

## PARAMETER IDENTIFICATION FOR TWO NONLINEAR MODELS OF SHIP ROLLING USING NEURAL NETWORKS

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### ABSTRACT

The present paper employs a neural network based approach for fitting a roll motion model to experimental data. Two multivariable nonlinear models are used to describe the nonlinear forced roll motion of a ship at sea. One, more traditional model, is based on ordinary differential equations (Soliman & Thompson, 1991), and the other on fractional differential equations (Spyrou, *et al*, 2008), which introduced a fractional derivative term to present added hydrodynamic inertia and traditional damping terms. The neural network method is tested using both numerically simulated data and experimental data. It is shown that this method produced good results using either nonlinear model.

**Keywords:** ship, nonlinear rolling motion, neural networks, ordinary differential equation, fractional differential equation

## 1. EQUATIONS OF MOTION

### 1.1. Ordinary Differential Equation of Motion

The rolling motion of a ship at sea could be mathematically modeled, at least approximately, by the following second-order ordinary, nonlinear differential equation of motion:

$$I\ddot{\varphi} + N(\dot{\varphi}) + K(\varphi) = M(t) \quad (1)$$

Here  $\varphi$  is the rolling angle of the ship,  $I$  is the total roll moment of inertia (including added moment of inertia),  $N(\dot{\varphi})$  is the nonlinear damping moment which is a function of roll velocity,  $K(\varphi)$  is the nonlinear restoring

moment which is a function of roll angle, and  $M(t)$  is the roll excitation moment. A dot over the variable  $\varphi$  indicates differentiation with respect to time.

By dividing equation (1) by the roll moment of inertia  $I$ , we obtain

$$\ddot{\varphi} + B(\dot{\varphi}) + C(\varphi) = F(t) \quad (2)$$

where  $B=N/I$ ,  $C=K/I$ ,  $F=M/I$ .

For the damping moment per unit virtual moment of inertia of the ship, Dalzell (1978) showed that the linear-plus-cubic is quite reasonable approximation form for large amplitude roll,

$$B(\dot{\varphi}) = b_1\dot{\varphi} + b_3\dot{\varphi}^3 \quad (3)$$

where  $b_1$  and  $b_3$  are the linear and nonlinear damping coefficients respectively.

A parametric form of the hydrostatic restoring moment per unit virtual moment of inertia of the ship,  $C$  can also be approximated by linear-plus-cubic model for large angle of roll (Haddara, 1984),

$$C(\varphi) = c_1\varphi + c_3\varphi^3 \quad (4)$$

where  $c_1$  and  $c_3$  are the linear and cubic stiffness coefficients respectively.

The exciting moment per unit virtual moment of inertia of the ship is assumed in the following form

$$F(t) = f \sin(\omega_e t + \phi_0) + f_0 \quad (5)$$

where  $f_0$  is a constant moment due to some mean bias (wind, loading, etc...),  $\phi_0$  is a phase angle,  $\omega_e$  is the encounter frequency of the exciting moment.

On combining equations (2) to (5), the parametric form of the ordinary differential equation of rolling motion is

$$\ddot{\varphi} + b_1\dot{\varphi} + b_3\dot{\varphi}^3 + c_1\varphi + c_3\varphi^3 = f \sin(\omega_e t + \phi_0) + f_0 \quad (6)$$

## 1.2 Fractional Differential Equation of Motion

We also use a novel model of ship roll similar to that proposed in Spyrou et al. (2008), which is a fractional differential equation (7). Added hydrodynamic moment of inertia and potential damping were accounted through a single fractional derivative of the roll angle  $\varphi$ . (Spyrou et al., 2008).

$$\ddot{\varphi} + bD^a\varphi + c_1\varphi + c_3\varphi^3 = f \sin(\omega_e t + \phi_0) + f_0 \quad (7)$$

where  $D^a\varphi$  indicates a fractional derivative of the roll angle  $\varphi$  where  $a$  is the fractional derivative order between 1 and 2.

