

STEEPEST DESCENT METHOD. RESOLVING AN OLD PROBLEM

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ABSTRACT

New designs increase the need to evaluate the stability at larger angles and with increasing interest in the righting arms beyond the maximum on the righting arm curve. With this trend, we find with increasing frequency abnormalities that are caused by the simplifications in the existing methods. Of these abnormalities, the most disconcerting we call fading stability. In the research on fading stability we have found that conventional “free-to-trim” stability calculation can lead to incorrect results and that the steepest descent method (SDM) is the correct methodology.

This paper will present the progress of the methods applied to stability analysis, the weakness of previous and current methods, the rational and mathematical basis for the proposed methodology. Lessons learned with SDM and the benefits of the application to the analysis to all types of hulls will be discussed as a way to trace the future research in this field.

Keywords: Stability, righting arm, energy to roll

1. EVOLUTION AND RATIONAL OF STABILITY ANALYSIS METHODS

New tools and higher understanding of the motions of a vessel in a seaway continues our journey to the ultimate way of evaluating the stability of our designs. It seems however, that the more we know, the more elusive the ultimate method appears to be. Until then, we will continue to adopt methods that are a simplified version of reality, and use them to compare with successful designs.

This approach has been extraordinarily successful, as relatively simple ways consistent with the available technology, have allowed us to design stable vessels, within the time available in the ship-design-construction cycle. We have painfully learned that approach is only applicable as long as the fundamental parameters remain constant. The lessons learned have been costly, in human and material loss, but we have incorporated them to our practice to avoid recurrence.

Most Rules and statutes, include intact stability standards. In most cases, at the core of the method, is a simplified way to predict and limit the extreme motions under the action of the environmental loads. These standards are primarily focused on two goals.

1. prevent submergence of openings that lead to downflooding, and
2. prevent capsizing.

In most cases, these standards are successful on the basis of substantial margin before reaching any of the terminal events. These margins are borne from three aspects of the standards:

1. reliability of the simplified method,
2. stochastic nature of the environment.
3. stochastic nature of vessel motions of a vessel on the seaway.

Before the advent of computers, the numerical methods we have applied required extensive calculations, often requiring large teams to draw conclusions in the limited time available. The similarity of the vessels that kept shipyards busy at any given period of history, allowed simplified comparative methods that dismissed minor risks and differentials of higher order.

2. FIXED-TRIM METHOD

Part of such simplification is the acceptance that, on conventional vessels, transverse stability is the governing criteria. With that principle the analysis was reduced to evaluating stability as the hull rotated around a longitudinal horizontal line that maintained a constant displacement. Vessels that did not fall within the parameters of conventional ships, require different approach.

The method described is commonly known as “Fixed-trim Method”. A more accurate name for the method is “Zero-trim Method”. The approach has the weakness in that most hulls do not maintain a zero trim as the hull rolls to its side. This is because the center of gravity will shift longitudinally as the buoyant body changes with the angle of heel. Naturally, the simplification of the Zero-trim carries a number of inaccuracies of which, the prediction of the point of submergence of downflooding points could be the most critical. This, as may be expected is a concern for openings closer to the end that without the simplification, would trim into the waterplane. It is also true, that such concern was resolved by enforcing very high coamings for ventilators that could otherwise lead to flooding.

Damage stability has also progressed with the advent of computers but most criteria published by the statutes are a static determination of the position of the hull after flooding. This simplified evaluation does account for trim, heel and draft and establish if

downflooding points or the Margin Lines are indeed submerged.

3. FREE-TO-TRIM METHOD

In a fixed-trim analysis, two moments are created when the hull is rotated; the heeling moment about the longitudinal axis, and a trimming moment about a transverse axis. With the advent of computers, the methods improved dramatically and the “Fixed-Trim” methodology was soon abandoned in favor of the Free-to-trim method. This method corrects the trim of the rotation axis to maintain the centre of buoyancy in line with the center of gravity. With the adjustment of trim, the righting moment, represented with a vector parallel to the axis of rotation is the only moment. This methodology allowed for new and more sophisticated ways to evaluate stability. The possibility of effectively accounting for trim made the prediction of downflooding more accurate.

The Free-to-trim does not resolve all issues because the hull is restrained to heel about the same axis. The weaknesses of the Free-to-trim method can be dismissed as the disparity with more rigorous methods can be regarded as “differentials of higher order”. This is true for conventional ship hulls and the predicted motions in a quasi-static method are “accurate enough.”

