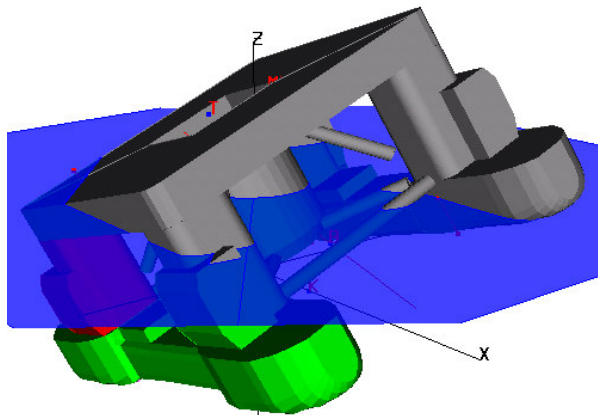


Figure 26- Sstab evaluation of flooding using measured drafts.

Figure 27 shows a configuration where there was a fail with the valves that control the oil inside the cargo tanks. Sstab was able to reproduce the situation connecting the fluid inside some tanks during the equilibrium computation. Sstab also generated the data for the dynamic analysis made with the heeled configuration.

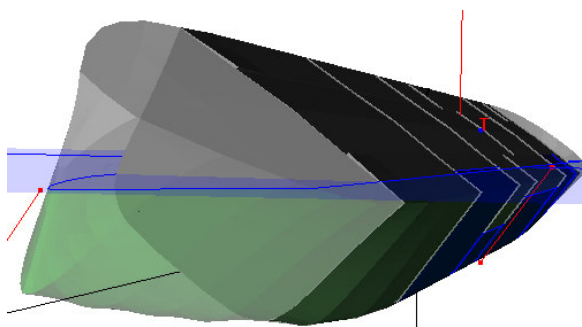


Figure 27- An accident with tank valves producing heeling.

Figure 28 shows the platform of Figure 27 after the recovering procedures, at the second day of operations. Every adopted strategy was simulated with Sstab.



Figure 28- Unit recovered with Sstab support.

8. CONCLUSIONS

We have built a system that is capable of helping naval architects, barge operators, and emergency personal in their tasks related to stability analysis, daily reports and decision-taking during emergencies.

Sstab is being used to provide support for the Emergency Program of Petrobras.

Sstab is also being used to prepare dynamic data for motion analyses.

We are working in a new algorithm that is capable of restoring the stability of damaged systems using the ballast potential.

9. ACKNOWLEDGMENTS

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