## 2. IDENTIFICATION OF COEFFICIENTS USING A SPECIALIZED NEURAL NETWORKS SYSTEM

The neural network is very useful to deal with non-linear dynamical systems with unknown non-linearities (Youlal et al., 1994). In this research, a specialized Neural Networks system has been introduced and applied to identification of the parameters of the two models to best fit the simulated data and experimental/-real data.

Figure 1 shows the specialized neural networks control system in which the equations of ship rolling motion are mapped into 'plant' as a part of the neural networks system. The input vector is time history of roll angle and roll velocity. The output vector is also time history of estimated roll angle and roll velocity. The learning schemes use the error (Equation 8) between the past inputs and outputs of the plant to identify and to adapt the plant inverse model.

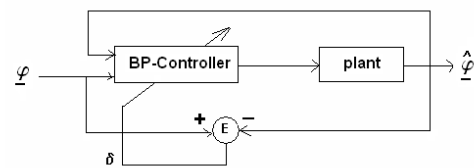


Figure 1. A simplified bloc diagram of the specified neural networks structure.

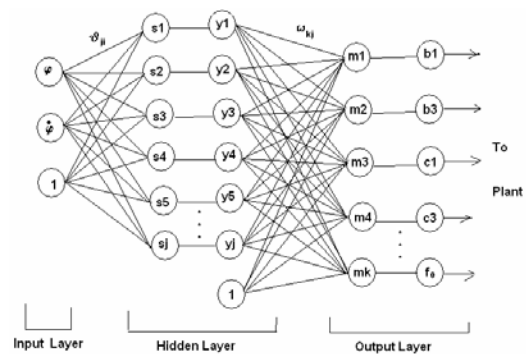


Figure 2. Structure of the Back-Propagation Neural Networks Controller.

Figure 2 shows the structure of the BP-Controller for the ODE given in Equation 6. The specific structure of neural networks we consider herein consists of only two hidden layers with sigmoid type neurons where neurons inter-connections occur only between adjacent layers. The number of neurons in the hidden layer is dependent on the number of input data, and the number of neurons in the output layer equals the number of parameters in the rolling equations. One neuron from the inputs to the first layer and one neuron from the hidden layer to the output layer have been set to 1 in order to add a constant bias to the weighted sum. The weights of the neural networks are trained by the gradient steepest descent algorithm such that the error between the past inputs and outputs of the plant approaches zero (Levin, Gewirtzman, and Inbar, 1991). The logistic activation function was used at the inside of the hidden layer and the tangential activation function was at the inside of the output layer. Both of logistic function and tangential function are the continuous functions. The Delta learning rule was applied to reduce the error between objective and the estimated output values through a steepest descent gradient along the error surface (Huang, 2004).

The error function is

$$E = \frac{1}{2} \sum_i (\varphi_i - \hat{\varphi}_i)^2 \quad (8)$$

To minimize E, the weight  $\omega_{kj}$  in the output layer should be adjusted in the direction of the maximum negative gradient of E, that is (Schiffmann & Geffers, 1993),

$$\Delta \omega_{kj} = -\eta \frac{\partial E}{\partial \omega_{kj}} \quad (9)$$

where  $\eta$  is the learning rate.

The momentum filtering term is recommended to apply too, which keeps the change of weights somewhat in the old update

direction. So, from the hidden layer to the output layer, the weight adjustments are

$$\omega_{kj}(t+1) = \omega_{kj}(t) - \eta_1 \frac{\partial E}{\partial \omega_{kj}(t)} + \alpha_1 \Delta \omega_{kj}(t) \quad (10)$$

where  $\eta_1$  is the learning rate and  $\alpha_1$  is a momentum factor affecting the change  $\Delta \omega_{kj}(t)$  of the network's weights at the  $t^{\text{th}}$  iteration. This can help avoid oscillations in the proximity of a minimum and accelerate convergence when the gradient is very small in wide plateau (Schiffmann & Geffers, 1993). From the input layer to the hidden layer, the weight adjustments are given by Equation (11):

$$v_{ji}(t+1) = v_{ji}(t) - \eta_2 \frac{\partial E}{\partial v_{ji}(t)} + \alpha_2 \Delta v_{ji}(t) \quad (11)$$

The weight matrices  $\omega$  and  $v$  are updated by these two formulas (10) and (11) after which a training cycle is complete (Schiffmann & Geffers, 1993).