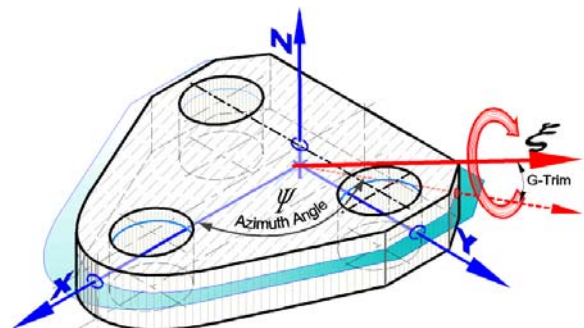


Figure 1. Typical axis of evaluation on a jack-up.

The difference between a Free-to-Trim and a more rigorous method is notable on hulls of unusual shape. By and large, we can say that

non-slender hulls, and hulls with significant difference in the shape of the forward half from the aft half are the most likely to require a more rigorous method of analysis.

For offshore structures, the stability analysis must be carried out for “all directions”. This is done by analyzing the inclination of the hull about several axes other than the longitudinal. This rotation is made as to emulate the environmental loads than may approach the hull from any direction. The rotation about a selected axis (ξ axis in the illustration to the right), is no longer the conventional heel, and the trimming of the rotation axis is no longer the conventional trim. To distinguish these angles we have adopted the concept of Generalized-Heel (G-heel), and Generalized-Trim (G-trim) as the rotation about a selected axis (ξ) and the trimming of this axis respectively. The axis of rotation is defined by its direction or azimuth angle.

This analysis for “all” orientation creates high trimming moments and the angle of trim cannot be dismissed as it has a major impact on the results. Frequently, the angles of G-trim are of the same magnitude as the G-heel.

While the adoption of the free-to-trim approach seems to resolve the problem of inaccurate prediction of the angle of heel. However, in the application of the method to stability analysis from all direction, the calculations often yield unexpected results and conclusions opposed to common sense.

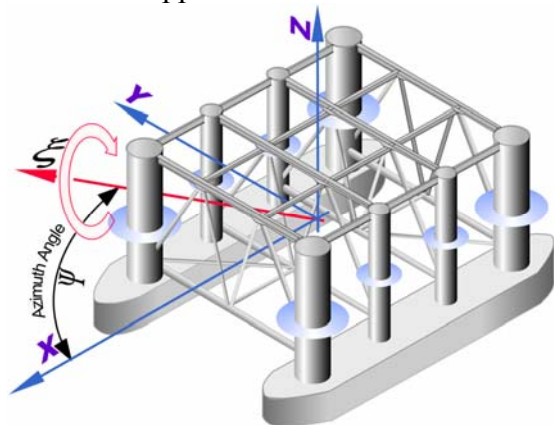


Figure 2. Typical axis of evaluation on Semisubmersible.

For most hull forms, a righting arm calculation about a selected (and constant) free-to-trim axis will not reach the range of G-heel angles from zero to ninety degrees G Heel. This is true for almost any azimuth angle. If extended to large angles the typical righting arm calculation will fail to complete the required range of angles. This phenomenon which we call a fading stability curve, is the inadequate principle on which the free-to-trim approach is based. In simple terms, the method of calculating stability curves follows a sequence of draft-heel-trim combinations that is impossible to satisfy. However, most of the conventional calculations required to satisfy statutory standards do not extend to angles where this is evident. Further, in conventional vessels, where stability is verified only about a longitudinal axis, the fading stability may not be experienced.

In offshore units and in highly stepped high speed hulls this is a frequent occurrence and the results may be incongruent and even incorrect. Thus, the problems created by the fixed nature of the axis of rotation can range from negligible for some hull forms to notable on others. Because this problem is not detected unless fading stability is experienced, inaccuracy in the stability curves can go undetected and a less than accurate sequence of Draft, heel, trim will predict the wrong angle of downflooding. Where the stability of most offshore units and many naval vessels is governed by the “angle of downflooding”, the results of the analysis will also be incorrect and sometimes in conflict with common sense.

4. STEEPEST DESCENT METHOD

The solution to this problem is to “release” the axis of rotation allowing the azimuth angle to vary as a function of the angle of G-heel. By changing this azimuth the only moment created is about the heeling axis (ξ) thus maintain a zero moment to G-trim. The relationship between the instant azimuth angle and the angle of G-heel must be established as there are



unlimited number of instant direction of rotations for each angle of G-heel. Of all options, the axis of G-heel that provides for the maximum increase of the potential energy is the most rational for a quasi-static analysis of stability. The rationale for this choice is presented in Ref [1].

5. INTRODUCTION

In the particular case of self-elevating (jack-up), and semisubmersible offshore units, the anomalies borne from the Fixed-trim, and the free-to-trim are magnified by the progress in design criteria and technology. The transverse direction of stability for conventional hulls is arguably, the critical direction. While this may be acceptable for conventional ships, offshore units must be excluded from this simplification and stability analysis must be done for different rotation directions. The reason for this is that it is not obvious what rotation direction that will be critical with respect to the applicable stability criteria. For most stability criteria, the analysis requires that a set of GZ-curves that covers rotations in all directions is generated to serve as a basis for the analysis.

It is in this context that we have experienced the above noted anomalies and shortcomings that so far have been neglected. The most frequent and visible anomaly are the fading stability curves. Typically, stability curves are calculated to a given range of angles of heel. Experience shows that, given a large enough range, most calculation will terminate before reaching the full range of angles. It is this “cropping” of the stability curve that has raised questions regarding the relevance of analysis results.

A rather simple remedy for these problems exists in the form of a change of the basic principles on how the GZ-curve is generated. The required modification is implemented in the proposed Steepest Descent (SD) method.

The analysis that follows will not only describe the mathematical and aspects of this method but also will show how conventional fixed-trim and free-to-trim methods are often inadequate for stability evaluation for offshore units, and other hulls of unusual proportions, but that the limited modification that the SD method constitutes could improve the existing evaluation procedures to ensure that results will be physically relevant and reproducible.

6. DEFINITIONS

The following definitions will be used throughout this document.

Heel angle σ , is the angle of rotation of the vessel about its longitudinal axis.

Trim angle τ , is the angle of rotation of the vessel about its transverse axis \hat{y} . The Oxyz-system is a right-handed system

Evaluation axis \hat{e} , is used as a reference axis for the Free-to-trim GZ-curve. It controls the direction of the righting moment vector.

Evaluation angle α , is the angle from the x-axis to the evaluation axis

Generalized Heel angle σ' , is the angle of rotation of the vessel about the \hat{s} -axis. The \hat{s} -axis is the projection of the evaluation axis in the water-plane. If the evaluation angle is zero, the generalized heel is identical to the heel and follows the definition used in shipbuilding.

Generalized Trim angle τ' , is the angle of rotation of the vessel about the \hat{t}' -axis. It is the angle of the evaluation axis with respect to the horizontal.

Inclination θ , is the angle between the hull base plane and the horizontal.

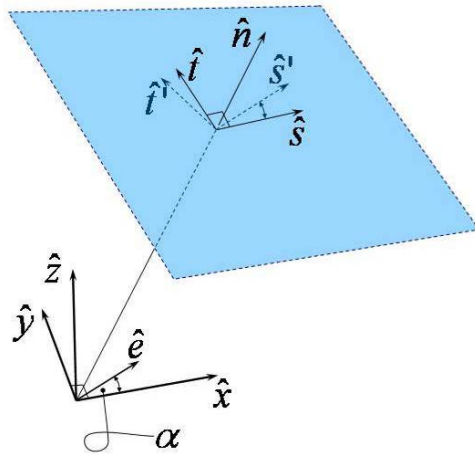


Figure 3. The discussion that follows will study the moment vector resulting from the gravity – buoyancy force pair. For the purpose of this discussion, we introduce a coordinate system $Ostn$; a right-handed Cartesian coordinate system where the n -axis coincides with the water plane normal. The s -axis is parallel to the projection of the vessel's longitudinal axis in the water plane. For the purpose of studying Free-to-trim GZ-curves we will also be using a second coordinate system $Os't'n$. In this system the s' -axis is parallel to the projection of the evaluation-axis in the water plane.

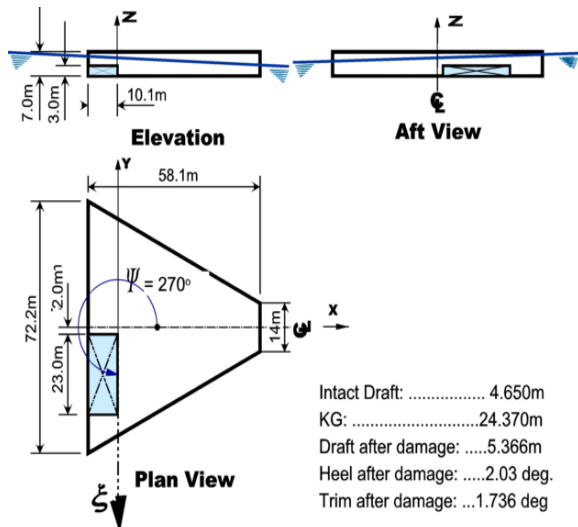


Figure 4. Model used for calculations and illustration.

7. TEST MODEL

To illustrate we use a model of a fictitious jack-up rig. The model contains one single tank that is damaged and flooded in the example condition.

The dimensions and relevant particulars of this test geometry are evident from the figure 4 and table 1 below:

Table 1.

Intact Draft	4.650 m
KG	23.498 m
Draft after damage	5.349 m
Heel after damage	1.681 deg
Trim after damage	-1.968 deg

8. THE FREE-TO-TRIM (FT) METHOD

Although not always explicitly specified in rule texts the de facto standard procedure used to generate GZ-curves is what we here refer to as the Free-to-trim (FT) method.