## RESULTS OF SIMULATED DATA AND EXPERIMENTAL DATA

### 3.1 Application to Numerically Simulated Data

A rolling motion equation could be written as (Soliman & Thompson, 1991)

$$\begin{aligned} &\ddot{\phi} + b_1 \dot{\phi} + b_2 \left| \dot{\phi} \right| \dot{\phi} + c_1 \phi \\ &+ c_2 \left| \phi \right| \phi + c_3 \phi^3 + c_4 \left| \phi \right| \phi^3 \\ &+ c_5 \phi^5 = F \sin(\omega t) \end{aligned} \quad (12)$$

where  $b_1=0.0043$ ,  $b_2=0.0225$ ,  $c_1=0.384$ ,  $c_2=0.1296$ ,  $c_3=1.0368$ ,  $c_4=4.059$ ,  $c_5=2.4052$ ,  $\omega=0.527$ ,  $F=0.0195$ . For this case, 200s are

simulated with roll and roll velocity initial conditions both equal to zero. MATLAB's built-in "ode45" solver, which is the fourth-order Runge-Kutta method, was used to solve equation (12) numerically in the time step 0.01 second to obtain simulated data (Fig. 3). Part of the output time series of this equation was treated as though it was an unknown roll/roll velocity "experimental" data set, which was the input data set of the neural networks.

The process is outlined as follows; the first 20 seconds data of roll angle and roll velocity are used as training data for the proposed neural networks system with equation (6) mapped into the plant. The initial conditions are set as  $\phi_0=0$ ,  $\dot{\phi}_0=0$  and  $\omega_e=0.527$  in equation (6), minimize the error of sum square of both roll angle and roll velocity, obtain the estimated results  $b_1=0.002576$ ,  $b_3=-0.004437$ ,  $c_1=0.3950$ ,  $c_3=0.6348$ , and  $f=0.01928$ . The tracks of estimating parameters and the error of sum square in 1000 iterations are shown in Figure 4. The comparisons of the objective values and estimated values of roll angle and roll velocity are shown in Figure 5. The prediction results are shown in Figure 6, where the data in red line before dash line are used in neural network system training to estimate the parameters, and the data in blue line after dash line are the predicted further 20 seconds results.

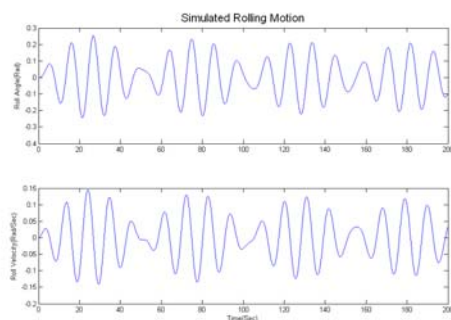


Figure 3. Simulated Rolling Motion.

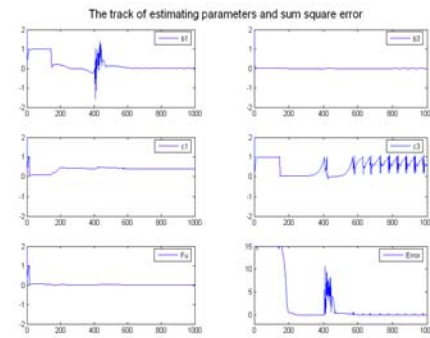


Figure 4. The Track of Estimating Parameters and SSE.

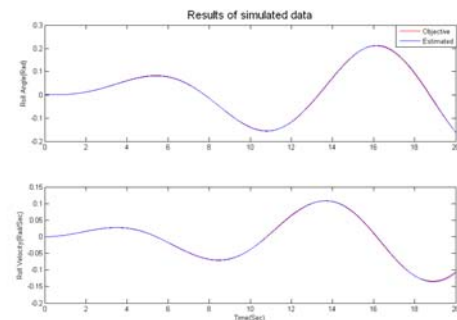


Figure 5. Results of Simulated Data.