One often used way to describe this method is to say that the vessel, while rotated to generate the GZ-curve, is forced to heel about a fixed axis, normally called the heel axis, and then allowed to trim freely about the perpendicular trim axis. As explained above in section 2 we here use the term evaluation axis instead of heel axis to avoid any ambiguities. Heel is used for rotations about the longitudinal axis and generalized heel is used for rotation about the evaluation axis.

The moment vector is perpendicular to the normal of the water plane. Using the symbols defined in section 2 we can therefore use the following expression for the moment vector:

$$\bar{M} = M_{s'} \epsilon' + M_{t'} \epsilon \quad (1)$$

Since the vessel is allowed to trim freely about the generalized trim axis, $M_{t'}$ is zero for all angles as the vessel is rotated to generate the free trim GZ-curve and we get the following expression for the moment:

$$\bar{M} = |\bar{M}| \mathbf{\bar{\epsilon}} \quad (2)$$

i.e. the moment is parallel to the axis $\mathbf{\bar{\epsilon}}$.

This yields the following expression for the build-up of potential energy:

$$\Delta E_p = \int \bar{M} \circ d\bar{\xi} = \int |\bar{M}| d\sigma' \quad (3)$$

From equations 4.2 and 4.3 we see that the GZ-function is proportional to the moment at any angle σ' and also that the integral of the function is proportional to the build up of potential energy within any angle interval. These properties qualify the free-to-trim GZ-curve as a suitable candidate for a physically meaningful evaluation.

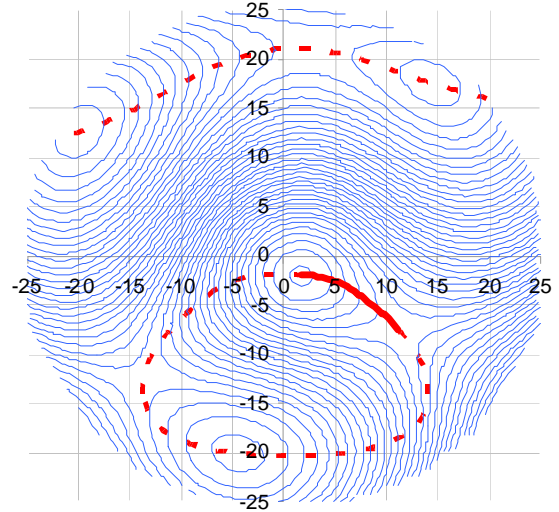
How does this rotation that puts restrictions on the moment look? If we turn to the example geometry described above in section 7, we can plot the solution to the equation $M_{t'} = 0$ in the σ - τ plane. Figure 5 shows this solution for the evaluation direction zero degrees.



Figure 5.

Part of this trace forms the rotation path associated with the GZ-curve. To visualize the general stability characteristics of the condition, we can superimpose a graphical representation of the potential energy (E_p) onto this diagram.

Figure 6. Rotation path corresponding to the



FT GZ-curve for the evaluation direction zero degrees.

The components of the moment vector are the partial derivatives of the potential energy.

$$\frac{\partial E_p}{\partial \sigma} = M_s \quad (4)$$

$$\frac{\partial E_p}{\partial \tau} = M_t$$

Therefore all intercepts of the GZ-curve will always occur at local extremes or saddle points of the E_p -function. At the stable equilibrium the potential energy has a local minimum. In the example the heel and trim angles are at 1.681 and -1.968 degrees respectively. At this point the GZ-curve has its first intercept and the potential energy has a local minimum.

Figure 7 shows the GZ curve related to this rotation path.

The following diagram shows the set of rotation paths corresponding to evaluation directions for every fifth degree in the range 0 to 360 degrees.

In a general case each of these paths corresponds to a GZ-curve that needs to be analyzed with respect to the applicable stability criteria to establish the critical direction and maximum allowable KG.

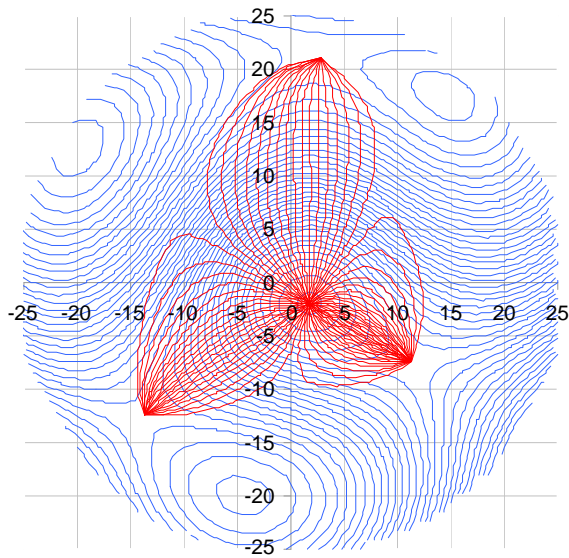
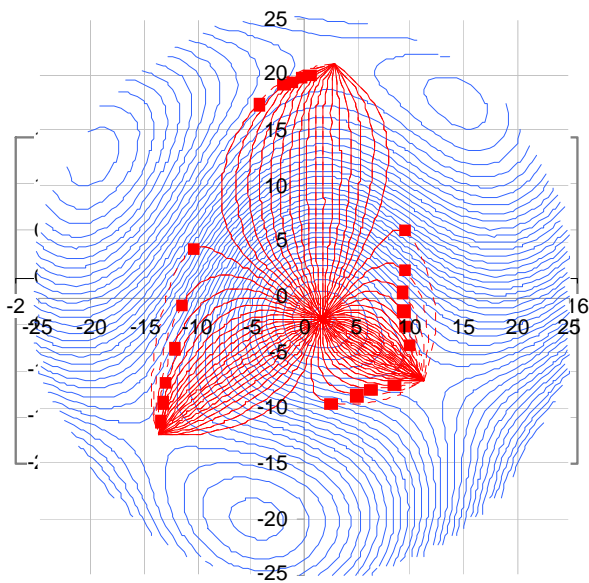


Figure 7. FT GZ-curve for the evaluation direction 0 degrees.

Figure 8. FT GZ-rotation paths corresponding to evaluation axes at every fifth degree.

9. STABILITY ANALYSIS USING FT

When we attempt to generate FT GZ-curves corresponding to the above set of rotation paths



it soon becomes obvious that the process is not at all straight-forward.

As we have seen in section 4, the GZ curve is a function of the generalized heel angle σ' . This means that each generalized heel angle value should correspond to exactly one heeling arm value. We can thus generate this function as we move along the rotation path only as long as the generalized heel is increasing. The definition range of the GZ-curve is limited where the generalized heel angle reaches a local maximum. In this example this happens before the GZ curve's second intercept for the evaluation angles in the intervals $[30, 50]$, $[120, 170]$ and $[255, 270]$ degrees as shown in the following figure.

Figure 9. Limits of the definition range of FT GZ-curves. The red squares mark the upper limit of the definition range for GZ-curves where this limit occurs before the second intercept.

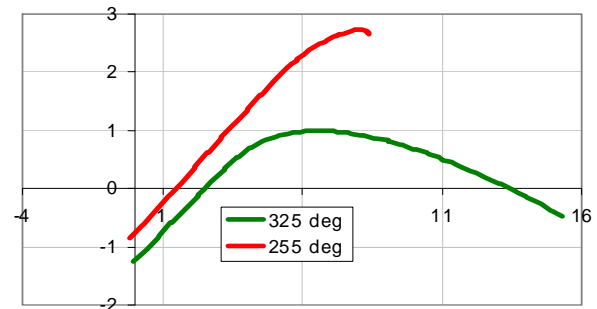


Figure 10.

For the evaluation angles 255 and 325 degrees the GZ-curve will look as follows.

The fact that the definition range for the GZ-curve at the evaluation axis 255 degrees is limited leads to logical problems when this GZ-curve is to be used to evaluate various stability criteria. Almost all stability criteria currently in use require the existence of a GZ-curve over a certain range. When the GZ-function doesn't exist over this range it is not clear how the criteria should be interpreted.

If we for example consider the damage stability criterion proposed by ABS for self-elevating units, namely that ...



The unit after damage must have a residual stability with a minimum range of stability (*RoS*) of:

$$RoS \geq 7^\circ + 1.5\varphi_s \quad \text{or} \quad (5)$$

$$RoS \geq 10^\circ \quad \text{whichever is greater}$$

φ_s is the static angle of heel after damage and the *RoS* is the range of stability evaluated as the difference between the second and first intercepts of the GZ-curve.

...and attempt to evaluate our test condition we run into problems.

From figure 10 it is quite clear that the GZ-curve corresponding to the evaluation 255 degrees, from a purely logical perspective, doesn't satisfy the above criterion. This in turn would imply that the evaluation direction 255 degrees is more critical than 325 degrees and that the KG needs to be reduced to satisfy the above stability criterion. A sufficient reduction of the KG value will result in a situation where the definition range for all the GZ-curves would match what is required for the evaluation.

This result is not the expected, and it is not clear what should be the conclusion. The details are discussed in some more details in section 7.1.1 where this method is compared to the evaluation method proposed in the next section.

There may be more than one problem involved here affecting the evaluation and resulting in questionable results. However the most important and fundamental issue is the fact that some FT GZ-curves have a limited definition range.

It is important to note that this is a very general problem not associated with the

particular geometry or criterion used in this example.