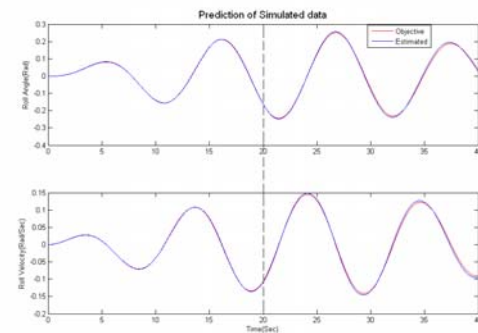


Figure 6. Prediction of Simulated Data.

### 3.2 Application to experimental data

Following the successful application to the numerical test case described above, the proposed neural networks method is then applied to analysis of experimental data for DTMB Hull 5514 dynamic stability tests detailed in Hayden *et al*, (2006). The model

tests are for a 1/46.6<sup>th</sup> scale notional destroyer in regular seas. Figure 7 gives an example of the experimental data. The portion of the data indicated in red was used for this study.

### 3.2.1 Estimation results by ODE equation (6)

Applying the ODE-based neural network approach described previously to this sample portion of data, the estimated values of the parameters in equation (6) are found as  $b_1=0.003263$ ,  $b_3=0.0002442$ ,  $c_1=12.8620$ ,  $c_3=9.8709$ ,  $f=0.4405$ ,  $\omega_e=2.1252$ ,  $\phi_0=-2.6272$ , and  $f_0=0.9106$ . The comparisons of the real values and estimated values of roll angle and roll velocity by ODE equation (6) are shown in Figure 8. The tracks of the error of sum square in 400 iterations are shown in Figure 9. The minimum error shown in figure 9 was 0.4348 using the ODE model for the sample data. The tracks of estimating parameters are shown in Figure 10.

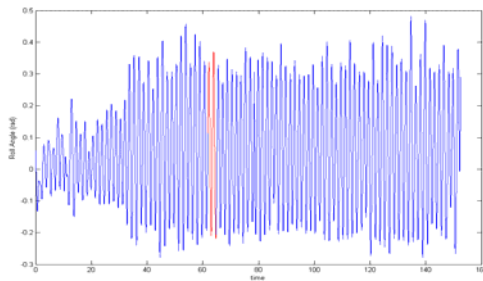


Figure 7. One Example of Experimental Data.

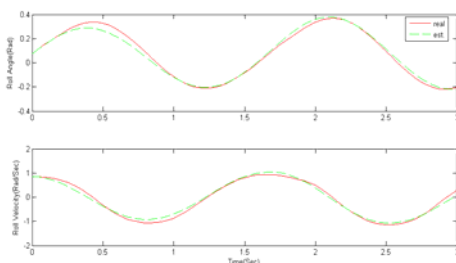


Figure 8. Results of Experimental Data by ODE

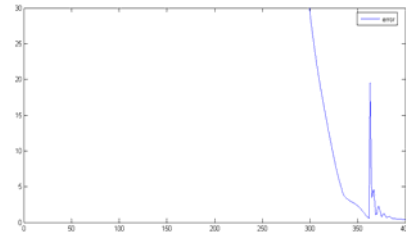


Figure 9. The track of SSE by ODE.

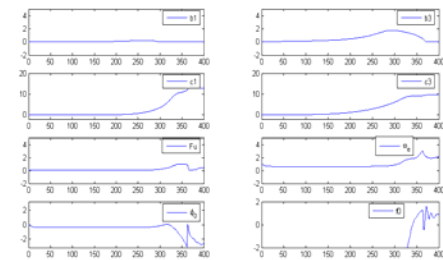


Figure 10. The track of estimating parameters in ODE.

### 3.2.2 Estimation results by FDE equation (7)

Applying the ODE-based neural network approach described previously to this sample portion of data, the estimated values of the parameters in FDE equation (7) are  $a=1.8772$ ,  $b=0.4409$ ,  $c_1=21.6147$ ,  $c_3=17.8551$ ,  $f=1.8308$ ,  $\omega_e=3.4652$ ,  $\phi_0=1.1971$ , and  $f_0=1.6502$ . The comparisons of the real values and estimated values of roll angle and roll velocity by FDE equation (7) are shown in Figure 11. The tracks of the error of sum square in 900 iterations are shown in Figure 12. The minimum error shown in figure 12 was 0.2286 using the FDE model for the sample data. The tracks of estimating parameters are shown in Figure 13.