In the above example it would maybe be possible to disregard the troublesome directions to arrive at more relevant results.

10. STEEPEST DESCENT (SD) METHOD

The findings described in the preceding section give a strong reason to investigate alternatives to the commonly used FT GZ-curve as the basis for the stability evaluation. In the following we describe what seems to be the most natural option.

As long as we stick to the basic approach of using GZ-curves as the basis for the evaluation, it is important to preserve the following two properties of the curve to maintain its physical significance:

- 1) The curve should be proportional to the righting moment at all angles
- 2) The area under the curve should be proportional to the build-up in potential energy in any angle interval.

We have seen that the FT GZ-curve satisfies these conditions, but we also have a quite obvious option. If we rotate the vessel along a path where the rotation vector is always parallel to the moment we get the following equation for the build-up of potential energy:

$$\Delta E = \int \overline{M} \odot d\overline{\xi} = \int |\overline{M}| |d\overline{\xi}| = \int |\overline{M}| d\xi \quad (6)$$

The paths generated according to this principle are displayed in the following figure 11.

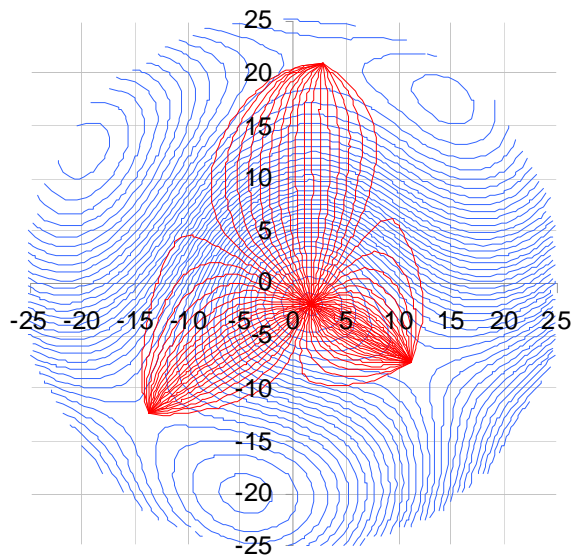


Figure 11. SD evaluation paths at every five degrees.

The paths in figure 11 are all perpendicular to the iso-energy lines. This follows from the fact that the rotation is parallel to the moment vector and the components of this vector are the partial derivatives of the potential energy.

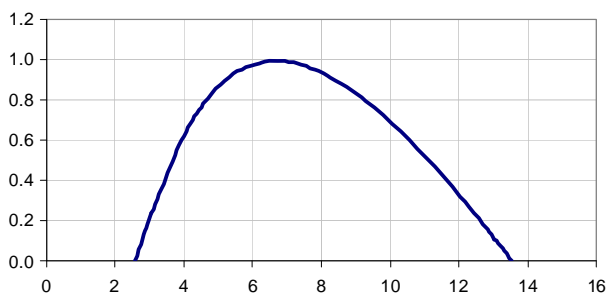


Figure 12. The critical SD GZ-curve.

Since the rotation paths are parallel to the moment vector at all angles, these paths will always follow the direction in which the energy changes most rapidly. This principle of selecting direction when varying the independent variables is similar to the one used in the numerical method commonly known as the “Steepest Descent Method” that can be applied to find local minima of a function. It therefore seems appropriate to let this method lend its name to the method proposed here for generating rotation paths and GZ-curves. By Steepest Descent (SD) GZ-curve we thus mean a curve that corresponds to a rotation path

where the rotation direction is always kept parallel to the moment.

The SD GZ-curve plots the righting lever as a function of the rotation (not the generalized heel as in the FT GZ-curve). This rotation equals the length of the rotation path and is measured from some reference angle where the rotation is set to zero. The reference angle can be chosen arbitrarily and here we have selected it in such a way that the first intercept of the GZ-curve always equals the inclination of the vessel in the equilibrium under consideration. The parameter value will always increase as we move along the rotation path and therefore there is no limitation to the definition range of the GZ-curve.

This is the simple key feature of the proposed SD method. This property guarantees that the curve can be defined over any range which is really a prerequisite for a successful GZ-curve based stability evaluation.

The difference between the FT and SD righting arm-curve is that while the FT-curve gives the righting arm as a function of generalized heel the SD-curve gives it as a function of rotation. This difference calls for a special attention.