Figure 14 shows prediction and comparison of the results of two models: fractional differential equation and ordinary differential equation. The red line shows real data; the blue line is the result of the model FDE, and the green line is the result of the 8 unknowns ODE. The data in red line before the dash line were used to training in the neural networks to



obtain the parameters values, and the data after the dash line were the next 1 second predicted result based on the estimated parameters of those equations.

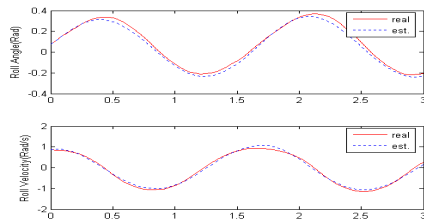


Figure 11. Results of Experimental Data by FDE.

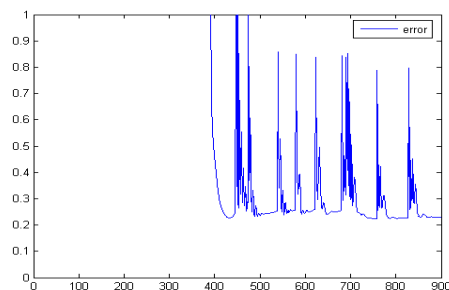


Figure 12. The track of SSE by FDE.

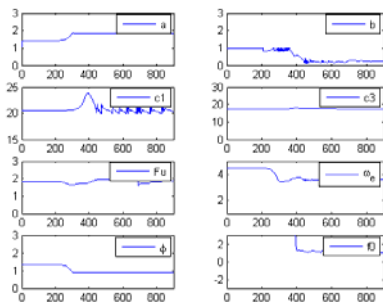


Figure 13. The track of estimating parameters in FDE.

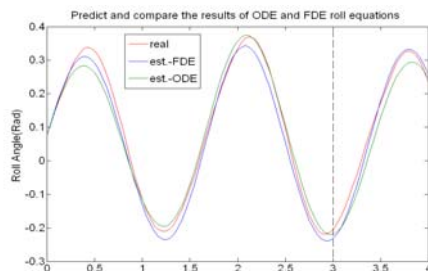


Figure 14. Comparison of the results of two models.

### 3. DISCUSSION AND CONCLUSION

In the above, we have developed and described a scheme for the estimation of the parameters in an equivalent roll equation of motion using only the roll response (roll angle and roll velocity), which could be relatively easily obtained for a ship sailing at sea. It has been shown, using numerically simulated as well as experimental data, that the method produces good estimates for all relevant parameters including damping, restoring and exciting moment parameters. There are several advantages of this method by comparison to other existing methods in the literature. These advantages can be outlined as follow:

1. The parameters in the equation of rolling motion are estimated using the roll response only. A priori knowledge of the input is not needed. This makes this method appealing for use on ships at sea for estimating equivalent instantaneous parameters values.
2. All the parameters in the equation of motion, including the magnitude and phase of excitation, can be estimated using this method. This makes this method particularly useful when simultaneous value for the excitation and the response for the excitation.
3. There is no assumption for the external excitation. The method can be used to predict a ship roll motion in real sea.

Tremendous successes have been seen using more traditional neural network based approaches (Hess, Faller, Fu and Ammeen 2006; Kimura and Amagai 2003; Haddara and Hinchey 1995). The fundamental aim of this approach is to yield a physical equation with which one might then apply analytical approaches for rapid analysis of stability boundaries and/or couple with a neural network to determine an ideal reduced order model to fit the current vessel motion conditions. This latter aspect of future work would allow captains and handlers to not only have predictions of the future state of their vessel, but to also

understand the phenomena affecting their vessel. That is, rather than simply giving a captain a warning indication, one might be able to give a warning of a specific type of dynamic instability. This is all exciting future work to be studied.

#### 4. ACKNOWLEDGMENTS

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