The actual value of the rotation at a certain intercept is normally not a very relevant parameter to consider in stability evaluation. Therefore criteria that involve requirements on angles or angle ranges need to be interpreted a little differently from what is the case in the normal FT evaluation. The following interpretation is proposed:

When a stability criterion puts a requirement on an angle, it is interpreted as a requirement on the inclination at that intercept (or more generally as the angle between a desired optimum normal in the intact condition and the normal in the actual intact or damaged condition)

i.e. a requirement of the type $\varphi > req.$ is interpreted as ...

$$\theta(\xi) > req \quad (7)$$

(or more general

$$\arccos(\mathbf{n}(\xi) \circ \mathbf{n}_0) > req$$

When a stability criterion puts a requirement on the difference between two angles, it is interpreted as a requirement on the angle between the two normal vectors at these two rotations.

i.e. a requirement of the type $\varphi_2 - \varphi_1 > req.$ is interpreted as ...

$$\arccos(\mathbf{n}(\xi_1) \circ \mathbf{n}(\xi_2)) > req \quad (8)$$

11. A METHOD COMPARISON

11.1 Stability evaluation in practice

11.1.1 Stability evaluation using the SD method

Rule texts usually stipulate that the stability criteria should be satisfied for rotations about the critical axis. We are not aware of any explicit definition of the concept critical axis, but a rather obvious approach to evaluating the stability is to check all directions. If the criterion is satisfied for all directions we must conclude that it is satisfied also for the critical direction and if not we conclude that the critical axis is among the directions for which the criterion is not satisfied, and the requirement thus not met.

If we consider the paths in figure 11 it is quite obvious that the path that reaches the closest saddle corresponds to the critical direction. As expected this path corresponds to rotation that increases the heel and reduces the trim and thus submerges the damaged compartment.

As described in the above section the criterion at hand is interpreted as follows:

$$\arccos(\mathbf{n}(\xi_1) \circ \mathbf{n}(\xi_2)) > \max(7 + 1.5\theta(\xi_1), 10) \quad (9)$$

Evaluating the criterion for $KG = 23.498$ m (the KG of the example condition) yields the following results:

Table 2

Crit. direction candidate	Obtained RoS	Required RoS
KG = 23.498		
Closest Saddle	10.883	10.883

The maximum allowable KG is thus 23.498 m for the example damage case.

The vessel can be rotated from the damaged equilibrium any angle up to the limit specified by the criterion about any axis and still be subject to a restoring moment for all KG values less than or equal to the maximum allowed KG . Also, the maximum allowed KG is the highest KG value for which this is true. The solution is illustrated graphically in figure 13. This is the intended interpretation of this criterion.

A little outside the scope of this discussion, we can note that this specific criterion could be evaluated independent of any set of GZ-curves. Both obtained and required values can be calculated once the extreme points of the potential energy are known.

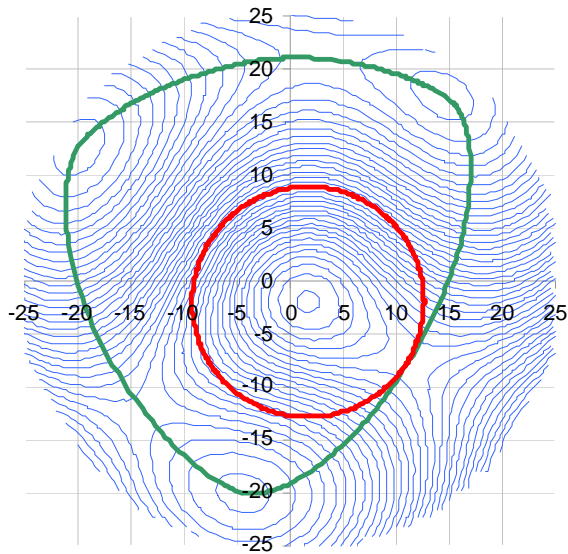


Figure 13. Graphical illustration of evaluation of the criterion . For all heel and trim angle combinations inside the green path the moment resulting from gravitation and buoyancy will tend to rotate the vessel back to the initial damaged equilibrium. For all heel and trim combinations outside this path it will tend to rotate it away. The red path shows the required range of stability. The two paths touch at the critical saddle point and therefore we have found the maximum allowable KG.

11.1.2 Stability evaluation using the FT method

In the case of FT analysis the criterion is interpreted as ...

$$\sigma_2 - \sigma_1 > \max(7 + 1.5\sigma_1, 10) \quad (9)$$

This is the commonly used and most direct interpretation of the criterion text.

The full analysis required to establish the maximum permissible KG involves a lot of calculations and table 3 below just shows a few sample values to support the following discussion.

Table 3.

Crit. Dir. candidate	Obtained RoS	Required RoS
KG = 23.498 m		
270	5.904 ²⁾	10.000
310.46 ¹⁾	10.349	10.883
325	10.909	10.759
KG = 23.05 m		
305	10.433	10.808
310.42 ¹⁾	10.826	10.825
355	10.408	10.000
0	9.957	10.000
KG = 17.18 m		
265	10.003 ²⁾	10.000

- 1) This direction coincides with the inclination axis of the damage condition
- 2) The definition range of the GZ-curve does not reach the second intercept. The obtained value is calculated up to the upper limit of the definition range.

What can be concluded regarding the maximum allowable KG from the above table 3?

If we use the straightforward approach to require that the criteria should be satisfied for all evaluation directions, we can see that the maximum allowable KG is not much higher than 17.18 m. (A full analysis shows that 265 degrees is close to critical (in the sense described above) so the max. allowable KG would actually be close to 17.18 m). It could be argued that even this KG value doesn't satisfy the criteria. Although there is a range of positive GZ that match the requirement, a second intercept does not exist for all directions.

The evaluation direction 265 degrees is almost perpendicular to the inclination axis of the damaged condition and to the weakest axis of the vessel, axes that are expected to be close the critical axis. This result is very difficult to digest and would probably in reality in most cases be disregarded as being physically non-



relevant. Also the results from the SD analysis tell us that the result is wrong.

At the same time this is the result that we do obtain by logically applying the rule guide lines using the de facto standard FT-based evaluation method. This is an example of a quite frequent situation where it is difficult to see how the FT method should be used to generate relevant results.

If we accept that it is in order to disregard certain evaluation axes from the analysis as being physically non-relevant, it remains to find some principle on which evaluations axes that should be considered as candidates for the critical axis.

Characteristic directions in the example condition

Table 4.

Inclination axis at first intercept	310.46
Inclination axis at second intercept	326.62
Weakest axis at damaged equilibrium	1.04

In a normal case we would for physical reasons expect that the critical axis would be close to the directions listed in table 4. A quick glance at any of the diagrams above confirms that this is the case for our example.

An option that has been used in practice for damage stability evaluation is to assume that the inclination axis in the damaged equilibrium can be used as the critical axis (for the particular example criterion this assumption is not completely out of the blue since the required value will have its maximum value in this direction). Table 3 shows that the maximum allowable KG is around 23.05 m under this assumption. However, if we decide to verify this assumption by including some more directions (e.g. 305 deg.) close to this axis we can see that the maximum allowable KG will be reduced.

This discussion shows that it is very difficult to find any convincing arguments on what is the correct and relevant maximum allowable KG based on FT-analysis.

The very root to this problem is the basic free-to-trim assumption that leads to GZ-curves with limited definition ranges. The stability evaluation assumes that we always will be able to find a GZ-curve extending over a sufficient angle range. When this assumption doesn't hold true we have seen that the method may yield irrelevant results if applied strictly. If we on the other hand try to work around the problems we will inevitably have to make some quite arbitrary assumptions regarding what is the critical direction.

The conclusion is that FT based analysis is not a very suitable tool to be used when analyzing stability for offshore units.

11.2 Basic differences

The rotation paths in e.g. figures 9 and 11 are not actually describing a rotation of the vessel in a real world situation. The GZ-curves are merely tools to be used to evaluate the stability.

As we have seen above the FT and SD method are equivalent in this respect. The very significant difference between the two method is that the FT method will generate GZ-curves with limited definition range while the GZ-curves generated using the SD-method will have no such limitation.

The FT method keeps the righting moment vector parallel to a specific vessel-fixed reference axis. The SD method does not fix the direction of the moment vector. The direction will vary relative the vessel as it is rotated. For offshore units where the righting energy is evaluated against the wind overturning energy, the SD method brings the need to calculate wind heeling moments from a changing

orientation dictated by the direction of the righting moment vector.

This difference should be considered when heeling arm curves are generated for the SD analysis so that the assumed wind direction always is consistent with the direction of the righting moment.

For the FT method the second intercept of the GZ-curve will always occur on a saddle point of the E_p function. For the SD method there will be one singular GZ-curve for each saddle point and all remaining curves will have their second intercepts on local E_p -maxima.

The SD-paths will cover all points in the σ - τ plane, the FT-paths will not. This implicit restriction of the FT-method is non-physical since it is obvious that the vessel can be dislocated to any point in the σ - τ plane if disturbed by environmental forces. It is conceivable that e.g. critical down-flooding could occur at attitudes not considered by the FT-paths.

12. CONCLUSIONS

Traditional simplified methods to evaluate stability such as fixed-trim, or free-to trim methods are valid as long as they are used as a comparison tool. Comparisons are valid as long as the fundamental parameters of the reference and the compared hull are similar. Such methods have resulted in anomalies, inaccurate results, and erroneous conclusions when applied to compare dissimilar cases. Such conflicts, found frequently when calculating righting arms and the corresponding sequence of draft-trim-heel in offshore units and hulls of unusual proportions are well resolved with the application of the Steepest Descent Method. The fading of the stability curves may be misunderstood as a stability failure, and inadequate sequence of waterlines may fail to predict angles of downflooding.

Because the Steepest Descent Method is technically more rational, it is applicable to all kinds of hulls, and will provide accurate results that the conventional methods

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